Cálculo de integrales indefinidas o técnicas de integración

Hemos decidido llamar a este inciso "técnicas de integración", debido a que a continuación desarrollamos un conjunto de saberes y anificios (procedimientos o técnicas) que nos permitan obtener las primitivas deseadas en términos de funciones. Esto, al igual que en cualquier técnica, requiere destreza algebraica e ingenio. Algunas de las técnicas desarrolladas han nacido de la prueba y el error, y han mejorado con la práctica; además, cada estudiante le imprimirá su acervo personal o particular.

Propiedades de la integral indefinida.

1. $\int k\,f(x)dx = k\int f(x)dx \qquad (k = const.,\ \ k \neq 0)$

2. $\int \big(f(x) \pm g(x)\big)dx = \int f(x)dx \pm \int g(x)dx$

Formulas de integrales inmediatas.

1. $\int dx = x + c$

2. $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c \qquad (n \neq 1)$

3. $\int \dfrac{dx}{x} = ln|x| + c \qquad (x \neq 0)$

4. $\int e^x dx = e^x + c$

5. $\int a^x dx = \dfrac{a^x}{ln\,a} + c \qquad (a > 0 \ \ a \neq 1)$

6. $\int sen\,x\,dx = -cos\,x + c$

7. $\int cos\,x\,dx = sen\,x + c$

8. $\int \dfrac{dx}{cos^2 x} = tg\,x + c \qquad \left(x \neq \dfrac{\pi}{2} + \pi\,n,\ \ n \in \mathbb{Z}\right)$

9. $\int \dfrac{dx}{sen^2 x} = -ctg\,x + c \qquad \big(x \neq \pi\,n,\ \ n \in \mathbb{Z}\big)$

10. $\int \dfrac{dx}{\sqrt{1-x^2}} = arcsen\,x + c \qquad (|x| < 1)$

11. $\int \dfrac{dx}{\sqrt{a^2-x^2}} = arcsen\,\dfrac{x}{2} + c \qquad (|x| < |a|,\ \ a \neq 0)$

12. $\int \dfrac{dx}{1+x^2} = arctg\,x + c$

13. $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\,arctg\,\dfrac{x}{a} + c \qquad (a \neq 0)$

14. $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\,ln\left|\dfrac{x-a}{x+a}\right| + c \qquad (a \neq 0,\ \ |x| < |a|)$

15. $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = ln\big|x + \sqrt{x^2 \pm a^2}\,\big| + c \qquad (a \neq 0,\ \ |x| > |a|)$

Directas (integrales directas). El método para hallar las integrales directas, consiste en tratar de descomponer el integrando como la suma algebraica de varias funciones y luego aplicar las propiedades y formulas básicas de integración enunciadas. Este método es llamado "método de integración por descomposición". En ciertas funciones, descomponer la función en sumas parciales no es tarea fácil, pues depende de la experiencia, habilidad y práctica del que calcula.

La íntegración se considerará entonces a partir de este punto como un procedimiento esencialmente de ensayos. Por ello, para facilitar el trabajo, es conveniente que el alumno elabore

o adquiera una tabla de integrales ya conocidas (como las que se presentan). Esta tabla de integrales inmediatas permitirá realizar alguna integración únicamente de la comparación de la expresión diferencial que se desea obtener, con alguna en la tabla. Si se encuentra escrita,, entonces ya conocemos la integral. Si no es asi, probaremos algunas manipulaciones algebraicas para reducirla a una de las fórmulas de las tablas.

Calcular las siguientes integrales idefinidas inmediatas:

9.1. $\int (x^4 - 2x^3 - 6x^2 + 8x + 7)dx$

Solución:

$$\begin{aligned}
\int (x^4 - 2x^3 - 6x^2 + 8x + 7)dx &= \int x^4 dx - \int 2x^3 dx - \int 6x^2 dx + \int 8x dx + 7\int dx \\
&= \int x^4 dx - 2\int x^3 dx - 6\int x^2 dx + 8\int x dx + 7\int dx \\
&= \frac{x^{4+1}}{4+1} - 2\frac{x^{3+1}}{3+1} - 6\frac{x^{2+1}}{2+1} + 8\frac{x^{1+1}}{1+1} + 7x + c \\
&= \frac{x^5}{5} - 2\frac{x^4}{4} - 6\frac{x^3}{3} + 8\frac{x^2}{2} + 7x + c \\
&= \frac{1}{5}x^5 - \frac{2}{4}x^4 - \frac{6}{3}x^3 + \frac{8}{2}x^2 + 7x + c \\
&= \frac{1}{5}x^5 - \frac{1}{2}x^4 - 2x^3 + 4x^2 + 7x + c
\end{aligned}$$

9.2. $\int \frac{3x^4 - 2x^3 + 5x^2 - 7x + 8}{x^2} dx$

Solución:

$$\begin{aligned}
\int \frac{3x^4 - 2x^3 + 5x^2 - 7x + 8}{x^2} dx &= \int \left(\frac{3x^4}{x^2} - \frac{2x^3}{x^2} + \frac{5x^2}{x^2} - \frac{7x}{x^2} + \frac{8}{x^2}\right) dx \\
&= \int \left(\frac{3x^{\not 4}}{\not x^{\not 2}} - \frac{2x^{\not 3}}{\not x^{\not 2}} + \frac{5\not x^{\not 2}}{\not x^{\not 2}} - \frac{7\not x}{x^{\not 2}} + 8x^{-2}\right) dx \\
&= \int \left(3x^2 - 2x + 5 - \frac{7}{x} + 8x^{-2}\right) dx \\
&= \int 3x^2 dx - \int 2x dx + \int 5 dx - \int \frac{7}{x} dx + \int 8x^{-2} dx \\
&= 3\int x^2 dx - 2\int x dx + 5\int dx - 7\int \frac{dx}{x} + 8\int x^{-2} dx \\
&= 3\,\frac{x^{2+1}}{2+1} - 2\,\frac{x^{1+1}}{1+1} + 5x - 7\ln\, x + 8\,\frac{x^{-2+1}}{-2+1} + c \\
&= 3\,\frac{x^3}{3} - 2\,\frac{x^2}{2} + 5x - 7\,ln\, x + 8\,\frac{x^{-1}}{-1} + c \\
&= \frac{3}{3}x^3 - \frac{2}{2}x^2 + 5x - 7\,ln\, x - 8\,\frac{1}{x} + c \\
&= x^3 - x^2 + 5x - 7\,ln\, x - \frac{8}{x} + c
\end{aligned}$$

9.3. $\int \left(x^2 - 2x + \frac{1}{x^2}\right) dx$

Solución:

$$\begin{aligned}
\int \left(x^2 - 2x + \frac{1}{x^2}\right) dx &= \int x^2 dx - \int 2x dx + \int x^{-2} dx \\
&= \int x^2 dx - 2\int x dx + \int x^{-2} dx \\
&= \frac{x^{2+1}}{2+1} - 2\,\frac{x^{1+1}}{1+1} + \frac{x^{-2+1}}{-2+1} + c \\
&= \frac{x^3}{3} - 2\,\frac{x^2}{2} + \frac{x^{-1}}{-1} + c \\
&= \frac{1}{3}x^3 - \frac{2}{2}x^2 - \frac{1}{x} + c \\
&= \frac{1}{3}x^3 - x^2 - \frac{1}{x} + c
\end{aligned}$$

9.4. $\int (a+bx)^2 dx$

Solución:

$$\begin{aligned}
\int (a+bx)^2 dx &= \int (a^2 + 2abx + b^2x^2) dx \\
&= \int a^2 dx + \int 2abx dx + \int b^2x^2 dx \\
&= a^2 \int dx + 2ab \int x dx + b^2 \int x^2 dx \\
&= a^2 x + 2ab\,\frac{x^{1+1}}{1+1} + b^2\,\frac{x^{2+1}}{2+1} \\
&= a^2 x + 2ab\,\frac{x^2}{2} + b^2\,\frac{x^3}{3} \\
&= a^2 x + \frac{2ab}{2}x^2 + \frac{b^3}{3}x^3 + c \\
&= a^2 x + \frac{\cancel{2}ab}{\cancel{2}}x^2 + \frac{b^3}{3}x^3 + c \\
&= a^2 x + abx^2 + \frac{b^3}{3}x^3 + c
\end{aligned}$$

9.5. $\int (a-bx)^3 dx$

Solución:

$$\begin{aligned}
\int (a-bx)^3 dx &= \int (a^3 - 3a^2bx + 3ab^2x^2 - b^3x^3) dx \\
&= \int a^3 dx - \int 3a^2bx dx + \int 3ab^2x^2 dx - \int b^3x^3 dx \\
&= a^3 \int dx - 3a^2b \int x dx + 3ab^2 \int x^2 dx - b^3 \int x^3 dx \\
&= a^3 x - 3a^2b\,\frac{x^{1+1}}{1+1} + 3ab^2\,\frac{x^{2+1}}{2+1} - b^3\,\frac{x^{3+1}}{3+1} + c \\
&= a^3 x - 3a^2b\,\frac{x^2}{2} + 3ab^2\,\frac{x^3}{3} - b^3\,\frac{x^4}{4} + c \\
&= a^3 x - \frac{3a^2b}{2}x^2 + \frac{3ab^2}{3}x^3 - \frac{b^3}{4}x^4 + c
\end{aligned}$$

$$= a^3x - \frac{3a^2b}{2}x^2 + ab^2x^3 - \frac{b^3}{4}x^4 + c$$

9.6. $\displaystyle\int \frac{x-2}{x^3}dx$

Solución:

$$\int \frac{x-2}{x^3}dx = \int (x-2)x^{-3}dx = \int (x^{-2} - 2x^{-3})dx$$
$$= \int x^{-2}dx - \int 2x^{-3}dx = \int x^{-2}dx - 2\int x^{-3}dx$$
$$= \frac{x^{-2+1}}{-2+1} - 2\,\frac{x^{-3+1}}{-3+1} + c = \frac{x^{-1}}{-1} - 2\,\frac{x^{-2}}{-2} + c$$
$$= -\frac{1}{x} + \frac{2}{2}\cdot\frac{1}{x^2} + c = -\frac{1}{x} + \frac{1}{x^2} + c$$

9.7. $\displaystyle\int \frac{(x-1)(x^2+6)}{3x^2}dx$

Solución:

$$\int \frac{(x-1)(x^2+6)}{3x^2}dx = \frac{1}{3}\int \frac{(x-1)(x^2+6)}{x^2}dx$$
$$= \frac{1}{3}\int (x-1)(x^2+6)x^{-2}dx$$
$$= \frac{1}{3}\int (x+6x+x^2+6)x^{-2}dx$$
$$= \frac{1}{3}\int (x+6x^{-1}+1+6x^{-2})dx$$
$$= \frac{1}{3}\left(\int xdx + \int \frac{6}{x}dx + \int dx + \int 6x^{-2}dx\right)$$
$$= \frac{1}{3}\left(\int xdx + 6\int \frac{dx}{x} + \int dx + 6\int x^{-2}dx\right)$$
$$= \frac{1}{3}\left(\frac{x^{1+1}}{1+1} + 6\,ln\,x + x + 6\,\frac{x^{-2+1}}{-2+1}\right) + c$$
$$= \frac{1}{3}\left(\frac{x^2}{2} + 6\,ln\,x + x + 6\,\frac{x^{-1}}{-1}\right) + c$$
$$= \frac{1}{3}\left(\frac{x^2}{2} + 6\,ln\,x + x - \frac{6}{x}\right) + c$$
$$= \frac{1}{6}x^2 + 6\,ln\,x + \frac{1}{3}x - \frac{2}{x} + c$$

9.8. $\displaystyle\int \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}\right)dx$

Solución:

$$\int \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}\right)dx = \int \left(\frac{1}{x} + 2x^{-2} + 3x^{-3}\right)dx$$
$$= \int \frac{dx}{x} + \int 2x^{-2}dx + \int 3x^{-3}dx$$

$$= \int \frac{dx}{x} + 2\int x^{-2}dx + 3\int x^{-3}dx$$

$$= ln \mid x \mid +2\,\frac{x^{-2+1}}{-2+1} + 3\,\frac{x^{-3+1}}{-3+1} + c$$

$$= ln \mid x \mid +2\,\frac{x^{-1}}{-1} + 3\,\frac{x^{-2}}{-2} + c$$

$$= ln \mid x \mid -\frac{2}{x} - \frac{3}{2x^2} + c$$

9.9. $\int \left(\frac{1}{x^2} - \frac{3}{x^4} + \frac{5}{x^6} - \frac{7}{x^6}\right) dx$

Solución:

$$\int \left(\frac{1}{x^2} - \frac{3}{x^4} + \frac{5}{x^6} - \frac{7}{x^6}\right) dx = \int (x^{-2} - 3x^{-4}5x^{-6} - 7x^{-8})dx$$

$$= \int x^{-2}dx - \int 3x^{-4}dx + \int 5x^{-6}dx - \int 7x^{-8}dx$$

$$= \int x^{-2}dx - 3\int x^{-4}dx + 5\int x^{-6}dx - 7\int x^{-8}dx$$

$$= \frac{x^{-2+1}}{-2+1} - 3\,\frac{x^{-4+1}}{-4+1} + 5\,\frac{x^{-6+1}}{-6+1} - 7\,\frac{x^{-8+1}}{-8+1} + c$$

$$= \frac{x^{-1}}{-1} - 3\,\frac{x^{-3}}{-3} + 5\,\frac{x^{-5}}{-5} - 7\,\frac{x^{-7}}{-7} + c$$

$$= -\frac{1}{x} + \frac{3}{3x^3} - \frac{5}{5x^5} + \frac{7}{7x^7} + c$$

$$= -\frac{1}{x} + \frac{1}{x^3} - \frac{1}{x^5} + \frac{1}{x^7} + c$$

9.10. $\int \left(\frac{x^4}{5} - \frac{5}{x^4}\right) dx$

Solución:

$$\int \left(\frac{x^4}{5} - \frac{5}{x^4}\right) dx = \int \left(\frac{x^4}{5} - 5x^{-4}\right) dx$$

$$= \int \frac{x^4}{5}\,dx - \int 5x^{-4}\,dx$$

$$= \frac{1}{5}\int x^4dx - 5\int x^{-4}dx$$

$$= \frac{1}{5}\cdot\frac{x^{4+1}}{4+1} - 5\,\frac{x^{-4+1}}{-4+1} + c$$

$$= \frac{1}{5}\cdot\frac{x^5}{5} - 5\,\frac{x^{-3}}{-3} + c$$

$$= \frac{1}{25}x^5 + \frac{5}{3x^3} + c$$

9.11. $\int \left(1 + \frac{1}{x}\right)^3 dx$

Solución:

$$\int\left(1+\frac{1}{x}\right)^{3}dx=\int\left(1+\frac{3}{x}+\frac{3}{x^{2}}+\frac{1}{x^{3}}\right)dx$$
$$=\int\left(1+\frac{3}{x}+3x^{-2}+x^{-3}\right)dx$$
$$=\int dx+\int\frac{3dx}{x}+\int 3x^{-2}dx+\int x^{-3}dx$$
$$=\int dx+3\int\frac{dx}{x}+3\int x^{-2}dx+\int x^{-3}dx$$
$$=x+3\,ln\mid x\mid+3\frac{x^{-2+1}}{-2+1}+\frac{x^{-3+1}}{-3+1}+c$$
$$=x+3\,ln\mid x\mid+3\frac{x^{-1}}{-1}+\frac{x^{-2}}{-2}+c$$
$$=x+3\,ln\mid x\mid-\frac{3}{x}-\frac{1}{2x^{2}}+c$$

9.12. $\int\frac{9x^{5}+12x^{4}-6x^{3}+7x^{2}-4x+2}{x^{3}}dx$

Solución:

$$\int\frac{9x^{5}+12x^{4}-6x^{3}+7x^{2}-4x+2}{x^{3}}dx=\int(9x^{5}+12x^{4}-6x^{3}+7x^{2}-4x+2)x^{-3}dx$$
$$=\int(9x^{2}+12x-6+7x^{-1}-4x^{-2}+2x^{-3})dx$$
$$=\int\left(9x^{2}+12x-6+\frac{7}{x}-4x^{-2}+2x^{-3}\right)dx$$
$$=\int 9x^{2}dx+\int 12xdx-\int 6dx+\int\frac{7dx}{x}-\int 4x^{-2}dx+\int 2x^{-3}dx$$
$$=9\int x^{2}dx+12\int xdx-6\int dx+7\int\frac{dx}{x}-4\int x^{-2}dx+2\int x^{-3}dx$$
$$=9\,\frac{x^{2+1}}{2+1}+12\,\frac{x^{1+1}}{1+1}-6x+7\,ln\mid x\mid-4\,\frac{x^{-2+1}}{-2+1}+2\,\frac{x^{-3+1}}{-3+1}+c$$
$$=9\,\frac{x^{3}}{3}+12\,\frac{x^{2}}{2}-6x+7\,ln\mid x\mid-4\,\frac{x^{-1}}{-1}+2\,\frac{x^{-2}}{-2}+c$$
$$=\frac{9}{3}x^{3}+\frac{12}{2}x^{2}-6x+7\,ln\mid x\mid+\frac{4}{x}-\frac{2}{2x^{2}}+c$$
$$=3x^{3}+6x^{2}-6x+7\,ln\mid x\mid+\frac{4}{x}-\frac{1}{x^{2}}+c$$

9.13. $\int\frac{(x-1)(x^{3}-1)}{x^{2}}dx$

Solución:

$$\int\frac{(x-1)(x^{3}-1)}{x^{2}}dx=\int(x-1)(x^{3}-1)x^{-2}dx$$
$$=\int(x^{4}-x-x^{3}+1)x^{-2}dx$$
$$=\int(x^{2}-x^{-1}-x+x^{-2})dx$$

$$= \int \left(x^2 - \frac{1}{x} - x + x^{-2} \right) dx$$

$$= \int x^2 dx - \int \frac{dx}{x} - \int x dx + \int x^{-2} dx$$

$$= \frac{x^{2+1}}{2+1} - ln \mid x \mid - \frac{x^{1+1}}{1+1} + \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{x^3}{3} - ln \mid x \mid - \frac{x^2}{2} + \frac{x^{-1}}{-1} + c$$

$$= \frac{1}{3}x^3 - ln \mid x \mid - \frac{1}{2}x^2 - \frac{1}{x} + c$$

9.14. $\int \frac{(x^2+x+1)^2}{x^3} dx$

Solución:

$$\int \frac{(x^2+x+1)^2}{x^3} dx = \int (x^2+x+1)^2 x^{-3} dx$$

$$= \int (x^4 + 2x^3 + 3x^2 + 2x + 1) x^{-3} dx$$

$$= \int (x + 2 + 3x^{-1} + 2x^{-2} + x^{-3}) dx$$

$$= \int \left(x + 2 + \frac{3}{x} + 2x^{-2} + x^{-3} \right) dx$$

$$= \int x dx + \int 2 dx + \int \frac{3dx}{x} + \int 2x^{-2} dx + \int x^{-3} dx$$

$$= \int x dx + 2 \int dx + 3 \int \frac{dx}{x} + 2 \int x^{-2} dx + \int x^{-3} dx$$

$$= \frac{x^{1+1}}{1+1} + 2x + 3\, ln \mid x \mid + 2\, \frac{x^{-2+1}}{-2+1} + \frac{x^{-3+1}}{-3+1} + c$$

$$= \frac{x^2}{2} + 2x + 3\, ln \mid x \mid + 2\, \frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + c$$

$$= \frac{1}{2}x^2 + 2x + 3\, ln \mid x \mid - \frac{2}{x} - \frac{1}{2x^2} + c$$

9.15. $\int \frac{x^2 - 9}{x+3} dx$

Solución:

$$\int \frac{x^2-9}{x+3} dx = \int \frac{x^2 - 3^2}{x+3} dx = \int \frac{(x-3)(x+3)}{x+3} dx$$

$$= \int \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} dx = \int (x-3) dx$$

$$= \int x dx - \int 3 dx = \int x dx - 3 \int dx$$

$$= \frac{x^{1+1}}{1+1} - 3x + c = \frac{1}{2}x^2 - 3x + c$$

9.16. $\int \frac{dx}{4x^2 - 9}$

Solución:

$$\int \frac{dx}{4x^2-9} = \int \frac{dx}{4\left(x^2-\frac{9}{4}\right)} = \frac{1}{4}\int \frac{dx}{x^2-\frac{3^2}{2^2}} = \frac{1}{4}\int \frac{dx}{x^2-\left(\frac{3}{2}\right)^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{3}{2}} ln\left|\frac{x-\frac{3}{2}}{x+\frac{3}{2}}\right| + c$$

$$= \frac{1}{4}\cdot\frac{1}{\not{2}\cdot\frac{3}{\not{2}}} ln\left|\frac{\frac{2x-3}{2}}{\frac{2x+3}{2}}\right| + c = \frac{1}{4}\cdot\frac{1}{3} ln\left|\frac{(x-3)2}{2(x+3)}\right| + c = \frac{1}{12} ln\left|\frac{(x-3)\not{2}}{\not{2}(x+3)}\right| + c = \frac{1}{12} ln\left|\frac{x-3}{x+3}\right| + c$$

9.17. $\int \frac{dx}{5-x^2}$

Solución:

$$\int \frac{dx}{5-x^2} = \int \frac{dx}{-x^2+5} = \int \frac{dx}{-(x^2-5)} = -\int \frac{dx}{x^2-5} = -\int \frac{dx}{x^2-(\sqrt{5}\,)^2} = -\frac{1}{2\sqrt{5}} ln\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + c$$

9.18. $\int \frac{dx}{2x^2+8}$

Solución:

$$\int \frac{dx}{2x^2+8} = \int \frac{dx}{2(x^2+4)} = \frac{1}{2}\int \frac{dx}{x^2+4} = \frac{1}{2}\int \frac{dx}{x^2+2^2} = \frac{1}{2}\cdot\frac{1}{2} arctg\,\frac{x}{2} + c = \frac{1}{4} arctg\,\frac{x}{2} + c$$

9.19. $\int \frac{dx}{4x^2+1}$

Solución:

$$\int \frac{dx}{4x^2+1} = \int \frac{dx}{4\left(x^2+\frac{1}{4}\right)} = \frac{1}{4}\int \frac{dx}{x^2+\frac{1}{4}} = \frac{1}{4}\int \frac{dx}{x^2+\frac{1^2}{2^2}} = \frac{1}{4}\int \frac{dx}{x^2+\left(\frac{1}{2}\right)^2} = \frac{1}{4}\cdot\frac{1}{\frac{1}{2}} arctg\,\frac{x}{\frac{1}{2}} + c$$

$$= \frac{1}{4}\cdot\frac{2}{1} arctg\,\frac{2x}{1} + c = \frac{2}{4} arctg\,2x + c = \frac{1}{2} arctg\,2x + c$$

9.20. $\int \frac{x^3+9x-2}{x^2+9}dx$

Solución:

$$\int \frac{x^3+9x-2}{x^2+9}dx = \int \left(x+\frac{3}{x^2+9}\right)dx = \int xdx + \int \frac{3dx}{x^2+9} = \int xdx + 3\int \frac{dx}{x^2+3^2}$$

$$= \frac{x^{1+1}}{1+1} + 3\cdot\frac{1}{2\cdot 3} arctg\,\frac{x}{3} + c = \frac{x^2}{2} + \frac{3}{6} arctg\,\frac{x}{3} + c = \frac{1}{2}x^2 + \frac{1}{2} arctg\,\frac{x}{3} + c$$

9.21. $\int \frac{x^2+4}{x^2-1}dx$

Solución:

$$\int \frac{x^2+4}{x^2-1}dx = \int \left(1+\frac{5}{x^2-1}\right)dx = \int dx + \int \frac{5dx}{x^2-1} = \int dx + 5\int \frac{dx}{x^2-1} = \int dx + 5\int \frac{dx}{x^2-1^2}$$

$$= x + 5\cdot\frac{1}{2\cdot 1} ln\left|\frac{x-1}{x+1}\right| + c = x + \frac{5}{2} ln\left|\frac{x-1}{x+1}\right| + c$$

9.22. $\int \frac{x^2}{x^2+1}dx$

Solución:

$$\int \frac{x^2}{x^2+1}dx = \int \left(1-\frac{1}{x^2+1}\right)dx = \int dx - \int \frac{dx}{x^2+1} = \int dx - \int \frac{dx}{x^2+1^2} = x - \frac{1}{2\cdot 1} arctg\,\frac{x}{1} + c$$

$$= x - \frac{1}{2} arctg\, x + c$$

9.23. $\int \frac{x^2}{x^2-1}dx$

Solución:

$$\int \frac{x^2}{x^2-1}dx = \int \left(1+\frac{1}{x^2-1}\right)dx = \int dx + \frac{dx}{x^2-1} = \int dx + \frac{dx}{x^2-1^2} = x + \frac{1}{2\cdot 1} ln\left|\frac{x-1}{x+1}\right| + c$$

$$= x + \frac{1}{2} ln\left|\frac{x-1}{x+1}\right| + c$$

9.24. $\int \frac{x^4}{x^2+1}dx$

Solución:

$$\int \frac{x^4}{x^2+1}dx = \int \left(x^2-1+\frac{1}{x^2+1}\right)dx = \int x^2dx - \int dx + \int \frac{dx}{x^2+1} = \frac{x^{2+1}}{2+1} - x + \frac{1}{2\cdot 1} arctg\, \frac{x}{1} + c$$

$$= \frac{1}{3}x^3 - x + \frac{1}{2} arctg\, x + c$$

9.25. $\int \frac{x^2}{x^2-a^2}dx$

Solución:

$$\int \frac{x^2}{x^2-a^2}dx = \int \left(1+\frac{a^2}{x^2-a^2}\right)dx = \int dx + \int \frac{a^2dx}{x^2-a^2} = \int dx + a^2 \int \frac{dx}{x^2-a^2}$$

$$= x + a^2 \cdot \frac{1}{2\cdot a} ln\left|\frac{x-a}{x+a}\right| + c = x + \frac{a^2}{2a} ln\left|\frac{x-a}{x+a}\right| + c = x + \frac{a^{\not 2}}{2\not a} ln\left|\frac{x-a}{x+a}\right| + c$$

$$= x + \frac{a}{2} ln\left|\frac{x-a}{x+a}\right| + c$$

9.26. $\int \frac{2-x^4}{1+x^2}dx$

Solución:

$$\int \frac{2-x^4}{1+x^2}dx = \int \frac{-x^4+2}{x^2+1}dx = \int \frac{-(x^4-2)}{x^2+1}dx = -\int \frac{x^4-2}{x^2+1}dx = -\int \left(x^2-1-\frac{1}{x^2+1}\right)dx$$

$$= -\left(\int x^2dx - \int dx - \int \frac{dx}{x^2+1}\right) = -\left(\frac{x^{2+1}}{2+1} - x - \frac{1}{2\cdot 1} arctg\, \frac{x}{1}\right) + c$$

$$= -\left(\frac{x^3}{3} - x - \frac{1}{2} arctg\, x\right) + c = -\frac{1}{3}x^3 + x + \frac{1}{2} arctg\, x + c$$

9.27. $\int \frac{1+2x^2}{x^2(1+x^2)}dx$

Solución:

$$\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int \frac{1+x^2+x^2}{x^2(1+x^2)}dx = \int \left(\frac{1+x^2}{x^2(1+x^2)} + \frac{x^2}{x^2(1+x^2)}\right)dx$$

$$= \int \left(\frac{\cancel{1+x^2}}{x^2\cancel{(1+x^2)}} + \frac{\cancel{x^2}}{\cancel{x^2}(1+x^2)}\right)dx = \int \left(\frac{1}{x^2} + \frac{1}{1+x^2}\right)dx$$

$$= \int \left(x^{-2} + \frac{1}{1+x^2} \right) dx = \int x^{-2} dx + \int \frac{dx}{1+x^2}$$

$$= \frac{x^{-2+1}}{-2+1} + arctg\, x + c = \frac{x^{-1}}{-1} + arctg\, x + c$$

$$= -\frac{1}{x} + arctg\, x + c$$

9.28. $\int (3x - \sqrt{x}\,)^2 dx$

Solución:

$$\int (3x - \sqrt{x}\,)^2 dx = \int \left((3x)^2 - 2 \cdot 3x\sqrt{x} + (\sqrt{x}\,)^2 \right) dx = \int (9x^2 - 6x \cdot x^{\frac{1}{2}} + x) dx = \int (9x^2 - 6x^{\frac{3}{2}} + x) dx$$

$$= \int 9x^2 dx - \int 6x^{\frac{3}{2}} dx + \int x dx = 9 \int x^2 dx - 6 \int x^{\frac{3}{2}} dx + \int x dx = 9\, \frac{x^{2+1}}{2+1} - 6\, \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + x + c$$

$$= 9\, \frac{x^3}{3} - 6\, \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + x + c = \frac{9}{3} x^3 - \frac{12}{5} \sqrt{x^5} + x + c = 3x^3 - \frac{12}{5} x^2 \sqrt{x} + x + c$$

9.29. $\int (2\sqrt[3]{x} + \sqrt{3x} - 5) dx$

Solución:

$$\int (2\sqrt[3]{x} + \sqrt{3x} - 5) dx = \int (2\sqrt[3]{x} + \sqrt{3}\,\sqrt{x} - 5) dx = \int 2\sqrt[3]{x} dx + \int \sqrt{3}\,\sqrt{x} dx - \int 5 dx$$

$$= 2 \int \sqrt[3]{x} dx + \sqrt{3} \int \sqrt{x} dx - 5 \int dx = 2 \int x^{\frac{1}{3}} dx + \sqrt{3} \int x^{\frac{1}{2}} dx - 5 \int dx$$

$$= 2\, \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + \sqrt{3}\, \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 5x + c = 2\, \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \sqrt{3}\, \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$$

$$= \frac{6}{4} \sqrt[3]{x^4} + \frac{2\sqrt{3}}{3} \sqrt{x^3} - 5x + c = \frac{3}{2} x \sqrt[3]{x} + \frac{2\sqrt{3}}{3} x\sqrt{x} - 5x + c$$

9.30. $\int \left(\frac{3}{\sqrt{x}} + \frac{x\sqrt{x}}{3} \right) dx$

Solución:

$$\int \left(\frac{3}{\sqrt{x}} + \frac{x\sqrt{x}}{3} \right) dx = \int \left(\frac{3}{x^{\frac{1}{2}}} + \frac{x \cdot x^{\frac{1}{2}}}{3} \right) dx = \int \left(3x^{-\frac{1}{2}} + \frac{x^{\frac{3}{2}}}{3} \right) dx = \int 3x^{-\frac{1}{2}} dx + \int \frac{x^{\frac{3}{2}} dx}{3}$$

$$= 3 \int x^{-\frac{1}{2}} dx + \frac{1}{3} \int x^{\frac{3}{2}} dx = 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{1}{3} \cdot \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c = 3 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{1}{3} \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= \frac{6}{1} \sqrt{x^3} + \frac{2}{15} \sqrt{x^5} + c = 6x\sqrt{x} + \frac{2}{15} x^2 \sqrt{x} + c$$

9.31. $\int (\sqrt{x} + \sqrt[3]{x}\,) dx$

Solución:

$$\int (\sqrt{x} + \sqrt[3]{x}\,) dx = \int (x^{\frac{1}{2}} + x^{\frac{1}{3}}\,) dx = \int x^{\frac{1}{2}} dx + \int x^{\frac{1}{3}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$= \frac{2}{3} \sqrt{x^3} + \frac{3}{4} \sqrt[3]{x^4} + c = \frac{2}{3} x\sqrt{x} + \frac{3}{4} x \sqrt[3]{x} + c$$

9.32. $\int \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt[4]{x^3}} \right) dx$

Solución:

$$\int\left(\frac{1}{\sqrt{x}}-\frac{1}{\sqrt[4]{x^3}}\right)dx=\int\left(\frac{1}{x^{\frac{1}{2}}}-\frac{1}{x^{\frac{3}{4}}}\right)dx=\int(x^{-\frac{1}{2}}-x^{-\frac{3}{4}})dx=\int x^{-\frac{1}{2}}dx-\int x^{-\frac{3}{4}}dx$$

$$=\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}-\frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1}+c=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{x^{\frac{1}{4}}}{\frac{1}{4}}+c=\frac{2}{1}\sqrt{x}-\frac{4}{1}\sqrt[4]{x}+c=2\sqrt{x}-4\sqrt[4]{x}+x$$

9.33. $\displaystyle\int\frac{(\sqrt[3]{x}+1)^3}{x}dx$

Solución:

$$\int\frac{(\sqrt[3]{x}+1)^3}{x}dx=\int(\sqrt[3]{x}+1)^3x^{-1}dx=\int\left((\sqrt[3]{x})^3+3\cdot(\sqrt[3]{x})^2\cdot1+3\cdot\sqrt[3]{x}\cdot1^2+1^3\right)x^{-1}dx$$

$$=\int(x+3\sqrt[3]{x^2}+3\sqrt[3]{x}+1)x^{-1}dx=\int(x+3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+1)x^{-1}dx$$

$$=\int(1+3x^{-\frac{1}{3}}+3x^{-\frac{2}{3}}+x^{-1})dx=\int dx+\int 3x^{-\frac{1}{3}}dx+\int 3x^{-\frac{2}{3}}dx+\int x^{-1}dx$$

$$=\int dx+3\int x^{-\frac{1}{3}}dx+3\int x^{-\frac{2}{3}}dx+\int\frac{dx}{x}=x+3\,\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}+3\,\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+ln|x|+c$$

$$=x+3\,\frac{x^{\frac{2}{3}}}{\frac{2}{3}}+3\,\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+ln|x|+c=x+\frac{9}{2}\sqrt[3]{x^2}+\frac{9}{1}\sqrt[3]{x}+ln|x|+c$$

$$=x+\frac{9}{2}\sqrt[3]{x^2}+9\sqrt[3]{x}+ln|x|+c$$

9.34. $\displaystyle\int\frac{x+1}{\sqrt[3]{x^2}}dx$

Solución:

$$\int\frac{x+1}{\sqrt[3]{x^2}}dx=\int\frac{x+1}{x^{\frac{2}{3}}}dx=\int(x+1)x^{-\frac{2}{3}}dx=\int(x^{\frac{1}{3}}+x^{-\frac{2}{3}})dx=\int x^{\frac{1}{3}}dx+\int x^{-\frac{2}{3}}dx$$

$$=\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+c=\frac{x^{\frac{4}{3}}}{\frac{4}{3}}+\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+c=\frac{3}{4}\sqrt[3]{x^2}+\frac{3}{1}\sqrt[3]{x}+c=\frac{3}{4}\sqrt[3]{x^2}+3\sqrt[3]{x}+c$$

9.35. $\displaystyle\int\left(\sqrt{x^7}+\frac{1}{\sqrt[3]{x^5}}\right)dx$

Solución:

$$\int\left(\sqrt{x^7}+\frac{1}{\sqrt[3]{x^5}}\right)=\int\left(x^{\frac{7}{2}}+\frac{1}{x^{\frac{5}{3}}}\right)dx=\int(x^{\frac{7}{2}}+x^{-\frac{5}{3}})dx=\int x^{\frac{7}{2}}dx+\int x^{-\frac{5}{3}}dx=\frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1}+\frac{x^{-\frac{5}{3}+1}}{-\frac{5}{3}+1}+c$$

$$=\frac{x^{\frac{9}{2}}}{\frac{9}{2}}+\frac{x^{-\frac{2}{3}}}{-\frac{2}{3}}+c=\frac{2}{9}x^{\frac{9}{2}}-\frac{3}{2}\cdot\frac{1}{x^{\frac{2}{3}}}+c=\frac{2}{9}\sqrt{x^9}-\frac{3}{2}\cdot\frac{1}{\sqrt[3]{x^2}}+c=\frac{2}{9}x^4\sqrt{x}-\frac{3}{2\sqrt[3]{x^2}}+c$$

9.36. $\displaystyle\int\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^2dx$

Solución:

$$\int\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^2dx=\int\left((\sqrt{x})^2+2\cdot\sqrt{x}\cdot\frac{1}{\sqrt[3]{x}}+\left(\frac{1}{\sqrt[3]{x}}\right)^2\right)dx=\int\left(x+\frac{2\sqrt{x}}{\sqrt[3]{x}}+\frac{1}{\sqrt[3]{x^2}}\right)dx$$

$$=\int\left(x+\frac{2x^{\frac{1}{2}}}{x^{\frac{1}{3}}}+\frac{1}{x^{\frac{2}{3}}}\right)dx=\int(x+2x^{\frac{1}{2}}\cdot x^{-\frac{1}{3}}+x^{-\frac{2}{3}})dx=\int(x+2x^{\frac{1}{6}}+x^{-\frac{2}{3}})dx$$

$$= \int x dx + \int 2x^{\frac{1}{6}} dx + \int x^{-\frac{2}{3}} dx = \int x dx + 2\int x^{\frac{1}{6}} dx + \int x^{-\frac{2}{3}} dx$$

$$= \frac{x^{1+1}}{1+1} + 2\,\frac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1} + \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + c = \frac{x^2}{2} + 2\,\frac{x^{\frac{7}{6}}}{\frac{7}{6}} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$$

$$= \frac{1}{2}x^2 + \frac{12}{7}\sqrt[6]{x^7} + \frac{3}{1}\sqrt[3]{x} + c = \frac{1}{2}x^2 + \frac{12}{7}x\sqrt[6]{x} + 3\sqrt[3]{x} + c$$

9.37. $\displaystyle\int \left(2\sqrt{x} + \frac{3}{x}\right)^2 dx$

Solución:

$$\int \left(2\sqrt{x} + \frac{3}{x}\right)^2 dx = \int \left((2\sqrt{x})^2 + 2\cdot 2\sqrt{x}\cdot\frac{3}{x} + \left(\frac{3}{x}\right)^2\right) dx = \int \left(4x + \frac{12\sqrt{x}}{x} + \frac{9}{x^2}\right) dx$$

$$= \int (4x + 12x^{\frac{1}{2}}\cdot x^{-1} + 9x^{-2}) dx = \int (4x + 12x^{-\frac{1}{2}} + 9x^{-2}) dx$$

$$= \int 4x dx + \int 12x^{-\frac{1}{2}} dx + \int 9x^{-2} dx = 4\int x dx + 12\int x^{-\frac{1}{2}} dx + 9\int x^{-2} dx$$

$$= 4\,\frac{x^{1+1}}{1+1} + 12\,\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + 9\,\frac{x^{-2+1}}{-2+1} + c = 4\,\frac{x^2}{2} + 12\,\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 9\,\frac{x^{-1}}{-1} + c$$

$$= \frac{4}{2}x^2 + \frac{24}{1}x^{\frac{1}{2}} - 9\,\frac{1}{x} + c = 2x^2 + 24\sqrt{x} - \frac{9}{x} + c$$

9.38. $\displaystyle\int \frac{(1-x^n)^2}{\sqrt{x}} dx$

Solución:

$$\int \frac{(1-x^n)^2}{\sqrt{x}} dx = \int \frac{1^2 - 2\cdot 1\cdot x^n + (x^n)^2}{x^{\frac{1}{2}}} dx = \int (1 - 2x^n + x^{2n}) x^{-\frac{1}{2}} dx$$

$$= \int (x^{-\frac{1}{2}} - 2x^{\frac{2n-1}{2}} + x^{\frac{4n-1}{2}}) dx = \int x^{-\frac{1}{2}} dx - \int 2x^{\frac{2n-1}{2}} dx + \int x^{\frac{4n-1}{2}} dx$$

$$= \int x^{-\frac{1}{2}} dx - 2\int x^{\frac{2n-1}{2}} dx + \int x^{\frac{4n-1}{2}} dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 2\,\frac{x^{\frac{2n-1}{2}+1}}{\frac{2n-1}{2}+1} + \frac{x^{\frac{4n-1}{2}+1}}{\frac{4n-1}{2}+1} + c$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 2\,\frac{x^{\frac{2n+1}{2}}}{\frac{2n+1}{2}} + \frac{x^{\frac{4n+1}{2}}}{\frac{4n+1}{2}} + c = \frac{2}{1}x^{\frac{1}{2}} - \frac{4}{2n+1}x^{\frac{2n+1}{2}} + \frac{2}{4n+1}x^{\frac{4n+1}{2}} + c$$

$$= 2x^{\frac{1}{2}} - \frac{4}{2n+1}x^{\frac{2n}{2}+\frac{1}{2}} + \frac{2}{4n+1}x^{\frac{4n}{2}+\frac{1}{2}} + c = 2x^{\frac{1}{2}} - \frac{4}{2n+1}x^n\cdot x^{\frac{1}{2}} + \frac{2}{4n+1}x^{2n}\cdot x^{\frac{1}{2}} + c$$

$$= 2x^{\frac{1}{2}} - \frac{4x^n\cdot x^{\frac{1}{2}}}{2n+1} + \frac{2x^{2n}\cdot x^{\frac{1}{2}}}{4n+1} + c = x^{\frac{1}{2}}\left(2 - \frac{4x^n}{2n+1} + \frac{2x^{2n}}{4n+1}\right) + c$$

$$= \sqrt{x}\left(2 - \frac{4x^n}{2n+1} + \frac{2x^{2n}}{4n+1}\right) + c$$

9.39. $\displaystyle\int \sqrt{3ax}\, dx \qquad (a > 0)$

Solución:

$$\int \sqrt{3ax}\, dx = \int \sqrt{3a}\,\sqrt{x}\, dx = \sqrt{3a}\int \sqrt{x}\, dx = \sqrt{3a}\int x^{\frac{1}{2}} dx = \sqrt{3a}\,\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \sqrt{3a}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \sqrt{3a}\cdot\frac{2}{3}x^{\frac{3}{2}} + c = \frac{2}{3}\sqrt{3a}\,\sqrt{x^3} + c = \frac{2}{3}\sqrt{3a}\,x\sqrt{x} + c$$

9.40. $\displaystyle\int \frac{dx}{\sqrt{2-3x^2}}$

Solución:

$$\int \frac{dx}{\sqrt{2-3x^2}} = \int \frac{dx}{\sqrt{3\left(\frac{2}{3}-x^2\right)}} = \int \frac{dx}{\sqrt{3}\sqrt{\left(\left(\sqrt{\frac{2}{3}}\right)^2-x^2\right)}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{\left(\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2-x^2\right)}}$$

$$= \frac{1}{\sqrt{3}} arcsen\, \frac{x}{\frac{\sqrt{2}}{\sqrt{3}}} + c = \frac{1}{\sqrt{3}} arcsen\, \frac{\sqrt{3}\,x}{\sqrt{2}} + c$$

9.41. $\displaystyle\int \left(\frac{3}{1+x^2} - \frac{7}{\sqrt{1-x^2}}\right) dx$

Solución:

$$\int \left(\frac{3}{1+x^2} - \frac{7}{\sqrt{1-x^2}}\right) dx = \int \frac{3dx}{1+x^2} - \int \frac{7dx}{\sqrt{1-x^2}} = 3\int \frac{dx}{1+x^2} - 7\int \frac{dx}{\sqrt{1-x^2}}$$

$$= 3\, arctg\, x - 7\, arcsen\, x + c$$

9.42. $\displaystyle\int \left(\frac{5}{2+x^2} - \frac{7}{\sqrt{2-x^2}}\right) dx$

Solución:

$$\int \left(\frac{5}{2+x^2} - \frac{7}{\sqrt{2-x^2}}\right) dx = \int \left(\frac{5}{x^2+2} - \frac{7}{\sqrt{2-x^2}}\right) dx = \int \left(\frac{5}{x^2+(\sqrt{2}\,)^2} - \frac{7}{\sqrt{(\sqrt{2})^2-x^2}}\right) dx$$

$$= \int \frac{5dx}{x^2+(\sqrt{2}\,)^2} - \int \frac{7dx}{\sqrt{(\sqrt{2})^2-x^2}} = 5\int \frac{dx}{x^2+(\sqrt{2}\,)^2} - 7\int \frac{dx}{\sqrt{(\sqrt{2})^2-x^2}}$$

$$= 5\cdot\frac{1}{\sqrt{2}} arctg\, \frac{x}{\sqrt{2}} - 7\cdot\frac{1}{\sqrt{2}} arcsen\, \frac{x}{\sqrt{2}} + c = \frac{5}{\sqrt{2}} arctg\, \frac{x}{\sqrt{2}} - \frac{7}{\sqrt{2}} arcsen\, \frac{x}{\sqrt{2}} + c$$

9.43. $\displaystyle\int \left(\frac{2}{\sqrt{1-x^2}} + \frac{8}{1+x^2}\right) dx$

Solución:

$$\int \left(\frac{2}{\sqrt{1-x^2}} + \frac{8}{1+x^2}\right) dx = \int \frac{2dx}{\sqrt{1-x^2}} + \int \frac{8dx}{1+x^2} = 2\int \frac{dx}{\sqrt{1-x^2}} + 8\int \frac{dx}{1+x^2}$$

$$= 2\, arcsen\, x + 8\, arctg\, x + c$$

9.44. $\displaystyle\int \frac{3+\sqrt{1-x^2}}{\sqrt{1-x^2}} dx$

Solución:

$$\int \frac{3+\sqrt{1-x^2}}{\sqrt{1-x^2}} dx = \int \left(\frac{3}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right) dx = \int \left(\frac{3}{\sqrt{1-x^2}} + \frac{\cancel{\sqrt{1-x^2}}}{\cancel{\sqrt{1-x^2}}}\right) dx$$

$$= \int \left(\frac{3}{\sqrt{1-x^2}} + 1\right) dx = \int \frac{3dx}{\sqrt{1-x^2}} + \int dx$$

$$= 3\int \frac{dx}{\sqrt{1-x^2}} + \int dx = 3\, arcsen\, x + x + c$$

9.45. $\displaystyle\int \frac{5-\sqrt{1+x^2}}{\sqrt{1+x^2}} dx$

Solución:

$$\int \frac{5-\sqrt{1+x^2}}{\sqrt{1+x^2}}dx = \int \left(\frac{5}{\sqrt{1+x^2}} - \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)dx = \int \left(\frac{5}{\sqrt{1+x^2}} - \frac{\cancel{\sqrt{1+x^2}}}{\cancel{\sqrt{1+x^2}}}\right)dx$$

$$= \int \left(\frac{5}{\sqrt{1+x^2}} - 1\right)dx = \int \frac{5dx}{\sqrt{1+x^2}} - \int dx$$

$$= 5\int \frac{dx}{\sqrt{1+x^2}} - \int dx = 5\,ln|x+\sqrt{1+x^2}| + x + c$$

9.46. $\int \frac{\sqrt{3-x^2}+\sqrt{3+x^2}}{\sqrt{9-x^4}}dx$

Solución:

$$\int \frac{\sqrt{3-x^2}+\sqrt{3+x^2}}{\sqrt{9-x^4}}dx = \int \frac{\sqrt{3-x^2}+\sqrt{3+x^2}}{\sqrt{3^2-(x^2)^2}}dx = \int \frac{\sqrt{3-x^2}+\sqrt{3+x^2}}{\sqrt{(3-x^2)(3+x^2)}}dx$$

$$= \int \frac{\sqrt{3-x^2}+\sqrt{3+x^2}}{\sqrt{3-x^2}\,\sqrt{3+x^2}}dx = \int \left(\frac{\sqrt{3-x^2}}{\sqrt{3-x^2}\,\sqrt{3+x^2}} + \frac{\sqrt{3+x^2}}{\sqrt{3-x^2}\,\sqrt{3+x^2}}\right)dx$$

$$= \int \left(\frac{\cancel{\sqrt{3-x^2}}}{\cancel{\sqrt{3-x^2}}\,\sqrt{3+x^2}} + \frac{\cancel{\sqrt{3+x^2}}}{\sqrt{3-x^2}\,\cancel{\sqrt{3+x^2}}}\right)dx = \int \left(\frac{1}{\sqrt{3+x^2}} + \frac{1}{\sqrt{3-x^2}}\right)dx$$

$$= \int \frac{dx}{\sqrt{3+x^2}} + \int \frac{dx}{\sqrt{3-x^2}} = \int \frac{dx}{\sqrt{x^2+3}} + \int \frac{dx}{\sqrt{3-x^2}}$$

$$= \int \frac{dx}{\sqrt{x^2+(\sqrt{3}\,)^2}} + \int \frac{dx}{\sqrt{(\sqrt{3}\,)^2-x^2}} = ln\left|x+\sqrt{x^2+(\sqrt{3}\,)^2}\right| + arcsen\,\frac{x}{\sqrt{3}} + c$$

$$= ln\left|x+\sqrt{x^2+3}\right| + arcsen\,\frac{x}{\sqrt{3}} + c$$

9.47. $\int e^x\left(5+\frac{3e^{-x}}{x^4}\right)dx$

Solución:

$$\int e^x\left(5+\frac{3e^{-x}}{x^4}\right)dx = \int e^x(5+3e^{-x}x^{-4})dx = \int (5e^x+3e^x\cdot e^{-x}x^{-4})dx$$

$$= \int (5e^x+3e^{x-x}x^{-4})dx = \int (5e^x+3e^0\,x^{-4})dx$$

$$= \int (5e^x+3\cdot 1\cdot x^{-4})dx = \int (5e^x+3x^{-4})dx$$

$$= \int 5e^x dx + \int 3x^{-4}dx = 5\int e^x dx + 3\int x^{-4}dx$$

$$= 5e^2 + 3\,\frac{x^{-4+1}}{-4+1} + c = 5e^2 + 3\,\frac{x^{-3}}{-3} + c$$

$$= 5e^2 - \frac{3}{3}\cdot\frac{1}{x^3} + c = 5e^2 - \frac{1}{x^3} + c$$

9.48. $\int e^x\left(1-\frac{e^{-x}}{x^3}\right)dx$

Solución:

$$\int e^x\left(1-\frac{e^{-x}}{x^3}\right)dx = \int e^x(1-e^{-x}x^{-3})dx = \int (e^x-e^x\cdot e^{-x}x^{-3})dx$$

$$= \int (e^x - e^{x-x}x^{-3})dx = \int (e^x - e^0 x^{-3})dx$$
$$= \int (e^x - 1 \cdot x^{-3})dx = \int (e^x - x^{-3})dx$$
$$= \int e^x dx - \int x^{-3} dx = e^x - \frac{x^{-3+1}}{-3+1} + c$$
$$= e^x - \frac{x^{-2}}{-2} + c = e^x + \frac{1}{2x^2} + c$$

9.49. $\int \frac{e^{2x}-1}{e^x-1} dx$

Solución:

$$\int \frac{e^{2x}-1}{e^x-1} dx = \int \frac{(e^x)^2 - 1^2}{e^x-1} dx = \int \frac{(e^x-1)(e^x+1)}{e^x-1} dx$$
$$= \int \frac{\cancel{(e^x-1)}(e^x+1)}{\cancel{e^x-1}} dx = \int (e^x+1)dx$$
$$= \int e^x dx + \int dx = e^x + x + c$$

9.50. $\int 2^x \cdot 3^{2x} \cdot 5^{3x}\, dx$

Solución:

$$\int 2^x \cdot 3^{2x} \cdot 5^{3x}\, dx = \int (2 \cdot 3^2 \cdot 5^3)^x dx = \int (2 \cdot 9 \cdot 25)^x dx$$
$$= \int 2250^x dx = \frac{2250^x}{ln\, 2250} + c$$

9.51. $\int e^{3x} \cdot 3^x\, dx$

Solución:

$$\int e^{3x} \cdot 3^x\, dx = \int (e^3\, 3)^x\, dx = \frac{(e^3\, 3)^x}{ln\, e^3\, 3} + c$$
$$= \frac{e^{3x}\, 3^x}{ln\, e^3 + ln\, 3} + c = \frac{e^{3x}\, 3^x}{3ln\, e + ln\, 3} + c$$
$$= \frac{e^{3x}\, 3^x}{3 \cdot 1 + ln\, 3} + c = \frac{e^{3x}\, 3^x}{3 + ln\, 3} + c$$

9.52. $\int \frac{3^{2x}-5^x}{3^x} dx$

Solución:

$$\int \frac{3^{2x}-5^x}{3^x} dx = \int \left(\frac{3^{2x}}{3^x} - \frac{5^x}{3^x}\right) dx = \int \left(\left(\frac{3^2}{3}\right)^x - \left(\frac{5}{3}\right)^x\right) dx$$
$$= \int \left(\left(\frac{9}{3}\right)^x - \left(\frac{5}{3}\right)^x\right) dx = \int \left(3^x - \left(\frac{5}{3}\right)^x\right) dx$$
$$= \int 3^x dx - \int \left(\frac{5}{3}\right)^x dx = \frac{3^x}{ln\, 3} - \frac{\left(\frac{5}{3}\right)^x}{ln\, \frac{5}{3}} + c$$

9.53. $\int \frac{5^{x+2}+7^{x+1}}{35^x} dx$

Solución:

$$\int \frac{5^{x+2}+7^{x+1}}{35^x}dx = \int \frac{5^x\,5^2+7^x\,7}{(5\cdot 7)^x}dx = \int \frac{5^x\,5^2+7^x\,7}{5^x\,7^x}dx$$

$$= \int \left(\frac{5^x\,5^2}{5^x\,7^x}+\frac{7^x\,7}{5^x\,7^x}\right)dx = \int \left(\frac{\cancel{5^x}\,5^2}{\cancel{5^x}\,7^x}+\frac{\cancel{7^x}\,7}{5^x\,\cancel{7^x}}\right)dx$$

$$= \int \left(\frac{5^2}{7^x}+\frac{7}{5^x}\right)dx = \int (5^2\cdot 7^{-x}+7\cdot 5^{-x})dx$$

$$= \int \left(25(7^{-1})^x+7(5^{-1})^x\right)dx = \int 25(7^{-1})^x dx + \int 7(5^{-1})^x dx$$

$$= 25\int (7^{-1})^x dx + 7\int (5^{-1})^x dx = 25\,\frac{(7^{-1})^x}{ln\,7^{-1}} + 7\,\frac{(5^{-1})^x}{ln\,5^{-1}} + c$$

$$= 25\,\frac{7^{-x}}{-1\cdot ln\,7} + 7\,\frac{5^{-x}}{-1\cdot ln\,5} + c = -25\,\frac{\frac{1}{7^x}}{ln\,7} - 7\,\frac{\frac{1}{5^x}}{ln\,5} + c$$

$$= -25\,\frac{1}{7^x\,ln\,7} - 7\,\frac{1}{5^x\,ln\,5} + c = -\frac{25}{7^x\,ln\,7} - \frac{7}{5^x\,ln\,5} + c$$

9.54. $\int \frac{2^{x-1}+3^{x+1}}{15^x}dx$

Solución:

$$\int \frac{2^{x-1}+3^{x+1}}{15^x}dx = \int \frac{2^x\,2^{-1}+3^x\,3}{15^x}dx = \int \left(\frac{2^x\,2^{-1}}{15^x}+\frac{3^x\,3}{15^x}\right)dx$$

$$= \int \left(\frac{2^x}{15^x}\cdot\frac{1}{2}+\frac{3^x\,3}{(3\cdot 5)^x}\right)dx = \int \left(\frac{1}{2}\left(\frac{2}{15}\right)^x+\frac{3^x\,3}{3^x\,5^x}\right)dx$$

$$= \int \left(\frac{1}{2}\left(\frac{2}{15}\right)^x+\frac{\cancel{3^x}\,3}{\cancel{3^x}\,5^x}\right)dx = \int \left(\frac{1}{2}\left(\frac{2}{15}\right)^x+\frac{3}{5^x}\right)dx$$

$$= \int \left(\frac{1}{2}\left(\frac{2}{15}\right)^x+3\cdot 5^{-x}\right)dx = \int \left(\frac{1}{2}\left(\frac{2}{15}\right)^x+3(5^{-1})^x\right)dx$$

$$= \int \frac{1}{2}\left(\frac{2}{15}\right)^x dx + \int 3(5^{-1})^x dx = \frac{1}{2}\int \left(\frac{2}{15}\right)^x dx + 3\int (5^{-1})^x dx$$

$$= \frac{1}{2}\cdot\frac{\left(\frac{2}{15}\right)^x}{ln\,\frac{2}{15}} + 3\,\frac{(5^{-1})^x}{ln\,5^{-1}} + c = \frac{\left(\frac{2}{15}\right)^x}{2\,ln\,\frac{2}{15}} + \frac{3\cdot 5^{-x}}{-1\cdot ln\,5} + c$$

$$= \frac{\left(\frac{2}{15}\right)^x}{2\,ln\,\frac{2}{15}} - \frac{3}{5^x\,ln\,5} + c$$

9.55. $\int 2^x\left(1-\frac{2^{-x}}{\sqrt{x^5}}\right)dx$

Solución:

$$\int 2^x\left(1-\frac{2^{-x}}{\sqrt{x^5}}\right)dx = \int 2^x\left(1-\frac{2^{-x}}{x^{\frac{5}{2}}}\right)dx = \int 2^x(1-2^{-x}\cdot x^{-\frac{5}{2}})dx$$

$$= \int (2^x - 2^x \cdot 2^{-x} \cdot x^{-\frac{5}{2}})dx = \int (2^x - 2^{x-x} \cdot x^{-\frac{5}{2}})dx$$

$$= \int (2^x - 2^0 \cdot x^{-\frac{5}{2}})dx = \int (2^x - 1 \cdot x^{-\frac{5}{2}})dx$$

$$= \int (2^x - x^{-\frac{5}{2}})dx = \int 2^x dx - \int x^{-\frac{5}{2}} dx$$

$$= \frac{2^x}{ln\, 2} - \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + c = \frac{2^x}{ln\, 2} - \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + c$$

$$= \frac{2^x}{ln\, 2} + \frac{2}{3} \cdot \frac{1}{x^{\frac{3}{2}}} + c = \frac{2^x}{ln\, 2} + \frac{2}{3} \cdot \frac{1}{\sqrt{x^3}} + c$$

$$= \frac{2^x}{ln\, 2} + \frac{2}{3} \cdot \frac{1}{x\sqrt{x}} + c = \frac{2^x}{ln\, 2} + \frac{2}{3x\sqrt{x}} + c$$

9.56. $\int a^x \left(1 + \frac{a^{-x}}{\sqrt[4]{x^3}}\right) dx$

Solución:

$$\int a^x \left(1 + \frac{a^{-x}}{\sqrt[4]{x^3}}\right) dx = \int a^x \left(1 + \frac{a^{-x}}{x^{\frac{3}{4}}}\right) dx = \int a^x (1 + a^{-x} \cdot x^{-\frac{3}{4}})dx$$

$$= \int (a^x + a^x \cdot a^{-x} \cdot x^{-\frac{3}{4}})dx = \int (a^x + a^{x-x} \cdot x^{-\frac{3}{4}})dx$$

$$= \int (a^x + a^0 \cdot x^{-\frac{3}{4}})dx = \int (a^x + 1 \cdot x^{-\frac{3}{4}})dx$$

$$= \int (a^x + x^{-\frac{3}{4}})dx = \int a^x dx + \int x^{-\frac{3}{4}} dx$$

$$= \frac{a^x}{ln\, a} + \frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + c = \frac{a^x}{ln\, a} + \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + c$$

$$= \frac{a^x}{ln\, a} + \frac{4}{1} \cdot x^{\frac{1}{4}} + c = \frac{a^x}{ln\, a} + 4\sqrt[4]{x} + c$$

9.57. $\int cos^2 \frac{x}{2} dx$

Solución:

$$\int cos^2 \frac{x}{2} dx = \int \frac{1 + cos\, x}{2} dx = \int \left(\frac{1}{2} + \frac{cos\, x}{2}\right) dx$$

$$= \int \frac{dx}{2} + \int \frac{cos\, x\, dx}{2} = \frac{1}{2} \int dx + \frac{1}{2} \int cos\, x\, dx$$

$$= \frac{1}{2} x + \frac{1}{2} sen\, x + c$$

9.58. $\int tg^2 x\, dx$

Solución:

$$\int tg^2 x\, dx = \int \left(\frac{1}{cos^2 x} - 1\right) dx = \int \frac{dx}{cos^2 x} - \int dx = tg\, x - x + c$$

9.59. $\int ctg^2 x\, dx$

Solución:

$$\int ctg^2 x\, dx = \int \left(\frac{1}{sen^2 x} - 1\right) dx = \int \frac{dx}{sen^2 x} - \int dx = -ctg\, x - x + x$$

9.60. $\int (3\,sen\,x - 4\cos x)dx$

Solución:

$$\int (3\,sen\,x - 4\cos x)dx = \int 3\,sen\,x\,dx - \int 4\cos x\,dx = 3\int sen\,x\,dx - 4\int \cos x\,dx$$
$$= 3(-\cos x) - 4\,sen\,x + c = -3\cos x - 4\,sen\,x + c$$

9.61. $\int \left(3\,sen\,x + \frac{1}{cos^2 x}\right)dx$

Solución:

$$\int \left(3\,sen\,x + \frac{1}{cos^2 x}\right)dx = \int 3\,sen\,x\,dx + \int \frac{dx}{cos^2 x} = 3\int sen\,x\,dx + \int \frac{dx}{cos^2 x}$$
$$= 3(-sen\,x) + tg\,x + c = -3\,sen\,x + tg\,x + c$$

9.62. $\int \frac{8 - 7\,cos^3 x}{cos^2 x}\,dx$

Solución:

$$\int \frac{8 - 7\,cos^3 x}{cos^2 x}\,dx = \int \left(\frac{8}{cos^2 x} - \frac{7\,cos^3 x}{cos^2 x}\right)dx = \int \left(\frac{8}{cos^2 x} - \frac{7\,cos^{\not 3} x}{\cancel{cos^2 x}}\right)dx$$
$$= \int \left(\frac{8}{cos^2 x} - 7\cos x\right)dx = \int \frac{8dx}{cos^2 x} - \int 7\cos x\,dx$$
$$= 8\int \frac{dx}{cos^2 x} - 7\int \cos x\,dx = 8\,tg\,x - 7\,sen\,x + c$$

9.63. $\int \left(\frac{1}{cos^2 x} + 3\cos x - 1\right)dx$

Solución:

$$\int \left(\frac{1}{cos^2 x} + 3\cos x - 1\right)dx = \int \frac{dx}{cos^2 x} + \int 3\cos x\,dx - \int dx = \int \frac{dx}{cos^2 x} + 3\int \cos x\,dx - \int dx$$
$$= tg\,x + 3\,sen\,x - x + c$$

9.64. $\int \frac{\cos 2x}{4\,cos^2 x\,sen^2 x}\,dx$

Solución:

$$\int \frac{\cos 2x}{4\,cos^2 x\,sen^2 x}\,dx = \int \frac{cos^2 x - sen^2 x}{4\,cos^2 x\,sen^2 x}\,dx = \int \left(\frac{cos^2 x}{4\,cos^2 x\,sen^2 x} - \frac{sen^2 x}{4\,cos^2 x\,sen^2 x}\right)dx$$
$$= \int \left(\frac{\cancel{cos^2 x}}{4\cancel{cos^2 x}\,sen^2 x} - \frac{\cancel{sen^2 x}}{4\,cos^2 x\,\cancel{sen^2 x}}\right)dx = \int \left(\frac{1}{4\,sen^2 x} - \frac{1}{4\,cos^2 x}\right)dx$$
$$= \int \frac{dx}{4\,sen^2 x} - \int \frac{dx}{4\,cos^2 x} = \frac{1}{4}\int \frac{dx}{sen^2 x} - \frac{1}{4}\int \frac{dx}{cos^2 x}$$
$$= \frac{1}{4}(-ctg\,x) - \frac{1}{4}\,tg\,x + c = -\frac{1}{4}\,ctg\,x - \frac{1}{4}\,tg\,x + c$$

9.65. $\int \frac{2\cos 2x}{sen^2\,2x}\,dx$

Solución:

$$\int \frac{2\cos 2x}{sen^2\,2x}\,dx = 2\int \frac{\cos 2x}{(sen\,2x)^2}\,dx = 2\int \frac{cos^2 x - sen^2 x}{(2\,sen\,x\cos x)^2}\,dx$$
$$= 2\int \frac{cos^2 x - sen^2 x}{4\,sen^2 x\,cos^2 x}\,dx = \frac{2}{4}\int \frac{cos^2 x - sen^2 x}{sen^2 x\,cos^2 x}\,dx$$
$$= \frac{1}{2}\int \left(\frac{cos^2 x}{sen^2 x\,cos^2 x} - \frac{sen^2 x}{sen^2 x\,cos^2 x}\right)dx = \frac{1}{2}\int \left(\frac{\cancel{cos^2 x}}{sen^2 x\,\cancel{cos^2 x}} - \frac{\cancel{sen^2 x}}{\cancel{sen^2 x}\,cos^2 x}\right)dx$$

$$= \frac{1}{2}\int\left(\frac{1}{sen^2x} - \frac{1}{cos^2x}\right)dx = \frac{1}{2}\left(\int\frac{dx}{sen^2x} - \int\frac{dx}{cos^2x}\right)$$
$$= \frac{1}{2}\left(-ctg\,x - tg\,x\right) + c = -\frac{1}{2}\,ctg\,x - \frac{1}{2}\,tg\,x + c$$

9.66. $\int(x^2 + 2\,sen\,x - 3e^x\,)dx$

Solución:

$$\int(x^2 + 2\,sen\,x - 3e^x\,)dx = \int x^2dx + \int 2\,sen\,x\,dx - \int 3e^x\,dx$$
$$= \int x^2dx + 2\int sen\,x\,dx - 3\int e^x\,dx$$
$$= \frac{x^{2+1}}{2+1} + 2(-cos\,x) - 3e^x + c$$
$$= \frac{1}{3}\,x^3 - 2\,cos\,x - 3e^x + c$$

9.67. $\int\frac{3tg^2x + 4}{sen^2x}\,dx$

Solución:

$$\int\frac{3tg^2x+4}{sen^2x}\,dx = \int\left(\frac{3tg^2x}{sen^2x} + \frac{4}{sen^2x}\right)dx = \int\left(\frac{3\,\frac{sen^2x}{cos^2x}}{sen^2x} + \frac{4}{sen^2x}\right)dx$$
$$= \int\left(\frac{\frac{3\,sen^2x}{cos^2x}}{sen^2x} + \frac{4}{sen^2x}\right)dx = \int\left(\frac{3sen^2x}{sen^2x\,cos^2x} + \frac{4}{sen^2x}\right)dx$$
$$= \int\left(\frac{3\,\cancel{sen^2x}}{\cancel{sen^2x}\,cos^2x} + \frac{4}{sen^2x}\right)dx = \int\left(\frac{3}{cos^2x} + \frac{4}{sen^2x}\right)dx$$
$$= \int\frac{3dx}{cos^2x} + \int\frac{4dx}{sen^2x} = 3\int\frac{dx}{cos^2x} + 4\int\frac{dx}{sen^2x}$$
$$= 3\,tg\,x + 4(-ctg\,x) + c = 3\,tg\,x - 4\,ctg\,x + c$$

9.68. $\int\left(\frac{2}{sen^2x} + 3\,sen\,x - 5\right)dx$

Solución:

$$\int\left(\frac{2}{sen^2x} + 3\,sen\,x - 5\right)dx = \int\frac{2dx}{sen^2x} + \int 3\,sen\,x\,dx - \int 5dx$$
$$= 2\int\frac{dx}{sen^2x} + 3\int sen\,x\,dx - 5\int dx$$
$$= 2(-ctg\,x) + 3(-cos\,x) - 5x + c$$
$$= -\,2ctg\,x - 3\,cos\,x - 5x + c$$

9.69. $\int\frac{1 + cos^2x}{1 + cos\,2x}\,dx$

Solución:

$$\int\frac{1+cos^2x}{1+cos\,2x}\,dx = \int\frac{1+cos^2x}{2\,cos^2x}\,dx = \frac{1}{2}\int\frac{1+cos^2x}{cos^2x}\,dx$$
$$= \frac{1}{2}\int\left(\frac{1}{cos^2x} + \frac{cos^2x}{cos^2x}\right)dx = \frac{1}{2}\int\left(\frac{1}{cos^2x} + 1\right)dx$$
$$= \frac{1}{2}\left(\int\frac{dx}{cos^2x} + \int dx\right) = \frac{1}{2}(tg\,x + x) + c$$

$$= \frac{1}{2}\, tg\, x + \frac{1}{2}\, x + c$$

9.70. $\displaystyle\int \frac{dx}{1+cos\, 2x}$

Solución:

$$\int \frac{dx}{1+cos\, 2x} = \int \frac{dx}{2\, cos^2 x} = \frac{1}{2}\int \frac{dx}{cos^2 x} = \frac{1}{2}\, tg\, x + c$$