CAPITULO II

INTEGRACION POR PARTES

En esta sección veremos un nuevo método para transformar una integral en otra que sea más fácil de evaluar, se conoce como ***integración por partes.*** Este método se aplica generalmente a integrandos que contienen un producto de funciones algebraicas o trascendentes, por ejemplo:

$$x^2 \ln x, \quad x\,\mathrm{sen}x, \quad e^x \mathrm{arctg}x, \quad x^3\sqrt{x^2-5}$$

La formula de integración por partes es una consecuencia de la regla del producto para la diferenciación:

Sean *u* y *v* funciones derivables de *x*. En estas condiciones,

$$d(uv) = u\,dv + v\,du$$
$$u\,dv = d(uv) - v\,du$$
$$\int u\,dv = \int d(uv) - \int v\,du$$
$$\int u\,dv = uv - \int v\,du \qquad \text{(I)}$$

Esta forma expresa el integrando original en términos de otro integrando que puede ser más fácil de manejar, dependiendo generalmente de nuestra elección de *u* y *dv*, para aplicar (I) en la práctica, se separa el integrando en dos partes; una de ellas se iguala *u* y la otra, junto con *dx*, a *dv*, (Por esta razón, este método se denomina ***integración por partes***). Es conveniente tener en cuenta los tres criterios siguientes:

(a) La parte que se iguala a *dv* debe ser aquella que se pueda integrar fácilmente;
(b) Elegir *u* como una función que se simplifique por derivación pero que permita que *dv* se puede integrar;
(d) Tener cuidado en un solo ejercicio, a veces, se tiene que integrar por partes más de una vez, o pueda resultar una ***integral circular (o reiterada).***

FORMULAS DE REDUCCION

Las ***fórmulas de reducción*** permiten simplificar el cálculo cuando se haya de aplicar la integración por partes varias veces consecutivas. En general, una formula de reducción es aquella que da lugar a una nueva integral de la misma forma que la original, pero con un exponente menor. Una fórmula de reducción es útil si, finalmente, conduce a una integral que se pueda calcular fácilmente, algunas de las fórmulas más corrientes de reducción son:

(A) $\int \frac{dx}{(a^2 \pm x^2)^m} = \frac{1}{a^2}\left[\frac{x}{(2m-2)(a^2 \pm x^2)^{m-1}} + \frac{2m-3}{2m-2}\int \frac{dx}{(a^2 \pm x^2)^{m-1}}\right]$, $m \neq 1$

(B) $\int (a^2 \pm x^2)^m du = \frac{x(a^2 \pm x^2)^m}{2m+1} + \frac{2ma^2}{2m+1}\int (a^2 \pm x^2)^{m-1} dx$, $m \neq -\frac{1}{2}$

(c) $\int \frac{du}{(x^2 - a^2)^m} = -\frac{1}{a^2}\left[\frac{x}{(2m-2)(x^2 - a^2)^{m-1}} + \frac{2m-3}{2m-1}\int \frac{dx}{(x^2 - a^2)^{m-1}}\right]$, $m \neq 1$

(D) $\int (x^2 - a^2)^m dx = \frac{x(x^2 - a^2)^m}{2m+1} - \frac{2ma^2}{2m+1}\int (x^2 - a^2)^{m-1} dx$, $m \neq -1/2$

(E) $\int x^m e^{ax} dx = \frac{1}{a} x^m e^{ax} - \frac{m}{a}\int x^{m-1} e^{ax} dx$

(F) $\int \operatorname{sen}^m x\, dx = -\frac{\operatorname{sen}^{m-1} x \cos x}{m} + \frac{m-1}{m}\int \operatorname{sen}^{m-2} x\, dx$

(G) $\int \cos^m x\, dx = \frac{\cos^{m-1} x \operatorname{sen} x}{m} + \frac{m-1}{m}\int \cos^{m-2} x\, dx$

(H) $\int \operatorname{sen}^m x \cos^n x\, dx = \frac{\operatorname{sen}^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \operatorname{sen}^m x \cos^{n-2} x\, dx$

$\int \operatorname{sen}^m x \cos^n x\, dx = -\frac{\operatorname{sen}^{m-1} x \cos^{m+1} x}{m+n} + \frac{m-1}{m+n}\int \operatorname{sen}^{m-2} x \cos^n x\, dx$, $m \neq -n$

(I) $\int x^m \operatorname{sen} bx\, dx = -\frac{x^m}{b}\cos bx + \frac{m}{b}\int x^{m-1} \cos bx\, dx$

(J) $\int x^m \cos bx\, dx = \frac{x^m}{b} \operatorname{sen} bx - \frac{m}{b}\int x^{m-1} \operatorname{sen} bx\, dx$

PROBLEMAS RESUELTOS

13.

$\int x \cos x\, dx$

Sea: $u = x$ $\quad dv = \cos x\, dx$

$\int dv = \int \cos x\, dx$

$du = dx$ $\quad v = \operatorname{sen} x$

$$\int u\, dv = uv - \int v\, du$$

$$\int x \cos x\, dx = x \cdot \operatorname{sen} x - \int \operatorname{sen} x\, dx = x \operatorname{sen} x - (-\cos x) + c = x \operatorname{sen} x + \cos x + c$$

14.

$\int x \sec^2 3x\, dx$

Sea: $u = x$ $\quad dv = \sec^2 3x\, dx$

$\int dv = \int \sec^2 3x\, dx$

$du = dx$ $\quad v = \frac{1}{3} \operatorname{tg} 3x$

$$\int u\, dv = uv - \int v\, du$$

$$\int x \sec^2 3x\, dx = x \cdot \frac{1}{3} \operatorname{tg} 3x - \int \frac{1}{3} \operatorname{tg} 3x\, dx = \frac{x}{3} \operatorname{tg} 3x - \frac{1}{3}\int \operatorname{tg} 3x\, dx$$

$$= \frac{x}{3} \operatorname{tg} 3x - \frac{1}{3} \cdot \frac{1}{3} \ln|\sec 3x| + c = \frac{x}{3} \operatorname{tg} 3x - \frac{1}{9} \ln|\sec 3x| + c$$

15.

$\int \arccos 2x\, dx$

Sea: $u = \arccos 2x$ $\quad dv = dx$

$du = -\frac{1}{\sqrt{1-(2x)^2}} \cdot 2 \cdot dx$ $\quad \int dv = \int dx$

$du = \frac{-2\, dx}{\sqrt{1-4x^2}}$ $\quad v = x$

$$\int u\, dv = uv - \int v\, du$$

$$\int \arccos 2x\,dx = \arccos 2x \cdot x - \int x \cdot \frac{-2\,dx}{\sqrt{1-4x^2}} = x\arccos 2x - \int \frac{-2x\,dx}{\sqrt{1-4x^2}} = x\arccos 2x - \frac{1}{4}\int \frac{-8x\,dx}{\sqrt{1-4x^2}}$$

$$= x\arccos 2x - \frac{1}{4}\cdot 2\sqrt{1-4x^2} + c = x\arccos 2x - \frac{1}{2}\sqrt{1-4x^2}$$

16.

$\int \text{arctag}x\,dx$

Sea:

$$u = \text{arctag}x \qquad dv = dx$$

$$du = \frac{1}{1+x^2}dx \qquad \int dv = \int dx$$

$$du = \frac{dx}{1+x^2} \qquad v = x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \text{arctag}x\,dx = \text{arctg}x \cdot x - \int x \cdot \frac{dx}{1+x^2} = x\,\text{arcag}x - \int \frac{x\,dx}{1+x^2} = x\,\text{arctag}x - \frac{1}{2}\int \frac{2x\,dx}{1+x^2}$$

$$= x\,\text{arctag}x - \frac{1}{2}\ln|1+x^2| + c = x\,\text{arctag}x - \ln(1+x^2)^{1/2} + c$$

$$= x\,\text{arctag}x - \ln\sqrt{1+x^2} + c$$

17.

$\int x^2\sqrt{1-x}\,dx$

$\int x^2\sqrt{1-x}\,dx = \int x^2(1-x)^{1/2}\,dx$

Sea:

$$u = x^2 \qquad dv = (1-x)^{1/2}dx$$

$$\int dv = \int (1-x)^{1/2}dx$$

$$du = 2x\,dx \qquad v = -\frac{2}{3}(1-x)^{\frac{3}{2}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2\sqrt{1-x}\,dx = x^2[-\frac{2}{3}(1-x)^{3/2}] - \int -\frac{2}{3}(1-x)^{3/2}\cdot 2x\,dx$$

$$= -\frac{2}{3}x^2(1-x)^{3/2} + \frac{4}{3}\int x(1-x)^{3/2}dx \;.........\; (I)$$

resolveremos la integral $\int x(1-x)^{3/2}dx$ por partes:

sea:

$$u = x \qquad dv = (1-x)^{3/2}dx$$

$$\int dv = \int (1-x)^{3/2}dx$$

$$du = dx \qquad v = -\frac{2}{5}(1-x)^{5/2}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x(1-x)^{3/2}dx = x[-\frac{2}{5}(1-x)^{5/2}] - \int -\frac{2}{5}(1-x)^{5/2}\cdot dx$$

$$= -\frac{2}{5}x(1-x)^{5/2} + \frac{2}{5}\int (1-x)^{5/2}dx$$

$$= -\frac{2}{5}x(1-x)^{5/2} + \frac{2}{5}[-\frac{2}{7}(1-x)^{7/2}]$$

$$= -\frac{2}{5}x(1-x)^{5/2} - \frac{4}{35}(1-x)^{7/2} \;........\; (II)$$

reemplazando (II) en (I)

$$\int x^2\sqrt{1-x}\,dx = -\frac{2}{3}x^2(1-x)^{3/2} + \frac{4}{3}[-\frac{2}{5}x(1-x)^{5/2} - \frac{4}{35}(1-x)^{7/2}]$$

$$= -\frac{2}{3}x^2(1-x)^{3/2} - \frac{8}{15}x(1-x)^{5/2} - \frac{16}{105}(1-x)^{7/2}$$

$$= -2(1-x)^{3/2}[\frac{1}{3}x^2 + \frac{4}{15}x(1-x) + \frac{8}{105}(1-x)^2] + c$$

$$= -2(1-x)^{3/2}[\frac{35x^2 + 28x(1-x) + 8(1-2x+x^2)}{105}] + c$$

$$\int x^2\sqrt{1-x}\,dx = -\frac{2}{105}(1-x)^{3/2}(35x^2+28x-28x^2+8-16x+8x^2)+c$$
$$= -\frac{2}{105}(1-x)^{3/2}(15x^2+12x+8)+c$$

18.

$$\int \frac{xe^x dx}{(1+x)^2}$$

$$\int \frac{xe^x dx}{(1+x)^2} = \int xe^x(1+x)^{-2}dx$$

Sea:

$$u = xe^x \qquad dv = (1+x)^{-2}dx$$
$$du = (e^x + xe^x)dx \qquad \int dv = \int (1+x)^{-2}dx$$
$$du = (1+x)e^x dx \qquad v = -(1+x)^{-1}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{xe^x dx}{(1+x)^2} = xe^x[-(1+x)^{-1}] - \int -(1+x)^{-1}(1+x)e^x dx = -\frac{xe^x}{1+x} + \int \frac{e^x(1+x)dx}{1+x} = -\frac{xe^x}{1+x} + \int e^x dx$$
$$= -\frac{xe^x}{1+x} + e^x + c = \frac{-xe^x + e^x(1+x)}{1+x} + c = \frac{-xe^x + e^x + xe^x}{1+x} + c = \frac{e^x}{1+x} + c$$

19.

$$\int x\,\mathrm{arctg}x\,dx$$

$$\int x\,\mathrm{arctg}x\,dx = \int \mathrm{arctag}x \cdot x\,dx$$

sea:

$$u = \mathrm{arctag}x \qquad dv = x\,dx$$
$$du = \frac{1}{1+x^2}dx \qquad \int dv = \int x\,dx$$
$$du = \frac{dx}{1+x^2} \qquad v = \frac{1}{2}x^2$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\,\mathrm{arctg}x\,dx = \mathrm{arctag}x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{dx}{1+x^2} = \frac{1}{2}x^2\,\mathrm{arctag}x - \frac{1}{2}\int \frac{x^2 dx}{1+x^2}$$
$$= \frac{1}{2}x^2\,\mathrm{arctag}x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right)dx = \frac{1}{2}x^2\,\mathrm{arctag}x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1+x^2}$$
$$= \frac{1}{2}x^2\,\mathrm{arctg}x - \frac{1}{2}x + \frac{1}{2}\mathrm{arctag}x + c = \frac{1}{2}(x^2+1)\mathrm{arctg}x - \frac{1}{2}x + c$$

20.

$$\int x^2 e^{-3x}dx$$

Integrando por partes:

sea:

$$u = x^2 \qquad dv = e^{-3x}dx$$
$$\int dv = \int e^{-3x}dx$$
$$du = 2x\,dx \qquad v = -\frac{1}{3}e^{-3x}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 e^{-3x}dx = x^2\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{1}{3}e^{-3x}\cdot 2x\,dx$$
$$= -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}\int xe^{-3x}dx \;........\;(I)$$

Integrando nuevamente por partes la integral $\int xe^{-3x}dx$

sea:

$$u = x \qquad dv = e^{-3x}dx$$
$$\int dv = \int e^{-3x}dx$$
$$du = dx \qquad v = -\frac{1}{3}e^{-3x}$$

$$\int v\,du = uv - \int v\,du$$

$$\int x e^{-3x}dx = x(-\frac{1}{3}e^{-3x}) - \int -\frac{1}{3}e^{-3x}\cdot dx$$
$$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}dx$$
$$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}(-\frac{1}{3}e^{-3x})$$
$$= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} \text{ .... (II)}$$

reemplazando (II) en (I)

$$\int x^2 e^{-3x}dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}[-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}] = -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x}$$
$$= -\frac{1}{3}e^{-3x}(x^2 + \frac{2}{3}x + \frac{2}{9}) + c$$

21.

$\int sen^3x\,dx$

$\int sen^3x\,dx = \int sen^2x\,senx\,dx$

Integrando por partes:

sea:

$$u = sen^2x \qquad dv = senx\,dx$$
$$\int dv = \int senx\,dx$$
$$du = 2\,senx\,cosx\,dx \qquad v = -cosx$$
$$\int u\,dv = uv - \int v\,du$$

$$\int sen^3x\,dx = sen^2x(-cosx) - \int -cosx\cdot 2\,senx\,cosx\,dx$$
$$= -sen^2x\,cosx + 2\int cos^2x\,senx\,dx \text{ .... (I)}$$

resolveremos la integral $\int cos^2x\,senx\,dx$ por sustitución

$\int cos^2x\,senx\,dx = \int (cosx)^2\,senx\,dx$

hacemos el cambio de variable: $t = cosx$; diferenciando: $dt = -senx\,dx$; despejando $senx\,dx$: $-dt = senx\,dx$; en consecuencia:

$$\int cos^2x\,senx\,dx = \int t^2(-dt) = -\int t^2dt = -\frac{t^3}{3} = -\frac{1}{3}t^3$$
$$\int cos^2x\,senx\,dx = -\frac{1}{3}(cosx)^3 = -\frac{1}{3}cos^3x \text{ ..... (II)}$$

reemplazando (II) en (I)

$$\int sen^3x\,dx = -sen^2x\,cosx + 2(-\frac{1}{3}cos^3x) + c = -sen^2x\,cosx - \frac{2}{3}cos^3x + c$$

22.

$\int x^3\,senx\,dx$

integrando por partes:

sea:

$$u = x^3 \qquad dv = senx\,dx$$
$$\int dv = \int senx\,dx$$
$$du = 3x^2\,dx \qquad v = -cosx$$
$$\int u\,dv = uv - \int v\,du$$

$$\int x^3\,senx\,dx = x^3(-cosx) - \int -cosx\cdot 3x^2\,dx$$
$$= -x^3cosx + 3\int x^2\,cosx\,dx \text{ ...... (I)}$$

Integrando nuevamente por partes la integral $\int x^2\,cosx\,dx$

sea:

$$u = x^2 \qquad dv = cosx\,dx$$
$$\int dv = \int cosx\,dx$$
$$du = 2x\,dx \qquad v = senx$$
$$\int u\,dv = uv - \int v\,du$$

$$\int x^2\cos x\,dx = x^2\cdot \operatorname{sen}x - \int \operatorname{sen}x\cdot 2x\,dx$$
$$= x^2\operatorname{sen}x - 2\int x\operatorname{sen}x\,dx \;\ldots\ldots\; (II)$$

reemplazando (II) en (I)

$$\int x^3\operatorname{sen}x\,dx = -x^3\cos x + 3\,[x^2\operatorname{sen}x - 2\int x\operatorname{sen}x\,dx]$$
$$= -x^3\cos x + 3x^2\operatorname{sen}x - 6\int x\operatorname{sen}x\,dx \;\ldots\ldots\ldots\; (III)$$

calculando por integración por partes la integral $\int x\operatorname{sen}x\,dx$

sea: $u = x$ $\quad dv = \operatorname{sen}x\,dx$

$\int dv = \int \operatorname{sen}x\,dx$

$du = dx$ $\quad v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\operatorname{sen}x\,dx = x(-\cos x) - \int -\cos x\cdot dx$$
$$= -x\cos x + \int \cos x\,dx$$
$$= -x\cos x + \operatorname{sen}x \;\ldots\ldots\ldots (IV)$$

reemplazando (IV) en (III)

$$\int x^3\operatorname{sen}x\,dx = -x^3\cos x + 3x^2\operatorname{sen}x - 6\,[-x\cos x + \operatorname{sen}x] + c$$
$$= -x^3\cos x + 3x^2\operatorname{sen}x + 6x\cos x - 6\operatorname{sen}x + c$$

23.

$$\int \frac{x\,dx}{\sqrt{a+bx}}$$

$$\int \frac{x\,dx}{\sqrt{a+bx}} = \int \frac{x\,dx}{(a+bx)^{1/2}} = \int x(a+bx)^{-1/2}dx$$

Integrando por partes:

sea: $u = x$ $\quad dv = (a+bx)^{-1/2}dx$

$\int dv = \int (a+bx)^{-1/2}dx$

$du = dx$ $\quad v = \frac{2}{b}(a+bx)^{1/2}$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{x\,dx}{\sqrt{a+bx}} = x\cdot\frac{2}{b}(a+bx)^{1/2} - \int \frac{2}{b}(a+bx)^{1/2}\cdot dx = \frac{2}{b}x(a+bx)^{1/2} - \frac{2}{b}\int (a+bx)^{1/2}dx$$
$$= \frac{2}{b}x(a+bx)^{1/2} - \frac{2}{b}\cdot\frac{2}{3b}(a+bx)^{3/2} + c = \frac{2}{b}(a+bx)^{1/2}[x - \frac{2}{3b}(a+bx)] + c$$
$$= \frac{2}{b}(a+bx)^{1/2}[\frac{3bx - 2(a+bx)}{3b}] + c = \frac{2}{b}\sqrt{a+bx}\,[\frac{3bx - 2a - 2bx}{3b}] + c$$
$$= \frac{2}{b}\sqrt{a+bx}\,[\frac{bx - 2a}{3b}] + c = \frac{2(bx-2a)\sqrt{a+bx}}{3b^2} + c$$

24.

$$\int \frac{x^2\,dx}{\sqrt{1+x}}$$

$$\int \frac{x^2\,dx}{\sqrt{1+x}} = \int \frac{x^2\,dx}{(1+x)^{1/2}} = \int x^2(1+x)^{-1/2}dx$$

Integrando por partes:

sea: $u = x^2$ $\quad dv = (1+x)^{-1/2}dx$

$\int dv = \int (1+x)^{-1/2}dx$

$du = 2x\,dx$ $\quad v = 2(1+x)^{1/2}$

$$\int u\,dv = uv - \int u\,dv$$

$$\int \frac{x^2\,dx}{\sqrt{1+x}} = x^2\cdot 2(1+x)^{1/2} - \int 2(1+x)^{1/2}\cdot 2x\,dx$$
$$= 2x^2(1+x)^{1/2} - 4\int x(1+x)^{1/2}dx \;\ldots\ldots\ldots\; (I)$$

integrando por partes la integral $\int x(1+x)^{1/2}dx$

sea:

$$u = x \qquad dv = (1+x)^{1/2}dx$$
$$\int dv = \int (1+x)^{1/2}dx$$
$$du = dx \qquad v = \frac{2}{3}(1+x)^{3/2}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x(1+x)^{1/2}dx = x\cdot\frac{2}{3}(1+x)^{3/2} - \int\frac{2}{3}(1+x)^{3/2}\cdot dx$$
$$= \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\int(1+x)^{3/2}dx$$
$$= \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(1+x)^{5/2}$$
$$= \frac{2}{3}x(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} \;\ldots\ldots\; (II)$$

reemplazando (II) en (I)

$$\int\frac{x^2dx}{\sqrt{1+x}} = 2x^2(1+x)^{1/2} - 4\left[\frac{2}{3}x(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2}\right]$$
$$= 2x^2(1+x)^{1/2} - \frac{8}{3}x(1+x)^{3/2} - \frac{16}{15}(1+x)^{5/2} + c$$
$$= 2(1+x)^{1/2}\left[x^2 - \frac{4}{3}x(1+x) + \frac{8}{15}(1+x)^2\right] + c$$
$$= 2(1+x)^{1/2}\left[\frac{15x^2 - 20x(1+x) + 8(1+2x+x^2)}{15}\right] + c$$
$$= \frac{2}{15}\sqrt{1+x}\,(15x^2 - 20x - 20x^2 + 8 + 16x + 8x^2) + c$$
$$= \frac{2}{15}(3x^2 - 4x + 8)\sqrt{1+x} + c$$

25.

$\int x\,\text{arcsen}\,x^2dx$

$\int x\,\text{arcsen}\,x^2dx = \int \text{arcsen}\,x^2\cdot x\,dx$

Integrando por partes:

sea:

$$u = \text{arcsen}\,x^2 \qquad dv = x\,dx$$
$$du = \frac{2x\,dx}{\sqrt{1-(x^2)^2}} \qquad \int dv = \int x\,dx$$
$$du = \frac{2x\,dx}{\sqrt{1-x^4}} \qquad v = \frac{1}{2}x^2$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\,\text{arcsen}\,x^2dx = \text{arcsen}\,x^2\cdot\frac{1}{2}x^2 - \int\frac{1}{2}x^2\cdot\frac{2x\,dx}{\sqrt{1+x^2}} = \frac{1}{2}x^2\,\text{arcsen}\,x^2 - \int\frac{x^3dx}{\sqrt{1-x^4}}$$
$$= \frac{1}{2}x^2\,\text{arcsen}\,x^2 - \frac{1}{-4}\int\frac{-4x^3dx}{\sqrt{1-x^4}} = \frac{1}{2}x^2\,\text{arcsen}\,x^2 + \frac{1}{4}\cdot 2\sqrt{1-x^4} + c$$
$$= \frac{1}{2}x^2\,\text{arcsen}\,x^2 + \frac{1}{2}\sqrt{1-x^4} + c$$

26.

$\int \text{sen}\,x\,\text{sen}\,3x\,dx$

Integrando por partes:

sea:

$$u = \text{sen}\,x \qquad dv = \text{sen}\,3x\,dx$$
$$\int dv = \int \text{sen}\,3x\,dx$$
$$du = \cos x\,dx \qquad v = -\frac{1}{3}\cos 3x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \text{sen}\,x\,\text{sen}\,3x\,dx = \text{sen}\,x\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x\cdot\cos x\,dx$$
$$= -\frac{1}{3}\text{sen}\,x\cos 3x + \frac{1}{3}\int\cos x\cos 3x\,dx \;\ldots\ldots\; (I)$$

integrando nuevamente por partes la integral $\int \cos x \cos 3x\,dx$

sea:

$$u = \cos x \qquad dv = \operatorname{sen} 3x\,dx$$
$$\int dv = \int \operatorname{sen} 3x\,dx$$
$$du = -\operatorname{sen} x\,dx \qquad v = \frac{1}{3}\cos 3x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \cos x \cos 3x\,dx = \cos x \cdot \frac{1}{3}\operatorname{sen} 3x - \int \frac{1}{3}\operatorname{sen} 3x(-\operatorname{sen} x\,dx)$$
$$= \frac{1}{3}\operatorname{sen} 3x \cos x + \frac{1}{3}\int \operatorname{sen} x \operatorname{sen} 3x\,dx \;\ldots\ldots\; \text{(II)}$$

reemplazando (II) en (I)

$$\int \operatorname{sen} x \operatorname{sen} 3x\,dx = -\frac{1}{3}\operatorname{sen} x \cos 3x + \frac{1}{3}\left[\frac{1}{3}\operatorname{sen} 3x \cos x + \frac{1}{3}\int \operatorname{sen} x \operatorname{sen} 3x\,dx\right]$$
$$\int \operatorname{sen} x \operatorname{sen} 3x\,dx = -\frac{1}{3}\operatorname{sen} x \cos 3x + \frac{1}{9}\operatorname{sen} 3x \cos x + \frac{1}{9}\int \operatorname{sen} x \operatorname{sen} 3x\,dx$$
$$\int \operatorname{sen} x \operatorname{sen} 3x\,dx - \frac{1}{9}\int \operatorname{sen} x \operatorname{sen} 3x\,dx = -\frac{1}{3}\operatorname{sen} x \cos 3x + \frac{1}{9}\operatorname{sen} 3x \cos x + c$$
$$\left(1 - \frac{1}{9}\right)\int \operatorname{sen} x \operatorname{sen} 3x\,dx = \frac{1}{9}\operatorname{sen} 3x \cos x - \frac{1}{3}\operatorname{sen} x \cos 3x + c$$
$$\frac{8}{9}\int \operatorname{sen} x \operatorname{sen} 3x\,dx = \frac{1}{9}\operatorname{sen} 3x \cos x - \frac{1}{3}\operatorname{sen} x \cos 3x + c$$
$$\int \operatorname{sen} x \operatorname{sen} 3x\,dx = \frac{9}{8}\left(\frac{1}{9}\operatorname{sen} 3x \cos x - \frac{1}{3}\operatorname{sen} x \cos 3x\right) + c$$
$$\int \operatorname{sen} x \operatorname{sen} 3x\,dx = \frac{1}{8}\operatorname{sen} 3x \cos x - \frac{3}{8}\operatorname{sen} x \cos 3x + c$$

27.

$\int \operatorname{sen}(\ln x)\,dx$

Integrando por partes:

sea:

$$u = \operatorname{sen}(\ln x) \qquad dv = dx$$
$$du = \cos(\ln x)\cdot\frac{1}{x}\cdot dx \qquad \int dv = \int dx$$
$$du = \frac{1}{x}\cos(\ln x)\,dx \qquad v = x$$

$$\int u\,dv = u\,dv - \int v\,du$$

$$\int \operatorname{sen}(\ln x)\,dx = \operatorname{sen}(\ln x)\cdot x - \int x \cdot \frac{1}{x}\cos(\ln x)\,dx$$
$$= x \operatorname{sen}(\ln x) - \int \cos(\ln x)\,dx \;\ldots\ldots\; \text{(I)}$$

Integrando nuevamente por partes la integral $\int \cos(\ln x)\,dx$

sea:

$$u = \cos(\ln x) \qquad dv = dx$$
$$du = -\operatorname{sen}(\ln x)\cdot\frac{1}{x}\cdot dx \qquad \int dv = \int dx$$
$$du = -\frac{1}{x}\operatorname{sen}(\ln x)\,dx \qquad v = x$$

$$\int u\,dv = u\,dv - \int v\,du$$

$$\int \cos(\ln x)\,dx = \cos(\ln x)\cdot x - \int x\left[-\frac{1}{x}\operatorname{sen}(\ln x)\,dx\right]$$
$$\int \cos(\ln x)\,dx = x\cos(\ln x) + \int \operatorname{sen}(\ln x)\,dx \;\ldots\ldots\; \text{(II)}$$

reemplazando (II) en (I)

$$\int \operatorname{sen}(\ln x)\,dx = x\operatorname{sen}(\ln x) - \left[x\cos(\ln x) + \int \operatorname{sen}(\ln x)\,dx\right]$$
$$\int \operatorname{sen}(\ln x)\,dx = x\operatorname{sen}(\ln x) - x\cos(\ln x) - \int \operatorname{sen}(\ln x)\,dx$$
$$\int \operatorname{sen}(\ln x)\,dx + \int \operatorname{sen}(\ln x)\,dx = x\operatorname{sen}(\ln x) - x\cos(\ln x) + c$$
$$2\int \operatorname{sen}(\ln x)\,dx = x\left[\operatorname{sen}(\ln x) - \cos(\ln x)\right] + c$$
$$\int \operatorname{sen}(\ln x)\,dx = \frac{1}{2}x\left[\operatorname{sen}(\ln x) - \cos(\ln x)\right] + c$$

28.

$\int e^{ax}\cos bx\,dx$

Integrando por partes:
sea:

$$u = e^{ax} \qquad dv = \cos bx\,dx$$
$$du = e^{ax}\cdot a\cdot dx \qquad \int dv = \int \cos bx\,dx$$
$$du = a\,e^{ax}dx \qquad v = \frac{1}{b}\operatorname{sen} bx$$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{ax}\cos bx\,dx = e^{ax}\cdot\frac{1}{b}\operatorname{sen} bx - \int\frac{1}{b}\operatorname{sen} bx\cdot a\,e^{ax}dx$$
$$\int e^{ax}\cos bx\,dx = \frac{1}{b}e^{ax}\operatorname{sen} bx - \frac{a}{b}\int e^{ax}\operatorname{sen} bx\,dx \;\ldots\ldots\; (I)$$

Integrando nuevamente por partes la integral $\int e^{ax}\operatorname{sen} bx\,dx$
sea:

$$u = e^{ax} \qquad dv = \operatorname{sen} bx\,dx$$
$$du = e^{ax}\cdot a\cdot dx \qquad \int dv = \int \operatorname{sen} bx\,dx$$
$$du = a\,e^{ax}dx \qquad v = -\frac{1}{b}\cos bx$$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{ax}\operatorname{sen} bx\,dx = e^{ax}\left(-\frac{1}{b}\cos bx\right) - \int -\frac{1}{b}\cos bx\cdot a\,e^{ax}dx$$
$$\int e^{ax}\operatorname{sen} bx\,dx = -\frac{1}{b}e^{ax}\cos bx + \frac{a}{b}\int e^{ax}\cos bx\,dx \;\ldots\ldots\; (II)$$

reemplazando (II) en (I)

$$\int e^{ax}\cos bx\,dx = \frac{1}{b}e^{ax}\operatorname{sen} bx - \frac{a}{b}\left[-\frac{1}{b}e^{ax}\cos bx + \frac{a}{b}\int e^{ax}\cos bx\,dx\right]$$
$$\int e^{ax}\cos bx\,dx = \frac{1}{b}e^{ax}\operatorname{sen} bx + \frac{a}{b^2}e^{ax}\cos bx - \frac{a^2}{b^2}\int e^{ax}\cos bx\,dx$$
$$\int e^{ax}\cos bx\,dx + \frac{a^2}{b^2}\int e^{ax}\cos bx\,dx = \frac{1}{b}e^{ax}\operatorname{sen} bx + \frac{a}{b^2}e^{ax}\cos bx + c$$
$$\left(1+\frac{a^2}{b^2}\right)\int e^{ax}\cos bx\,dx = e^{ax}\left(\frac{1}{b}\operatorname{sen} bx + \frac{a}{b^2}\cos bx\right) + c$$
$$\frac{b^2+a^2}{b^2}\int e^{ax}\cos bx\,dx = e^{ax}\left(\frac{b\operatorname{sen} bx + a\cos bx}{b^2}\right) + c$$
$$\int e^{ax}\cos bx\,dx = \frac{b^2 e^{ax}(b\operatorname{sen} bx + a\cos bx)}{b^2(b^2+a^2)} + c$$
$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(b\operatorname{sen} bx + a\cos bx)}{b^2+a^2} + c$$

29.

$\int e^{ax}\operatorname{sen} bx\,dx$

Integrando por partes:
sea:

$$u = e^{ax} \qquad dv = \operatorname{sen} bx\,dx$$
$$du = e^{ax}\cdot a\cdot dx \qquad \int dv = \int \operatorname{sen} bx\,dx$$
$$du = a\,e^{ax}dx \qquad v = -\frac{1}{b}\cos bx$$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{ax}\operatorname{sen} bx\,dx = e^{ax}\left(-\frac{1}{b}\cos bx\right) - \int -\frac{1}{b}\cos bx\cdot a\,e^{ax}dx$$
$$\int e^{ax}\operatorname{sen} bx\,dx = -\frac{1}{b}e^{ax}\cos bx + \frac{a}{b}\int e^{ax}\cos bx\,dx \;\ldots\ldots\; (I)$$

Integrando nuevamente por partes la integral $\int e^{ax}\cos bx\,dx$

sea:

$$u = e^{ax} \qquad dv = \cos bx\,dx$$
$$du = e^{ax}\cdot a\cdot dx \qquad \int dv = \int \cos bx\,dx$$
$$du = a\,e^{ax}\,dx \qquad v = \frac{1}{b}\,\text{sen}\,bx$$
$$\int u\,dv = uv - \int v\,du$$

$$\int e^{ax}\cos bx\,dx = e^{ax}\cdot\frac{1}{b}\,\text{sen}\,bx - \int \frac{1}{b}\,\text{sen}\,bx\cdot a\,e^{ax}\,dx$$
$$\int e^{ax}\cos bx\,dx = \frac{1}{b}\,e^{ax}\,\text{sen}bx - \frac{a}{b}\int e^{ax}\,\text{sen}\,bx\,dx \;\ldots\ldots\; (II)$$

reemplazando (II) en (I)

$$\int e^{ax}\,\text{sen}\,bx\,dx = -\frac{1}{b}\,e^{ax}\cos dx + \frac{a}{b}\left[\frac{1}{b}\,e^{ax}\,\text{sen}\,bx - \frac{a}{b}\int e^{ax}\,\text{sen}\,bx\,dx\right]$$
$$\int e^{ax}\,\text{sen}\,bx\,dx = -\frac{1}{b}\,e^{ax}\cos bx + \frac{a}{b^2}\,e^{ax}\,\text{sen}\,bx - \frac{a^2}{b^2}\int e^{ax}\,\text{sen}\,bx\,dx$$
$$\int e^{ax}\,\text{sen}\,bx\,dx + \frac{a^2}{b^2}\int e^{ax}\,\text{sen}\,bx\,dx = -\frac{1}{b}\,e^{ax}\cos bx + \frac{a}{b^2}\,e^{ax}\,\text{sen}\,bx + c$$
$$\left(1+\frac{a^2}{b^2}\right)\int e^{ax}\,\text{sen}\,bx\,dx = e^{ax}\left(\frac{a}{b^2}\,\text{sen}\,bx - \frac{1}{b}\cos bx\right) + c$$
$$\frac{b^2+a^2}{b^2}\int e^{ax}\,\text{sen}\,bx\,dx = e^{ax}\left(\frac{a\,\text{sen}\,bx - b\cos bx}{b^2}\right) + c$$
$$\int e^{ax}\,\text{sen}\,bx\,dx = \frac{b^2 e^{ax}(a\,\text{sen}\,bx - b\cos bx)}{b^2(b^2+a^2)} + c$$
$$\int e^{ax}\,\text{sen}\,bx\,dx = \frac{e^{ax}(a\,\text{sen}\,bx - b\cos bx)}{b^2+a^2} + c$$

30. Deducir las formulas de reducción (A) y (B)

(A) $$\int \frac{dx}{(a^2\pm x^2)^m} = \frac{1}{a^2}\left[\frac{x}{(2m-2)(a^2\pm x^2)^{m-1}} + \frac{2m-3}{2m-2}\int\frac{dx}{(a^2\pm x^2)^{m-1}}\right], \quad m \neq -1$$

Demostración:

$$\int \frac{dx}{(a^2\pm x^2)^m} = \int \frac{1}{(a^2\pm x^2)^m}\,dx$$

multiplicando dividiendo por a^2

$$\int \frac{dx}{(a^2\pm x^2)^m} = \frac{a^2}{a^2}\int\frac{1}{(a^2\pm x^2)^m}\,dx = \frac{1}{a^2}\int\frac{a^2}{(a^2\pm x^2)^m}\,dx$$

sumando y restando x^2 en el numerador

$$\int \frac{dx}{(a^2\pm x^2)^m} = \frac{1}{a^2}\int\frac{a^2\pm x^2\mp x^2}{(a^2\pm x^2)^m}\,dx = \frac{1}{a^2}\int\left[\frac{a^2\pm x^2}{(a^2\pm x^2)^m} \mp \frac{x^2}{(a^2\pm x^2)^m}\right]dx$$
$$= \frac{1}{a^2}\int\frac{(a^2\pm x^2)\,dx}{(a^2\pm x^2)^m} \mp \frac{1}{a^2}\int\frac{x^2\,dx}{(a^2\pm x^2)^m}$$
$$= \frac{1}{a^2}\int\frac{dx}{(a^2\pm x^2)^m(a^2\pm x^2)^{-1}} \mp \frac{1}{a^2}\int\frac{x^2\,dx}{(a^2\pm x^2)^m}$$
$$= \frac{1}{a^2}\int\frac{dx}{(a^2\pm x^2)^{m-1}} \mp \frac{1}{a^2}\int\frac{x^2\,dx}{(a^2\pm x^2)^m} \;\ldots\ldots\ldots\ldots\; (I)$$

resolveremos la integral $\int\frac{x^2\,dx}{(a^2\pm x^2)^m}$ por partes

$$\int\frac{x^2\,dx}{(a^2\pm x^2)^m} = \int\frac{x\cdot x\,dx}{(a^2\pm x^2)^m} = \int x\cdot\frac{x\,dx}{(a^2\pm x^2)^m}$$

sea: $u = x$; diferenciando; $du = dx$

$$dv = \frac{x\,dx}{(a^2\pm x^2)^m} \;\rightarrow\; \int dv = \int\frac{x\,dx}{(a^2\pm x^2)^m} \;\rightarrow\; v = \int\frac{x\,dx}{(a^2\pm x^2)^2}$$

hacemos cambio: $z = a^2\pm x^2$; diferenciando: $dz = (0\pm 2x)\,dx \;\rightarrow\; dz = \pm 2x\,dx$;

despejando $x\,dx$: $\pm\frac{dz}{2} = x\,dx$; en consecuencia:

$$v = \int\frac{\pm\frac{dz}{2}}{z^m} = \int\pm\frac{dz}{2z^m} = \pm\frac{1}{2}\int\frac{dz}{z^m} = \pm\frac{1}{2}\int z^{-m}\,dz = \pm\frac{1}{2}\cdot\frac{z^{-m+1}}{-m+1} = \pm\frac{z^{-(m-1)}}{2[-(m-1)]}$$
$$v = \pm\frac{1}{-2(m-1)\,z^{m-1}} \;\rightarrow\; v = \mp\frac{1}{(2m-2)\,z^{m-1}} \;\rightarrow\; v = \mp\frac{1}{(2m-2)(a^2\pm x^2)^{m-1}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{x^2 dx}{(a^2 \pm x^2)^m} = x\,[\mp \frac{1}{(2m-2)(a^2 \pm x^2)^{m-1}}] - \int \mp \frac{1}{(2m-2)(a^2 \pm x^2)^{m-1}} \cdot dx$$

$$= \mp \frac{x}{(2m-2)(a^2 \pm x^2)^{m-1}} \pm \frac{1}{2m-2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}} \quad \ldots\ldots (II)$$

reemplazando (II) en (I)

$$\int \frac{dx}{(a^2 \pm x^2)^m} = \frac{1}{a^2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}} \mp \frac{1}{a^2} [\mp \frac{x}{(2m-2)(a^2 \pm x^2)^{m-1}} \pm \frac{1}{2m-2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}]$$

$$= \frac{1}{a^2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}} + \frac{x}{a^2 (2m-2)(a^2 \pm x^2)^{m-1}} - \frac{1}{a^2 (2m-2)} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}$$

$$= \frac{x}{a^2 (2m-2)(a^2 \pm x^2)^{m-1}} + \frac{1}{a^2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}} - \frac{1}{a^2 (2m-2)} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}$$

$$= \frac{x}{a^2 (2m-2)(a^2 \pm x^2)^{m-1}} + (1 - \frac{1}{2m-2}) \frac{1}{a^2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}$$

$$= \frac{x}{a^2 (2m-2)(a^2 \pm x^2)^{m-1}} + (\frac{2m-2-1}{2m-2}) \frac{1}{a^2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}$$

$$= \frac{x}{a^2 (2m-2)(a^2 \pm x^2)^{m-1}} + (\frac{2m-3}{2m-2}) \frac{1}{a^2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}$$

$$= \frac{x}{a^2 (2m-2)(a^2 \pm x^2)^{m-1}} + \frac{2m-3}{a^2 (2m-2)} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}$$

$$= \frac{1}{a^2} [\frac{x}{(2m-2)(a^2 \pm x^2)^{m-1}} + \frac{2m-3}{2m-2} \int \frac{dx}{(a^2 \pm x^2)^{m-1}}] \quad \text{l. q. q.d.}$$

(B) $\int (a^2 \pm x^2)^m dx = \frac{x(a^2 \pm x^2)^m}{2m+1} - \frac{2ma^2}{2m+1} \int (a^2 \pm x^2)^{m-1} dx \quad , \quad m \neq -1/2$

Demostración:

Integrando por partes:

sea:

$$u = (a^2 \pm x^2)^m \qquad dv = dx$$

$$dx = m(a^2 \pm x^2)^{m-1} (0 \pm 2x)\,dx \qquad \int dv = \int dx$$

$$du = \pm 2mx(a^2 \pm x^2)^{m-1} dx \qquad v = x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int (a^2 \pm x^2)^m dt = (a^2 \pm x^2)^m \cdot x - \int x \cdot [\pm 2mx(a^2 \pm x^2)^{m-1} dx]$$

$$\int (a^2 \pm x^2)^m dx = x(a^2 \pm x^2)^m \mp 2m \int x^2 (a^2 \pm x^2)^{m-1} dx$$

$$\int (a^2 \pm x^2)^m dx = x(a^2 \pm x^2)^m \mp 2m \int \frac{x^2 (a^2 \pm x^2)^m}{a^2 \pm x^2} dx$$

$$\int (a^2 \pm x^2)^m dx = x(a^2 \pm x^2)^m \mp 2m \int (\frac{x^2}{a^2 \pm x^2})(a^2 \pm x^2)^m dx$$

$$\int (a^2 \pm x^2)^m dx = x(a^2 \pm x^2)^m \mp 2m \int (\pm 1 \mp \frac{a^2}{a^2 \pm x^2})(a^2 \pm x^2)^m dx$$

$$\int (a^2 \pm x^2)^m dx = x(a^2 \pm x^2)^m - 2m \int (a^2 \pm x^2)^m dx + 2ma^2 \int \frac{(a^2 \pm x^2)^m}{a^2 \pm x^2} dx$$

$$2m \int (a^2 \pm x^2)^m dx + \int (a^2 \pm x^2)^m dx = x(a^2 \pm x^2)^m + 2ma^2 \int (a^2 \pm x^2)^m (a^2 \pm x^2)^{-1} dx$$

$$(2m+1) \int (a^2 \pm x^2)^m dx = x(a^2 \pm x^2)^m + 2ma^2 \int (a^2 \pm x^2)^{m-1} dx$$

$$\int (a^2 \pm x^2)^m dx = \frac{x(a^2 \pm x^2)^m}{2m+1} + \frac{2ma^2}{2m+1} \int (a^2 \pm x^2)^{m-1} dx \quad \text{l. q. q. d.}$$

31. Deducir las fórmulas de reducción (C)-(J).

(C) $\int \frac{dx}{(x^2 - a^2)^m} = -\frac{1}{a^2} [\frac{x}{(2m-2)(x^2 - a^2)^{m-1}} + \frac{2m-3}{2m-2} \int \frac{dx}{(x^2 - a^2)^{m-1}}] \quad , \quad m \neq -1$

Demostración:

$$\int \frac{dx}{(x^2 - a^2)^m} = \int \frac{1}{(x^2 - a^2)^m} dx$$

multiplicando dividiendo por $-a^2$

$$\int \frac{dx}{(x^2 - a^2)^m} = \frac{-a^2}{-a^2} \int \frac{1}{(x^2 - a^2)^m} dx = -\frac{1}{a^2} \int \frac{-a^2}{(x^2 - a^2)^m} dx$$

sumando y restando x^2 en el numerador

$$\int\frac{dx}{(x^2-a^2)^m}=-\frac{1}{a^2}\int\frac{x^2-x^2-a^2}{(x^2-a^2)^m}dx=-\frac{1}{a^2}\int[\frac{x^2-a^2}{(x^2-a^2)^m}-\frac{x^2}{(x^2-a^2)^m}]dx$$

$$=-\frac{1}{a^2}\int\frac{(x^2-a^2)dx}{(x^2-a^2)^m}+\frac{1}{a^2}\int\frac{x^2dx}{(x^2-a^2)^m}$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^m(x^2-a^2)^{-1}}+\frac{1}{a^2}\int\frac{x^2dx}{(x^2-a^2)^m}$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}+\frac{1}{a^2}\int\frac{x^2dx}{(x^2-a^2)^m}\ \ldots\ldots\ldots\ldots\ldots\ (I)$$

resolveremos la integral $\int\frac{x^2dx}{(x^2-a^2)^m}$ por partes

$$\int\frac{x^2dx}{(x^2-a^2)^m}=\int\frac{x\cdot x\,dx}{(x^2-a^2)^m}=\int x\cdot\frac{x\,dx}{(x^2-a^2)^m}$$

sea

$$u=x;\quad \text{diferenciando;}\quad du=dx$$

$$dv=\frac{x\,dx}{(x^2-a^2)^m};\quad \int dv=\int\frac{x\,dx}{(x^2-a^2)^m};\quad v=\int\frac{x\,dx}{(x^2-a^2)^m}$$

hacemos cambio: $z=x^2-a^2$; diferenciando: $dz=(2x-0)dx \rightarrow dz=2x\,dx$;

despejando $x\,dx$: $\frac{dz}{2}=x\,dx$; en consecuencia:

$$v=\int\frac{\frac{dz}{2}}{z^m}=\int\frac{dz}{2z^m}=\frac{1}{2}\int\frac{dz}{z^m}=\frac{1}{2}\int z^{-m}dz=\frac{1}{2}\cdot\frac{z^{-m+1}}{-m+1}=\frac{z^{-(m-1)}}{2[-(m-1)]}=\frac{1}{-2(m-1)z^{m-1}}$$

$$v=-\frac{1}{(2m-2)z^{m-1}}\rightarrow v=-\frac{1}{(2m-2)(x^2-a^2)^{m-1}}$$

$$\int u\,dv=uv-\int v\,du$$

$$\int\frac{x^2dx}{(x^2-a^2)^m}=x\cdot[-\frac{1}{(2m-2)(x^2-a^2)^{m-1}}]-\int-\frac{1}{(2m-2)(x^2-a^2)^{m-1}}\cdot dx$$

$$=-\frac{x}{(2m-2)(x^2-a^2)^{m-1}}+\frac{1}{2m-2}\int\frac{dx}{(x^2-a^2)^{m-1}}\ \ldots\ldots\ (II)$$

reemplazando (II) en (I)

$$\int\frac{dx}{(x^2-a^2)^m}=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}+\frac{1}{a^2}[-\frac{x}{(2m-2)(x^2-a^2)^{m-1}}+\frac{1}{2m-2}\int\frac{dx}{(x^2-a^2)^{m-1}}]$$

$$=-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}-\frac{x}{a^2(2m-2)(x^2-a^2)^{m-1}}+\frac{1}{a^2(2m-2)}\int\frac{dx}{(x^2-a^2)^{m-1}}$$

$$=-\frac{x}{a^2(2m-2)(x^2-a^2)^{m-1}}+\frac{1}{a^2(2m-2)}\int\frac{dx}{(x^2-a^2)^{m-1}}-\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}$$

$$=-\frac{x}{a^2(2m-2)(x^2-a^2)^{m-1}}+(\frac{1}{2m-2}-1)\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}$$

$$=-\frac{x}{a^2(2m-2)(x^2-a^2)^{m-1}}+(\frac{1-2m+2}{2m-2})\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}$$

$$=-\frac{x}{a^2(2m-2)(x^2-a^2)^{m-1}}+(\frac{-2m+3}{2m-1})\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}$$

$$=-\frac{x}{a^2(2m-2)(x^2-a^2)^{m-1}}+(\frac{-[2m-3]}{2m-2})\frac{1}{a^2}\int\frac{dx}{(x^2-a^2)^{m-1}}$$

$$=-\frac{x}{a^2(2m-2)(x^2-a^2)^{m-1}}-\frac{2m-3}{a^2(2m-2)}\int\frac{dx}{(x^2-a^2)^{m-1}}$$

$$=-\frac{1}{a^2}[\frac{x}{(2m-2)(x^2-a^2)^{m-1}}+\frac{2m-3}{2m-2}\int\frac{dx}{(x^2-a^2)^{m-1}}]\qquad \text{l. q. q.d.}$$

(D) $\int(x^2-a^2)^m dx=\frac{x(x^2-a^2)^m}{2m-1}-\frac{2ma^2}{2m-1}\int(x^2-a^2)^{m-1}dx\quad,\quad m\neq-1/2$

Demostración:

Integrando por partes:

sea:

$$u=(x^2-a^2)^m\qquad dv=dx$$

$$du=m(x^2-a^2)^{m-1}(2x-0)dx\qquad \int dv=\int dx$$

$$du=2mx(x^2-a^2)^{m-1}dx\qquad v=x$$

$$\int u\,dv=uv-\int v\,du$$

$$\int(x^2-a^2)^m dx = (x^2-a^2)^m\cdot x - \int x\cdot 2mx(x^2-a^2)^{m-1}dx$$
$$\int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2m\int x^2(x^2-a^2)^{m-1}dx$$
$$\int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2m\int x^2(x^2-a^2)^m(x^2-a^2)^{-1}dx$$
$$\int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2m\int \frac{x^2(x^2-a^2)^m}{x^2-a^2}dx$$
$$\int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2m\int (\frac{x^2}{x^2-a^2})(x^2-a^2)^m dx$$
$$\int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2m\int (1+\frac{a^2}{x^2-a^2})(x^2-a^2)^m dx$$
$$\int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2m\int (x^2-a^2)^m dx - 2ma^2\int\frac{(x^2-a^2)^m}{x^2-a^2}dx$$
$$2m\int(x^2-a^2)^m dx + \int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2ma^2\int(x^2-a^2)^m(x^2-a^2)^{-1}dx$$
$$(2m+1)\int(x^2-a^2)^m dx = x(x^2-a^2)^m - 2ma^2\int(x^2-a^2)^{m-1}dx$$
$$\int(x^2-a^2)^m dx = \frac{x(x^2-a^2)^m}{2m+1} - \frac{2ma^2}{2m+1}\int(x^2a^2)^{m-1}dx \quad \text{l. q. q. d.}$$

(E) $\int x^m e^{ax}dx = \frac{1}{a}x^m e^{ax} - \frac{m}{a}\int x^{m-1}e^{ax}dx$

Demostración:
Integrando por partes:
sea:

$$u = x^m \qquad dv = e^{ax}dx$$
$$\int dv = \int e^{ax}dx$$
$$du = mx^{m-1}dx \qquad v = \frac{1}{a}e^{ax}$$
$$\int u\,dv = uv - \int v\,du$$
$$\int x^m e^{ax}dx = x^m\cdot\frac{1}{a}e^{ax} - \int\frac{1}{a}e^{ax}\cdot mx^{m-1}dx$$
$$\int x^m e^{ax}dx = \frac{1}{a}x^m e^{ax} - \frac{m}{a}\int x^{m-1}e^{ax}dx \quad \text{l. q. q. d.}$$

(F) $\int sen^m x\,dx = -\frac{sen^{m-1}x\cos x}{m} + \frac{m-1}{m}\int sen^{m-2}x\,dx$

Demostración:
multiplicando y dividiendo por senx:
$$\int sen^m x\,dx = \int sen^m x\cdot\frac{senx}{senx}dx = \int sen^m x\,sen^{-1}x\,senx\,dx = \int sen^{m-1}x\,senx\,dx$$

Integrando por partes la integral $\int sen^{m-1}x\,senx\,dx$
sea:

$$u = sen^{m-1}x \qquad dv = senx\,dx$$
$$du = (m-1)sen^{m-1-1}x\cdot\cos x\cdot dx \qquad \int dv = \int senx\,dx$$
$$du = (m-1)sen^{m-2}x\cos x\,dx \qquad v = -\cos x$$
$$\int u\,dv = uv - \int v\,du$$
$$\int sen^m x\,dx = sen^{m-1}x(-\cos x) - \int -\cos x\cdot(m-1)sen^{m-2}x\cos x\,dx$$
$$\int sen^m x\,dx = -sen^{m-1}x\cos x + (m-1)\int sen^{m-2}x\cos^2 x\,dx$$
$$\int sen^m x\,dx = -sen^{m-1}x\cos x + (m-1)\int sen^{m-2}x(1-sen^2x)dx$$
$$\int sen^m x\,dx = -sen^{m-1}x\cos x + (m-1)\int sen^{m-2}x\,dx - (m-1)\int sen^m x\,dx$$
$$\int sen^m x\,dx + (m-1)\int sen^m x\,dx = -sen^{m-1}x\cos x + (m-1)\int sen^{m-2}x\,dx$$
$$(1+m-1)\int sen^m x\,dx = -sen^{m-1}x\cos x + (m-1)\int sen^{m-2}x\,dx$$
$$m\int sen^m x\,dx = -sen^{m-1}x\cos x + (m-1)\int sen^{m-2}x\,dx$$
$$\int sen^m x\,dx = -\frac{sen^{m-1}x\cos x}{m} + \frac{m-1}{m}\int sen^{m-2}x\,dx \quad \text{l. q. q. d.}$$

(G) $\int \cos^m x\,dx = \dfrac{\cos^{m-1}x\,\mathrm{sen}x}{m} + \dfrac{m-1}{m}\int \cos^{m-2}x\,dx$

Demostración:

multiplicando y dividiendo por cosx:

$\int \cos^m x\,dx = \int \cos^m x \cdot \dfrac{\cos x}{\cos x}\,dx = \int \cos^m x \cos^{-1}x \cos x\,dx = \int \cos^{m-1}x \cos x\,dx$

Integrando por partes la integral $\int \cos^{m-1}x \cos x\,dx$

sea: $u = \cos^{m-1}x \qquad dv = \cos x\,dx$

$du = (m-1)\cos^{m-1-1}x \cdot (-\mathrm{sen}x)\cdot dx \qquad \int dv = \int \cos x\,dx$

$du = -(m-1)\cos^{m-2}x\,\mathrm{sen}x\,dx \qquad v = \mathrm{sen}x$

$$\int u\,dv = uv - \int v\,du$$

$$\int \cos^m x\,dx = \cos^{m-1}x \cdot \mathrm{sen}x - \int \mathrm{sen}x\,[-(m-1)\cos^{m-2}x\,\mathrm{sen}x\,dx]$$

$$\int \cos^m x\,dx = \cos^{m-1}x\,\mathrm{sen}x + (m-1)\int \cos^{m-2}x\,\mathrm{sen}^2x\,dx$$

$$\int \cos^m x\,dx = \cos^{m-1}x\,\mathrm{sen}x + (m-1)\int \cos^{m-2}x(1-\cos^2x)\,dx$$

$$\int \cos^m x\,dx = \cos^{m-1}x\,\mathrm{sen}x + (m-1)\int \cos^{m-2}x\,dx - (m-1)\int \cos^m x\,dx$$

$$\int \cos^m x\,dx + (m-1)\int \cos^m x\,dx = \cos^{m-1}x\,\mathrm{sen}x + (m-1)\int \cos^{m-2}x\,dx$$

$$(1+m-1)\int \cos^m x\,dx = \cos^{m-1}x\,\mathrm{sen}x + (m-1)\int \cos^{m-2}x\,dx$$

$$m\int \cos^m x\,dx = \cos^{m-1}x\,\mathrm{sen}x + (m-1)\int \cos^{m-2}x\,dx$$

$$\int \cos^m x\,dx = \frac{\cos^{m-1}x\,\mathrm{sen}x}{m} + \frac{m-1}{m}\int \cos^{m-2}x\,dx$$

l. q. q. d.

(H.1) $\int \mathrm{sen}^m x \cos^n x\,dx = \dfrac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+n} + \dfrac{n-1}{m+n}\int \mathrm{sen}^m x\cos^{n-2}x\,dx \quad , \quad m \neq -n$

Demostración

Multiplicando y dividiendo por cosx

$\int \mathrm{sen}^m x\cos^n x\,dx = \int \mathrm{sen}^m x\cos^n x \cdot \dfrac{\cos x}{\cos x}\,dx = \int \mathrm{sen}^m x\cos^n x\cos^{-1}x\cos x\,dx$

$\int \mathrm{sen}^m x\cos^n x\,dx = \int \mathrm{sen}^m x\cos^{n-1}x\cos x\,dx = \int \cos^{n-1}x\,\mathrm{sen}^m x\cos x\,dx \ldots\ldots (I)$

resolveremos la integral $\int \cos^{n-1}x\,\mathrm{sen}^m x\cos x\,dx$ por partes

sea: $u = \cos^{n-1}x \rightarrow du = (n-1)\cos^{n-1-1}x(-\mathrm{sen}x)\,dx \rightarrow du = -(n-1)\mathrm{sen}x\cos^{n-2}x\,dx$

$dv = \mathrm{sen}^m x\cos x\,dx \rightarrow \int dv = \int \mathrm{sen}^m x\cos x\,dx \rightarrow v = \int (\mathrm{sen}x)^m\cos x\,dx$

hacemos el cambio: $z = \mathrm{sen}x$; diferenciando: $dz = \cos x$; entonces:

$v = \int z^m\,dz \rightarrow v = \dfrac{z^{m+1}}{m+1} \rightarrow v = \dfrac{(\mathrm{sen}x)^{m+1}}{m+1} \rightarrow v = \dfrac{\mathrm{sen}^{m+1}x}{m+1}$

$$\int u\,dv = uv - \int v\,du$$

$$\int \mathrm{sen}^m x\cos^n x\,dx = \cos^{m-1}x \cdot \frac{\mathrm{sen}^{m+1}x}{m+1} - \int \frac{\mathrm{sen}^{m+1}x}{m+1}[-(n-1)\mathrm{sen}x\cos^{n-2}x\,dx]$$

$$\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int \mathrm{sen}^{m+2}x\cos^{n-2}x\,dx$$

$$\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int \mathrm{sen}^m x\,\mathrm{sen}^2x\cos^{n-2}x\,dx$$

$$\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int \mathrm{sen}^m x(1-\cos^2x)\cos^{n-2}x\,dx$$

$$\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\left[\int \mathrm{sen}^m x\cos^{n-2}x\,dx - \int \mathrm{sen}^m x\cos^{n-2+2}x\,dx\right]$$

$$\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int \mathrm{sen}^m x\cos^{n-2}x\,dx - \frac{n-1}{m+1}\int \mathrm{sen}^m x\cos^n x\,dx$$

$$\int \mathrm{sen}^m x\cos^n x\,dx + \frac{n-1}{m+1}\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int \mathrm{sen}^m x\cos^{n-2}x\,dx$$

$$\left(1 + \frac{n-1}{m+1}\right)\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int \mathrm{sen}^m x\cos^{n-2}x\,dx$$

$$\left(\frac{m+1+n-1}{m+1}\right)\int \mathrm{sen}^m x\cos^n x\,dx = \frac{\mathrm{sen}^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int \mathrm{sen}^m x\cos^{n-2}x\,dx$$

$$\frac{m+n}{m+1}\int sen^m x\cos^n x\,dx = \frac{sen^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int sen^m x\cos^{n-2}x\,dx$$

$$\int sen^m x\cos^n x\,dt = \frac{m+1}{m+n}\left[\frac{sen^{m+1}x\cos^{n-1}x}{m+1} + \frac{n-1}{m+1}\int sen^m x\cos^{n-2}x\,dx\right]$$

$$\int sen^m x\cos^n x\,dx = \frac{sen^{m+1}x\cos^{n-1}x}{m+n} + \frac{n-1}{m+n}\int sen^m x\cos^{n-2}x\,dx \qquad \text{l. q. q. d.}$$

(H.2) $\displaystyle\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{m+n} + \frac{m-1}{m+n}\int sen^{m-2}x\cos^n x\,dx \quad , \quad m \neq -n$

Demostración

multiplicando y dividiendo por senx

$$\int sen^m x\cos^n x\,dx = \int sen^m x\cdot\frac{senx}{senx}\cdot\cos^n x\,dx = \int sen^m x\, sen^{-1}x\, senx\cos^n x\,dx$$

$$\int sen^m x\cos^n x\,dx = \int sen^{m-1}x\, senx\cos^n x\,dx = \int sen^{m-1}x\cos^n x\, senx\,dx \;.......(I)$$

resolveremos la integral $\int sen^{m-1}x\cos^n x\, senx\,dx$ por partes:

$$u = sen^{m-1}x \;\rightarrow\; du = (m-1)sen^{m-1-1}x\cos x\,dx \;\rightarrow\; du = (m-1)sen^{m-2}x\cos x\,dx$$

$$dv = \cos^n x\, senx\,dx \;\rightarrow\; \int dv = \int \cos^n x\, senx\,dx \;\rightarrow\; v = \int(\cos x)^n senx\,dx$$

hacemos el cambio: $z = \cos x$; diferenciando: $dz = -senx\,dx$

despejando $senx\,dx$: $-dz = senx\,dx$; en consecuencia

$$v = \int z^n(-dz) \;\rightarrow\; v = -\int z^n dz \;\rightarrow\; v = -\frac{z^{n+1}}{n+1} \;\rightarrow\; v = -\frac{(\cos x)^{n+1}}{n+1} \;\rightarrow\; v = -\frac{\cos^{n+1}x}{n+1}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int sen^m x\cos^n x\,dx = sen^{m-1}x\left(-\frac{\cos^{n+1}x}{n+1}\right) - \int -\frac{\cos^{n+1}x}{n+1}\cdot(m-1)sen^{m-2}x\cos x\,dx$$

$$\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int sen^{m-2}x\cos^{n+2}x\,dx$$

$$\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x\cos^2 x\,dx$$

$$\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x(1-sen^2 x)\,dx$$

$$\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\left[\int sen^{m-2}x\cos^n x\,dx - \int sen^{m-2+2}x\cos^n x\,dx\right]$$

$$\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x\,dx - \frac{m-1}{n+1}\int sen^m x\cos^n x\,dx$$

$$\int sen^m x\cos^n x\,dx + \frac{m-1}{n+1}\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} - \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x\,dx$$

$$\left(1+\frac{m-1}{n+1}\right)\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} - \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x\,dx$$

$$\frac{n+1+m-1}{n+1}\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} - \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x\,dx$$

$$\frac{m+n}{n+1}\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{n+1} - \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x\,dx$$

$$\int sen^m x\cos^n x\,dx = \frac{n+1}{m-1}\left[-\frac{sen^{m-1}x\cos^{n+1}x}{n+1} - \frac{m-1}{n+1}\int sen^{m-2}x\cos^n x\,dx\right]$$

$$\int sen^m x\cos^n x\,dx = -\frac{sen^{m-1}x\cos^{n+1}x}{m+n} - \frac{m-1}{m+n}\int sen^{m-2}x\cos^n x\,dx \qquad \text{l. q. q. d.}$$

(I) $\displaystyle\int x^m senbx\,dx = -\frac{x^m}{b}\cos bx + \frac{m}{b}\int x^{m-1}\cos bx\,dx$

Demostración

Integrando por partes:

sea:

$$u = x^m \qquad dv = senbx\,dx$$

$$du = mx^{m-1}dx \qquad v = -\frac{1}{b}\cos bx$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^m senbx\,dx = x^m\left(-\frac{1}{b}\cos bx\right) - \int -\frac{1}{b}\cos bx\cdot mx^{m-1}dx$$

$$= -\frac{x^m}{b}\cos bx + \frac{m}{b}\int x^{m-1}\cos bx\,dx \qquad \text{l. q. q. d.}$$

(J) $\int x^m \cos bx\,dx = \frac{x^m}{b}\operatorname{sen} bx - \frac{m}{b}\int x^{m-1}\operatorname{sen} bx\,dx$

Demostración

Integrando por partes:

sea:

$$u = x^m \qquad dv = \cos bx\,dx$$
$$du = mx^{m-1}dx \qquad v = \frac{1}{b}\operatorname{sen} bx$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^m \cos bx\,dx = x^m\cdot\frac{1}{b}\operatorname{sen} bx - \int \frac{1}{b}\operatorname{sen} bx\cdot mx^{m-1}dx$$
$$= \frac{x^m}{b}\operatorname{sen} bx - \frac{m}{b}\int x^{m-1}\operatorname{sen} bx\,dx \quad \text{l. q. q. d.}$$

32.

$$\int \frac{dx}{(1-x^2)^3}$$

Haciendo cambio: $x = \operatorname{sen}\theta$; diferenciando: $dx = \cos\theta\,d\theta$

$$\int \frac{dx}{(1-x^2)^3} = \int \frac{\cos\theta\,d\theta}{[1-(\operatorname{sen}\theta)^2]^3} = \int \frac{\cos\theta\,d\theta}{(1-\operatorname{sen}^2\theta)^3} = \int \frac{\cos\theta\,d\theta}{(\cos^2\theta)^3} = \int \frac{\cos\theta\,d\theta}{\cos^6\theta} = \int \frac{d\theta}{\cos^5\theta}$$
$$= \int \frac{1}{\cos^5\theta}d\theta = \int \sec^5\theta\,d\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (I)$$

La integral $\int \sec^5\theta\,d\theta$ la resolveremos por partes:

$\int \sec^5\theta\,d\theta = \int \sec^3\theta\sec^2\theta\,d\theta$

sea:

$$u = \sec^3\theta \qquad dv = \sec^2\theta\,d\theta$$
$$du = 3\sec^2\theta\sec\theta\,\operatorname{tg}\theta\,d\theta \qquad v = \operatorname{tg}\theta$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \sec^5\theta\,d\theta = \sec^3\theta\,\operatorname{tg}\theta - \int \operatorname{tg}\theta\cdot 3\sec^2\theta\sec\theta\,\operatorname{tg}\theta\,d\theta$$
$$\int \sec^5\theta\,d\theta = \sec^3\theta\,\operatorname{tg}\theta - 3\int \sec^3\theta\,\operatorname{tg}^2\theta\,d\theta$$
$$\int \sec^5\theta\,d\theta = \sec^3\theta\,\operatorname{tg}\theta - 3\int \sec^3\theta(\sec^2\theta - 1)\,d\theta$$
$$\int \sec^5\theta\,d\theta = \sec^3\theta\,\operatorname{tg}\theta - 3\int (\sec^5\theta - \sec^3\theta)\,d\theta$$
$$\int \sec^5\theta\,d\theta = \sec^3\theta\,\operatorname{tg}\theta - 3\int \sec^5\theta\,d\theta + 3\int \sec^3\theta\,d\theta$$
$$\int \sec^5\theta\,d\theta + 3\int \sec^5\theta\,d\theta = \sec^3\theta\,\operatorname{tg}\theta + 3\int \sec^3\theta\,d\theta$$
$$4\int \sec^5\theta\,d\theta = \sec^3\theta\,\operatorname{tg}\theta + 3\int \sec^3\theta\,d\theta$$
$$\int \sec^5\theta\,d\theta = \frac{1}{4}(\sec^3\theta\,\operatorname{tg}\theta + 3\int \sec^3\theta\,d\theta)$$
$$\int \sec^5\theta\,d\theta = \frac{1}{4}\sec^3\theta\,\operatorname{tg}\theta + \frac{3}{4}\int \sec^3\theta\,d\theta \;\ldots\ldots (II)$$

reemplazando (II) en (I)

$$\int \frac{dx}{(1-x^2)^5} = \frac{1}{4}\sec^3\theta\,\operatorname{tg}\theta + \frac{3}{4}\int \sec^3\theta\,d\theta \;\ldots\ldots\ldots\ldots\ldots\ldots (III)$$

resolviendo la integral $\int \sec^3\theta\,d\theta$ por partes:

$\int \sec^3\theta\,d\theta = \int \sec\theta\sec^2\theta\,d\theta$

sea:

$$u = \sec\theta \qquad dv = \sec^2\theta\,d\theta$$
$$du = \sec\theta\,\operatorname{tg}\theta\,d\theta \qquad v = \operatorname{tg}\theta$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \sec^3\theta\,d\theta = \sec\theta\,\operatorname{tg}\theta - \int \operatorname{tg}\theta\sec\theta\,\operatorname{tg}\theta\,d\theta$$
$$\int \sec^3\theta\,d\theta = \sec\theta\,\operatorname{tg}\theta - \int \sec\theta\,\operatorname{tg}^2\theta\,d\theta$$
$$\int \sec^3\theta\,d\theta = \sec\theta\,\operatorname{tg}\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta$$
$$\int \sec^3\theta\,d\theta = \sec\theta\,\operatorname{tg}\theta - \int (\sec^3\theta - \sec\theta)\,d\theta$$

$$\int \sec^3\theta\, d\theta = \sec\theta\, tg\theta - \int \sec^3\theta\, d\theta - \int \sec\theta\, d\theta$$

$$\int \sec^3\theta\, d\theta + \int \sec^3\theta\, d\theta = \sec\theta\, tg\theta + \int \sec\theta\, d\theta$$

$$2\int \sec^3\theta\, d\theta = \sec\theta\, tg\theta + \ln|\sec\theta + tg\theta|$$

$$\int \sec^3\theta\, d\theta = \frac{1}{2}\left[\sec\theta\, tg\theta + \ln|\sec\theta + tg\theta|\right] + c$$

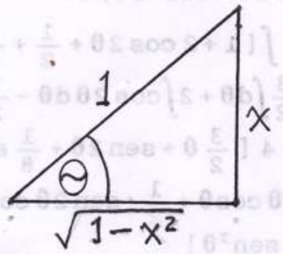
$$\int \sec^3\theta\, d\theta = \frac{1}{2}\sec\theta\, tg\theta + \frac{1}{2}\ln|\sec\theta + tg\theta| + c \ \ldots\ldots \text{(IV)}$$

reemplazando (IV) en (III)

$$\int \frac{dx}{(1-x^2)^2} = \frac{1}{4}\sec^3\theta\, tg\theta + \frac{3}{4}\left[\frac{1}{2}\sec\theta\, tg\theta + \frac{1}{2}\ln|\sec\theta + tg\theta|\right] + c$$

$$= \frac{1}{4}\sec^3\theta\, tg\theta + \frac{3}{8}\sec\theta\, tg\theta + \frac{3}{8}\ln|\sec\theta + tg\theta| + c \ \ldots\ldots \text{(V)}$$

de la gráfica obtenemos:



$$\sec\theta = \frac{1}{\sqrt{1-x^2}}$$

$$tg\theta = \frac{x}{\sqrt{1-x^2}}$$

sustituyendo en (V)

$$\int \frac{dx}{(1-x^2)^2} = \frac{1}{4}\left(\frac{1}{\sqrt{1-x^2}}\right)^3\left(\frac{x}{\sqrt{1-x^2}}\right) + \frac{3}{8}\left(\frac{1}{\sqrt{1-x^2}}\right)\left(\frac{x}{\sqrt{1-x^2}}\right) + \frac{3}{8}\ln\left|\frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}\right| + c$$

$$= \frac{x}{4\sqrt{(1-x^2)^3}\sqrt{1-x^2}} + \frac{3x}{8(\sqrt{1-x^2})^2} + \frac{3}{8}\ln\left|\frac{1+x}{\sqrt{1-x^2}}\right| + c$$

$$= \frac{x}{4(1-x^2)\sqrt{1-x^2}\sqrt{1-x^2}} + \frac{3x}{8(1-x^2)} + \frac{3}{8}\ln\left|\frac{1+x}{\sqrt{(1-x)(1+x)}}\right| + c$$

$$= \frac{x}{4(1-x^2)(\sqrt{1-x^2})^2} + \frac{3x}{8(1-x^2)} + \frac{3}{8}\ln\left|\frac{1+x}{\sqrt{1-x}\sqrt{1+x}}\right| + c$$

$$= \frac{x}{4(1-x^2)(1-x^2)} + \frac{3x}{8(1-x^2)} + \frac{3}{8}\ln\left|\frac{1+x}{(1-x)^{1/2}(1+x)^{1/2}}\right| + c$$

$$= \frac{x}{4(1-x^2)^2} + \frac{3x}{8(1-x^2)} + \frac{3}{8}\ln\left|\frac{(1+x)(1+x)^{-1/2}}{(1-x)^{1/2}}\right| + c$$

$$= \frac{2x + 3x(1-x^2)}{8(1-x^2)^2} + \frac{3}{8}\ln\left|\frac{(1+x)^{1/2}}{(1-x)^{1/2}}\right| + c = \frac{2x + 3x - 3x^3}{8(1-x^2)^2} + \frac{3}{8}\ln\left|\left(\frac{1+x}{1-x}\right)^{1/2}\right| + c$$

$$= \frac{5x - 3x^3}{8(1-x^2)^2} + \frac{3}{8}\cdot\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + c = \frac{x(5-3x^2)}{8(1-x^2)^2} + \frac{3}{16}\ln\left|\frac{1+x}{1-x}\right| + c$$

33.

$$\int \frac{dx}{(4+x^2)^{3/2}}$$

$$\int \frac{dx}{(4+x^2)^{3/2}} = \int \frac{dx}{\sqrt{(4+x^2)^3}} = \int \frac{dx}{(4+x^2)\sqrt{4+x^2}} = \int \frac{dx}{(2^2+x^2)\sqrt{2^2+x^2}}$$

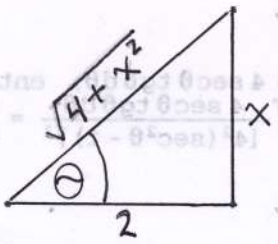
Hacemos el cambio: $x = 2\,tg\theta$; diferenciando: $dx = 2\sec^2\theta\, d\theta$; entonces:

$$\int \frac{dx}{(4+x^2)^{3/2}} = \int \frac{2\sec^2\theta\, d\theta}{[2^2 + (2\,tg\theta)^2]\sqrt{2^2 + (2\,tg\theta)^2}} = \int \frac{2\sec^2\theta\, d\theta}{(4 + 4\,tg^2\theta)\sqrt{4 + 4\,tg^2\theta}}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{4(1 + tg^2\theta)\sqrt{4(1 + tg^2\theta)}} = \int \frac{\sec^2\theta\, d\theta}{2\sec^2\theta\sqrt{4\sec^2\theta}} = \frac{1}{2}\int \frac{d\theta}{2\sec\theta}$$

$$= \frac{1}{4}\int \cos\theta\, d\theta = \frac{1}{4}\operatorname{sen}\theta + c \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

del gráfico tenemos:



$$2\,tg\theta = x$$

$$tg\theta = \frac{x}{2}$$

$$\operatorname{sen}\theta = \frac{x}{\sqrt{4+x^2}}$$

reemplazando en (I)

$$\int \frac{dx}{(4+x^2)^{3/2}} = \frac{1}{4}\cdot\frac{x}{\sqrt{4+x^2}} + c = \frac{x}{4\sqrt{4+x^2}} + c$$

34.

$$\int (4-x^2)^{3/2}\,dx$$

$$\int (4-x^2)^{3/2} = \int \sqrt{(4-x^2)^3}\,dx = \int (4-x^2)\sqrt{4-x^2}\,dx = \int (2^2-x^2)\sqrt{2^2-x^2}\,dx$$

Hacemos el cambio: $x = 2\,\text{sen}\theta$; diferenciando: $dx = 2\cos\theta\,d\theta$; entonces:

$$\int (4-x^2)^{3/2}dx = \int [2^2-(2\,\text{sen}\theta)^2]\sqrt{2^2-(2\,\text{sen}\theta)^2}\cdot 2\cos\theta\,d\theta = 2\int (4-4\,\text{sen}^2\theta)\sqrt{4-4\,\text{sen}^2\theta}\cdot\cos\theta\,d\theta$$

$$= 2\int 4(1-\text{sen}^2\theta)\sqrt{4(1-\text{sen}^2\theta)}\;\cos\theta\,d\theta = 8\int \cos^2\theta\cdot 2\;\sqrt{\cos^2\theta}\;\cos\theta\,d\theta$$

$$= 16\int \cos^3\theta\cdot\cos\theta\,d\theta = 16\int \cos^4\theta\,d\theta = 16\int (\cos^2\theta)^2\,d\theta = 16\int [\tfrac{1}{2}(1+\cos 2\theta)]^2\,d\theta$$

$$= 16\int \frac{1}{4}\,(1+\cos 2\theta)^2\,d\theta = \frac{16}{4}\int (1+2\cos 2\theta+\cos^2 2\theta)\,d\theta$$

$$= 4\int [1+2\cos 2\theta+\tfrac{1}{2}(1+\cos 4\theta)]\,d\theta = 4\int [1+2\cos 2\theta+\tfrac{1}{2}+\tfrac{1}{2}\cos 4\theta]\,d\theta$$

$$= 4\int [\tfrac{3}{2}+2\cos 2\theta+\tfrac{1}{2}\cos 4\theta]\,d\theta = 4\,[\tfrac{3}{2}\int d\theta+2\int\cos 2\theta\,d\theta-\tfrac{1}{2}\int\cos 4\theta\,d\theta]$$

$$= 4\,[\tfrac{3}{2}\theta+2\cdot\tfrac{1}{2}\,\text{sen}\,2\theta+\tfrac{1}{2}\cdot\tfrac{1}{4}\,\text{sen}\,4\theta] = 4\,[\tfrac{3}{2}\theta+\text{sen}\,2\theta+\tfrac{1}{8}\,\text{sen}\,4\theta)$$

$$= 6\theta+4\,\text{sen}2\theta+\frac{1}{2}\,\text{sen}\,4\theta = 6\theta+4\cdot 2\,\text{sen}\theta\cos\theta+\frac{1}{2}\cdot\text{sen}\,2\theta\cos 2\theta$$

$$= 6\theta+8\,\text{sen}\theta\cos\theta+2\,\text{sen}\theta\cos\theta\,[\cos^2\theta-\text{sen}^2\theta]$$

$$= 6\theta+8\,\text{sen}\theta\cos\theta+2\,\text{sen}\theta\cos\theta\,[(\cos\theta)^2-(\text{sen}\theta)^2]$$

del gráfico tenemos

$$2\,\text{sen}\theta = x$$

$$\text{sen}\theta = \frac{x}{2}$$

$$\theta = \text{arcsen}\,\frac{x}{2}$$

$$\cos\theta = \frac{\sqrt{4-x^2}}{2}$$

$$\int (4-x^2)^{3/2}dx = 6\,\text{arcsen}\,\frac{x}{2}+8\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}+2\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\;[\,(\frac{\sqrt{4-x^2}}{2})^2-(\frac{x}{2})^2]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+\frac{8}{4}\,x\sqrt{4-x^2}+\frac{2}{4}\,x\sqrt{4-x^2}\,[\,\frac{4-x^2}{4}-\frac{x^2}{4}\,]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+2x\sqrt{4-x^2}+\frac{1}{2}\,x\sqrt{4-x^2}\,[\,\frac{4-x^2-x^2}{4}\,]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+2x\sqrt{4-x^2}+\frac{1}{2}\,x\sqrt{4-x^2}\,[\,\frac{4-2x^2}{4}\,]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+2x\sqrt{4-x^2}+\frac{1}{2}\,x\sqrt{4-x^2}\,[\,\frac{2(2-x^2)}{4}\,]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+2x\sqrt{4-x^2}+\frac{1}{4}\,x\,(2-x^2)\sqrt{4-x^2}+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+x\sqrt{4-x^2}\,[\,2+\frac{2-x^2}{4}\,]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+x\sqrt{4-x^2}\,[\,\frac{8+2-x^2}{4}\,]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+x\sqrt{4-x^2}\,[\,\frac{10-x^2}{4}\,]+c$$

$$= 6\,\text{arcsen}\,\frac{x}{2}+\frac{1}{4}\,x\,(10-x^2)\sqrt{4-x^2}+c$$

35.

$$\int \frac{dx}{(x^2-16)^3}$$

$$\int \frac{dx}{(x^2-16)^3} = \int \frac{dx}{(x^2-4^2)^3}$$

Hacemos el cambio: $x = 4\sec\theta$; diferenciando: $dx = 4\sec\theta\,\text{tg}\theta\,d\theta$; entonces:

$$\int \frac{dx}{(x^2-16)^3} = \int \frac{4\sec\theta\,\text{tg}\theta\,d\theta}{[(4\sec\theta)^2-4^2]^3} = \int \frac{4\sec\theta\,\text{tg}\theta\,d\theta}{(4^2\sec^2\theta-4^2)^3} = \int \frac{4\sec\theta\,\text{tg}\theta\,d\theta}{[4^2(\sec^2\theta-1)]^3} = \int \frac{4\sec\theta\,\text{tg}\theta\,d\theta}{(4^2\cdot\text{tg}^2\theta)^3}$$

$$\int\frac{dx}{(x^2-16)^3}=\int\frac{4\sec\theta\, tg\theta\, d\theta}{4^6\, tg^6\theta}=\int\frac{\sec\theta\, d\theta}{4^5\, tg^5\theta}=\frac{1}{4^5}\int\frac{\frac{1}{\cos\theta}}{\frac{sen^5\theta}{\cos^5\theta}}d\theta=\frac{1}{1024}\int\frac{\cos^5\theta}{sen^5\theta\cos\theta}d\theta$$

$$=\frac{1}{1024}\int\frac{\cos^4\theta}{sen^4\theta\, sen\theta}d\theta=\frac{1}{1024}\int\frac{\cos^4\theta}{sen^4\theta}\cdot\frac{1}{sen\theta}d\theta=\frac{1}{1024}\int ctg^4\theta\csc\theta\, d\theta\ \dots\ (I)$$

resolveremos la integral $\int ctg^4\theta\csc\theta\, d\theta$ por partes:

$$\int ctg^4\theta\csc\theta\, d\theta=\int ctg^3\theta\, ctg\theta\csc\theta\, d\theta$$

sea:

$$u=ctg^3\theta \qquad dv=\csc\theta\, ctg\theta\, d\theta$$
$$du=3\,ctg^2\theta\,(-\csc^2\theta)\,d\theta \qquad \int dv=\int\csc\theta\, ctg\theta\, d\theta$$
$$du=-3\,ctg^2\theta\csc^2\theta\, d\theta \qquad v=-\csc\theta$$

$$\int u\,dv=uv-\int v\,du$$

$$\int ctg^4\theta\csc\theta\, d\theta=ctg^3\theta(-\csc\theta)-\int(-\csc\theta)(-3\,ctg^2\theta\csc^2\theta\, d\theta)$$
$$\int ctg^4\theta\csc\theta\, d\theta=-\csc\theta\, ctg^3\theta-3\int\csc^2\theta\, ctg^2\theta\csc\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta=-\csc\theta\, ctg^3\theta-3\int(1+ctg^2\theta)\, ctg^2\theta\csc\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta=-\csc\theta\, ctg^3\theta-3\int ctg^2\theta\csc\theta\, d\theta-3\int ctg^4\theta\csc\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta+3\int ctg^4\theta\csc\theta\, d\theta=-\csc\theta\, ctg^3\theta-3\int ctg^2\theta\csc\theta\, d\theta$$
$$4\int ctg^4\theta\csc\theta\, d\theta=-\csc\theta\, ctg^3\theta-3\int ctg^2\theta\, scs\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta=-\frac{1}{4}\csc\theta\, ctg^3\theta-\frac{3}{4}\int ctg^2\theta\csc\theta\, d\theta\ \dots\dots\dots\ (II)$$

resolveremos la integral $\int ctg^2\theta\csc\theta\, d\theta$ por partes:

$$\int ctg^2\theta\csc\theta\, d\theta=\int ctg\theta\, ctg\theta\csc\theta\, d\theta=\int ctg\theta\, scc\theta\, ctg\theta\, d\theta$$

sea:

$$u=ctg\theta \qquad dv=\csc\theta\, ctg\theta\, d\theta$$
$$du=-\csc^2\theta \qquad v=-\csc\theta$$

$$\int u\,dv=uv-\int v\,du$$

$$\int ctg^2\theta\csc\theta\, d\theta=ctg\theta(-\csc\theta)-\int(-\csc\theta)(-\csc^2\theta)\,d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta=-\csc\theta\, ctg\theta-\int\csc^2\theta\csc\theta\, d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta=-\csc\theta\, ctg\theta-\int(1+ctg^2\theta)\csc\theta\, d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta=-\csc\theta ctg\theta-\int\csc\theta\, d\theta-\int ctg^2\theta\csc\theta\, d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta+\int ctg^2\theta\csc\theta\, d\theta=-\csc\theta\, ctg\theta-\int\csc\theta\, d\theta$$
$$2\int ctg^2\theta\csc\theta\, d\theta=-\csc\theta\, ctg\theta-\ln|\csc\theta-ctg\theta|$$
$$\int ctg^2\theta\csc\theta\, d\theta=-\frac{1}{2}\csc\theta\, ctg\theta-\frac{1}{2}\ln|\csc\theta-ctg\theta|\ \dots\dots\ (III)$$

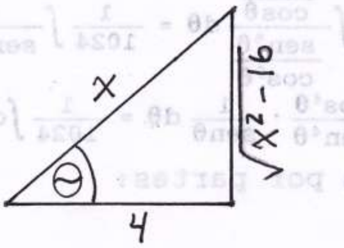
reemplazando (III) en (II)

$$\int ctg^4\theta\csc\theta\, d\theta=-\frac{1}{4}\csc\theta\, ctg^3\theta-\frac{3}{4}[-\frac{1}{2}\csc\theta\, ctg\theta-\frac{1}{2}\ln|\csc\theta-ctg\theta|]$$
$$=-\frac{1}{4}\csc\theta\, ctg^3\theta+\frac{3}{8}\csc\theta\, ctg\theta+\frac{3}{8}\ln|\csc\theta-ctg\theta|\ \dots\ (IV)$$

reemplazando (IV) en (I)

$$\int\frac{dx}{(x^2-16)^3}=\frac{1}{1024}\left[-\frac{1}{4}\csc\theta\, ctg^3\theta+\frac{3}{8}\csc\theta\, ctg\theta+\frac{3}{8}\ln|\csc\theta-ctg\theta|\right]+c$$

del gráfico tenemos



$$4\sec\theta = x \qquad \sec\theta = \frac{x}{4} \qquad \csc\theta = \frac{x}{\sqrt{x^2-16}} \qquad \mathrm{ctg}\,\theta = \frac{4}{\sqrt{x^2-16}}$$

$$\int \frac{dx}{(x^2-16)^3} = \frac{1}{1024}\left[-\frac{1}{4}\left(\frac{x}{\sqrt{x^2-16}}\right)\left(\frac{4}{\sqrt{x^2-16}}\right)^3 + \frac{3}{8}\left(\frac{x}{\sqrt{x^2-16}}\right)\left(\frac{4}{\sqrt{x^2-16}}\right) + \frac{3}{8}\ln\left|\frac{x}{\sqrt{x^2-16}} - \frac{4}{\sqrt{x^2-16}}\right|\right]$$

$$= \frac{1}{1024}\left[-\frac{64x}{4\sqrt{x^2-16}\sqrt{(x^2-16)^3}} + \frac{12x}{8(\sqrt{x^2-16})^2} + \frac{3}{8}\ln\left|\frac{x-4}{\sqrt{x^2-16}}\right|\right] + c$$

$$= \frac{1}{1024}\left[-\frac{16x}{\sqrt{x^2-16}\,(x^2-16)\sqrt{x^2-16}} + \frac{3x}{2(x^2-16)} + \frac{3}{8}\ln\left|\frac{x-4}{\sqrt{(x-4)(x+4)}}\right|\right] + c$$

$$= \frac{1}{1024}\left[-\frac{16x}{(x^2-16)(\sqrt{x^2-16})^2} + \frac{3x}{2(x^2-16)} + \frac{3}{8}\ln\left|\frac{x-4}{\sqrt{x-4}\sqrt{x+4}}\right|\right] + c$$

$$= \frac{1}{1024}\left[-\frac{16x}{(x^2-16)(x^2-16)} + \frac{3x}{2(x^2-16)} + \frac{3}{8}\ln\left|\frac{x-4}{(x-4)^{1/2}(x+4)^{1/2}}\right|\right] + c$$

$$= \frac{1}{1024}\left[-\frac{16x}{(x^2-16)^2} + \frac{3x}{2(x^2-16)} + \frac{3}{8}\ln\left|\frac{(x-4)(x-4)^{-1/2}}{(x+4)^{1/2}}\right|\right] + c$$

$$= \frac{1}{1024}\left[\frac{-32x+3x(x^2-16)}{2(x^2-16)^2} + \frac{3}{8}\ln\left|\frac{(x-4)^{1/2}}{(x+4)^{1/2}}\right|\right] + c$$

$$= \frac{1}{1024}\left[\frac{-32x+3x^3-48x}{2(x^2-16)^2} + \frac{3}{8}\ln\left|\left(\frac{x-4}{x+4}\right)^{1/2}\right|\right] + c$$

$$= \frac{1}{1024}\left[\frac{3x^3-80x}{2(x^2-16)^2} + \frac{3}{8}\cdot\frac{1}{2}\ln\left|\frac{x-4}{x+4}\right|\right] + c$$

$$= \frac{1}{1024}\cdot\frac{1}{2}\left[\frac{x(3x^2-80)}{x^2-16} + \frac{3}{8}\ln\left|\frac{x-4}{x+4}\right|\right] + c$$

$$= \frac{1}{2048}\left[\frac{x(3x^2-80)}{x^2-16} + \frac{3}{8}\ln\left|\frac{x-4}{x+4}\right|\right] + c$$

36.

$$\int (x^2-1)^{5/2}\,dx$$

$$\int (x^2-1)^{5/2}\,dx = \int\sqrt{(x^2-1)^5} = \int (x^2-1)^2\sqrt{x^2-1}\,dx$$

Hacemos el cambio: $x = \sec\theta$; **diferenciando:** $dx = \sec\theta\,\mathrm{tg}\,\theta\,d\theta$; **por lo tanto:**

$$\int (x^2-1)^{5/2}\,dx = \int[(\sec\theta)^2-1]^2\sqrt{(\sec\theta)^2-1}\cdot\sec\theta\,\mathrm{tg}\,\theta\,d\theta = \int(\sec^2\theta-1)^2\sqrt{\sec^2\theta-1}\cdot\sec\theta\,\mathrm{tg}\,\theta\,d\theta$$

$$= \int(\mathrm{tg}^2\theta)^2\sqrt{\mathrm{tg}^2\theta}\,\sec\theta\,\mathrm{tg}\,\theta\,d\theta = \int\mathrm{tg}^4\theta\,\mathrm{tg}\,\theta\sec\theta\,\mathrm{tg}\,\theta\,d\theta = \int\mathrm{tg}^6\theta\sec\theta\,d\theta \;\ldots\ldots\; \text{(I)}$$

integraremos la integral $\int\mathrm{tg}^6\theta\sec\theta\,d\theta$ por partes:

$$\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \int\mathrm{tg}^5\theta\,\mathrm{tg}\,\theta\sec\theta\,d\theta = \int\mathrm{tg}^5\theta\sec\theta\,\mathrm{tg}\,\theta\,d\theta$$

sea:

$$u = \mathrm{tg}^5\theta \qquad dv = \sec\theta\,\mathrm{tg}\,\theta\,d\theta$$
$$du = 5\,\mathrm{tg}^4\theta\sec^2\theta\,d\theta \qquad v = \sec\theta$$

$$\int u\,dv = uv - \int v\,du$$

$$\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \mathrm{tg}^5\theta\cdot\sec\theta - \int\sec\theta\cdot 5\,\mathrm{tg}^4\theta\sec^2\theta\,d\theta$$

$$\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \mathrm{tg}^5\theta\sec\theta - 5\int\sec^2\theta\,\mathrm{tg}^4\theta\sec\theta\,d\theta$$

$$\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \mathrm{tg}^5\theta\sec\theta - 5\int(\mathrm{tg}^2\theta+1)\,\mathrm{tg}^4\theta\sec\theta\,d\theta$$

$$\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \mathrm{tg}^5\theta\sec\theta - 5\int\mathrm{tg}^6\theta\sec\theta\,d\theta - 5\int\mathrm{tg}^4\theta\sec\theta\,d\theta$$

$$\int\mathrm{tg}^6\theta\sec\theta\,d\theta + 5\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \mathrm{tg}^5\theta\sec\theta - 5\int\mathrm{tg}^4\theta\sec\theta\,d\theta$$

$$6\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \mathrm{tg}^5\theta\sec\theta - 5\int\mathrm{tg}^4\theta\sec\theta\,d\theta$$

$$\int\mathrm{tg}^6\theta\sec\theta\,d\theta = \frac{1}{6}\,\mathrm{tg}^5\theta\sec\theta - \frac{5}{6}\int\mathrm{tg}^4\theta\sec\theta\,d\theta \;\ldots\ldots\ldots\; \text{(II)}$$

$$\int \frac{dx}{(x^2-16)^3} = \int \frac{4\sec\theta\, tg\theta\, d\theta}{4^6\, tg^6\theta} = \int \frac{\sec\theta\, d\theta}{4^5\, tg^5\theta} = \frac{1}{4^5}\int \frac{\frac{1}{\cos\theta}}{\frac{sen^5\theta}{\cos^5\theta}}\, d\theta = \frac{1}{1024}\int \frac{\cos^5\theta}{sen^5\theta\cos\theta}\, d\theta$$

$$= \frac{1}{1024}\int \frac{\cos^4\theta}{sen^4\theta\, sen\theta}\, d\theta = \frac{1}{1024}\int \frac{\cos^4\theta}{sen^4\theta}\cdot\frac{1}{sen\theta}\, d\theta = \frac{1}{1024}\int ctg^4\theta\csc\theta\, d\theta \; \ldots \text{(I)}$$

resolveremos la integral $\int ctg^4\theta\csc\theta\, d\theta$ por partes:

$$\int ctg^4\theta\csc\theta\, d\theta = \int ctg^3\theta\, ctg\theta\csc\theta\, d\theta$$

sea:

$$u = ctg^3\theta \qquad\qquad dv = \csc\theta\, ctg\theta\, d\theta$$
$$du = 3\, ctg^2\theta\,(-\csc^2\theta)\, d\theta \qquad\qquad \int dv = \int \csc\theta\, ctg\theta\, d\theta$$
$$du = -3\, ctg^2\theta\csc^2\theta\, d\theta \qquad\qquad v = -\csc\theta$$

$$\int u\, dv = u\, v - \int v\, du$$

$$\int ctg^4\theta\csc\theta\, d\theta = ctg^3\theta\,(-\csc\theta) - \int(-\csc\theta)\,(-3\, ctg^2\theta\csc^2\theta\, d\theta)$$
$$\int ctg^4\theta\csc\theta\, d\theta = -\csc\theta\, ctg^3\theta - 3\int \csc^2\theta\, ctg^2\theta\csc\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta = -\csc\theta\, ctg^3\theta - 3\int (1+ctg^2\theta)\, ctg^2\theta\csc\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta = -\csc\theta\, ctg^3\theta - 3\int ctg^2\theta\csc\theta\, d\theta - 3\int ctg^4\theta\csc\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta + 3\int ctg^4\theta\csc\theta\, d\theta = -\csc\theta\, ctg^3\theta - 3\int ctg^2\theta\csc\theta\, d\theta$$
$$4\int ctg^4\theta\csc\theta\, d\theta = -\csc\theta\, ctg^3\theta - 3\int ctg^2\theta\, scs\theta\, d\theta$$
$$\int ctg^4\theta\csc\theta\, d\theta = -\frac{1}{4}\csc\theta\, ctg^3\theta - \frac{3}{4}\int ctg^2\theta\csc\theta\, d\theta \;\ldots\ldots\ldots \text{(II)}$$

resolveremos la integral $\int ctg^2\theta\csc\theta\, d\theta$ por partes:

$$\int ctg^2\theta\csc\theta\, d\theta = \int ctg\theta\, ctg\theta\csc\theta\, d\theta = \int ctg\theta\, scc\theta\, ctg\theta\, d\theta$$

sea:

$$u = ctg\theta \qquad\qquad dv = \csc\theta\, ctg\theta\, d\theta$$
$$du = -\csc^2\theta \qquad\qquad v = -\csc\theta$$

$$\int u\, dv = u\, v - \int v\, du$$

$$\int ctg^2\theta\csc\theta\, d\theta = ctg\theta\,(-\csc\theta) - \int(-\csc\theta)\,(-\csc^2\theta)\, d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta = -\csc\theta\, ctg\theta - \int \csc^2\theta\csc\theta\, d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta = -\csc\theta\, ctg\theta - \int (1+ctg^2\theta)\csc\theta\, d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta = -\csc\theta\, ctg\theta - \int \csc\theta\, d\theta - \int ctg^2\theta\csc\theta\, d\theta$$
$$\int ctg^2\theta\csc\theta\, d\theta + \int ctg^2\theta\csc\theta\, d\theta = -\csc\theta\, ctg\theta - \int \csc\theta\, d\theta$$
$$2\int ctg^2\theta\csc\theta\, d\theta = -\csc\theta\, ctg\theta - \ln|\csc\theta - ctg\theta|$$
$$\int ctg^2\theta\csc\theta\, d\theta = -\frac{1}{2}\csc\theta\, ctg\theta - \frac{1}{2}\ln|\csc\theta - ctg\theta| \;\ldots\ldots \text{(III)}$$

reemplazando (III) en (II)

$$\int ctg^4\theta\csc\theta\, d\theta = -\frac{1}{4}\csc\theta\, ctg^3\theta - \frac{3}{4}\left[-\frac{1}{2}\csc\theta\, ctg\theta - \frac{1}{2}\ln|\csc\theta - ctg\theta|\right]$$
$$= -\frac{1}{4}\csc\theta\, ctg^3\theta + \frac{3}{8}\csc\theta\, ctg\theta + \frac{3}{8}\ln|\csc\theta - ctg\theta| \;\ldots \text{(IV)}$$

reemplazando (IV) en (I)

$$\int \frac{dx}{(x^2-16)^3} = \frac{1}{1024}\left[-\frac{1}{4}\csc\theta\, ctg^3\theta + \frac{3}{8}\csc\theta\, ctg\theta + \frac{3}{8}\ln|\csc\theta - ctg\theta|\right] + c$$

reemplazando (II) en (I)

$$\int (x^2-1)^{5/2}dx = \frac{1}{6}\,tg^5\theta\sec\theta - \frac{5}{6}\int tg^4\theta\sec\theta\,d\theta \;\ldots\ldots\ldots\ldots\ldots\ldots\ (III)$$

integraremos la integral $\int tg^4\theta\sec\theta\,d\theta$ por partes:

$$\int tg^4\theta\sec\theta\,d\theta = \int tg^3\theta\,tg\theta\sec\theta\,d\theta = \int tg^3\theta\sec\theta\,tg\theta\,d\theta$$

sea: $u = tg^3\theta$ $\quad dv = \sec\theta\,tg\theta\,d\theta$

$du = 3\,tg^2\theta\sec^2\theta\,d\theta$ $\quad v = \sec\theta$

$$\int u\,dv = uv - \int v\,du$$

$$\int tg^4\theta\sec\theta\,d\theta = tg^3\theta\sec\theta - \int \sec\theta\cdot 3\,tg^2\theta\sec^2\theta\,d\theta$$

$$\int tg^4\theta\sec\theta\,d\theta = tg^3\theta\sec\theta - 3\int \sec^2\theta\,tg^2\theta\sec\theta\,d\theta$$

$$\int tg^4\theta\sec\theta\,d\theta = tg^3\theta\sec\theta - 3\int (tg^2\theta+1)\,tg^2\theta\sec\theta\,d\theta$$

$$\int tg^4\theta\sec\theta\,d\theta = tg^3\theta\sec\theta - 3\int tg^4\theta\sec\theta\,d\theta - 3\int tg^2\theta\sec\theta\,d\theta$$

$$\int tg^4\theta\sec\theta\,d\theta + 3\int tg^4\theta\sec\theta\,d\theta = tg^3\theta\sec\theta - 3\int tg^2\theta\sec\theta\,d\theta$$

$$4\int tg^4\theta\sec\theta\,d\theta = tg^3\theta\sec\theta - 3\int tg^2\theta\sec\theta\,d\theta$$

$$\int tg^4\theta\sec\theta\,d\theta = \frac{1}{4}\,tg^3\theta\sec\theta - \frac{3}{4}\int tg^2\theta\sec\theta\,d\theta \;\ldots\ldots\ (IV)$$

reemplazando (IV) en (III)

$$\int (x^2-1)^{5/2}dx = \frac{1}{6}\,tg^5\theta\sec\theta - \frac{5}{6}\left[\frac{1}{4}\,tg^3\theta\sec\theta - \frac{3}{4}\int tg^2\theta\sec\theta\,d\theta\right]$$

$$= \frac{1}{6}\,tg^5\theta\sec\theta - \frac{5}{24}\,tg^3\theta\sec\theta + \frac{5}{8}\int tg^2\theta\sec\theta\,d\theta \;\ldots\ (V)$$

integraremos la integral $\int tg^2\theta\sec\theta\,d\theta$ por partes:

$$\int tg^2\theta\sec\theta\,d\theta = \int tg\theta\,tg\theta\sec\theta\,d\theta = \int tg\theta\sec\theta\,tg\theta\,d\theta$$

sea: $u = tg\theta$ $\quad dv = \sec\theta\,tg\theta\,d\theta$

$du = \sec^2\theta\,d\theta$ $\quad v = \sec\theta$

$$\int u\,dv = uv - \int u\,dv$$

$$\int tg^2\theta\sec\theta\,d\theta = tg\theta\sec\theta - \int \sec\theta\cdot\sec^2\theta\,d\theta$$

$$\int tg^2\theta\sec\theta\,d\theta = tg\theta\sec\theta - \int \sec^2\theta\sec\theta\,d\theta$$

$$\int tg^2\theta\sec\theta\,d\theta = tg\theta\sec\theta - \int (tg^2\theta+1)\sec\theta\,d\theta$$

$$\int tg^2\theta\sec\theta\,d\theta = tg\theta\sec\theta - \int tg^2\theta\sec\theta\,d\theta - \int \sec\theta\,d\theta$$

$$\int tg^2\theta\sec\theta\,d\theta + \int tg^2\theta\sec\theta\,d\theta = tg\theta\sec\theta - \int \sec\theta\,d\theta$$

$$2\int tg^2\theta\sec\theta\,d\theta = tg\theta\sec\theta - \ln|\sec\theta + tg\theta|$$

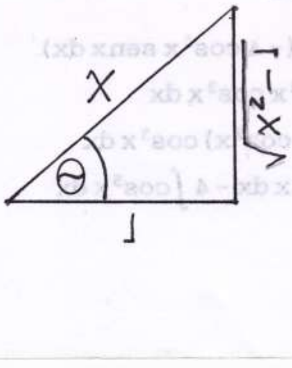
$$\int tg^2\theta\sec\theta\,d\theta = \frac{1}{2}\,tg\theta\sec\theta - \frac{1}{2}\ln|\sec\theta + tg\theta| \;\ldots\ (VI)$$

reemplazando (VI) en (V)

$$\int (x^2-1)^{5/2}dx = \frac{1}{6}\,tg^5\theta\sec\theta - \frac{5}{24}\,tg^3\theta\sec\theta + \frac{5}{8}\left[\frac{1}{2}\,tg\theta\sec\theta - \frac{1}{2}\ln|\sec\theta + tg\theta|\right] + c$$

$$= \frac{1}{6}\,tg^5\theta\sec\theta - \frac{5}{25}\,tg^3\theta\sec\theta + \frac{5}{16}\,tg\theta\sec\theta - \frac{5}{16}\ln|\sec\theta + tg\theta| + c \;\ldots\ (VII)$$

del gráfico tenemos



$\sec\theta = x$

$\sec\theta = \dfrac{x}{1}$

$tg\theta = \dfrac{\sqrt{x^2-1}}{1}$

$tg\theta = \sqrt{x^2-1}$

remplazando en (VII)

$$\int (x^2-1)^{5/2}dx = \frac{1}{6}(\sqrt{x^2-1})^5\cdot x - \frac{5}{24}(\sqrt{x^2-1})^3\cdot x + \frac{5}{16}\cdot\sqrt{x^2-1}\cdot x - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

$$= \frac{1}{6}x\sqrt{(x^2-1)^5} - \frac{5}{24}x\sqrt{(x^2-1)^3} + \frac{5}{16}x\sqrt{x^2-1} - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

$$= \frac{1}{6}x(x^2-1)^2\sqrt{x^2-1} - \frac{5}{24}x(x^2-1)\sqrt{x^2-1} + \frac{5}{16}x\sqrt{x^2-1} - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

$$= x\sqrt{x^2-1}\,[\frac{1}{6}(x^2-1)^2 - \frac{5}{24}(x^2-1) + \frac{5}{16}] - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

$$= x\sqrt{x^2-1}\,[\frac{8(x^2-1)^2-10(x^2-1)+15)}{48}] - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

$$= \frac{1}{48}x\sqrt{x^2-1}\,[8(x^4-2x^2+1)-10(x^2-1)+15] - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

$$= \frac{1}{48}x\sqrt{x^2-1}\,(8x^4-16x^2+8-10x^2+10+15) - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

$$= \frac{1}{48}x\sqrt{x^2-1}\,(8x^4-26x^2+33) - \frac{5}{16}\ln|x+\sqrt{x^2-1}|+c$$

37.

$\int sen^4x\,dx$

$\int sen^4x\,dx = \int sen^3x\,senx\,dx$

integrando por partes:

sea:

$u = sen^3x \qquad dv = senx\,dx$

$du = 3\,sen^2x\cos x\,dx \qquad v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$\int sen^4x\,dx = sen^3x(-\cos x) - \int -\cos x\cdot 3\,sen^2x\cos x\,dx$$

$$\int sen^4x\,dx = -sen^2x\cos x + 3\int sen^2x\cos^2x\,dx$$

$$\int sen^4x\,dx = -sen^2x\cos x + 3\int sen^2x(1-sen^2x)\,dx$$

$$\int sen^4x\,dx = -sen^2x\cos x + 3\int sen^2x\,dx - 3\int sen^4x\,dx$$

$$\int sen^4x\,dx + 3\int sen^4x\,dx = -sen^2x\cos x + 3\int \frac{1}{2}(1-\cos 2x)\,dx$$

$$4\int sen^4x\,dx = -sen^2x\cos x + \frac{3}{2}\int(1-\cos 2x)\,dx$$

$$\int sen^4x\,dx = \frac{1}{4}[-sen^2x\cos x + \frac{3}{2}\int dx - \frac{3}{2}\int\cos 2x\,dx]$$

$$\int sen^4x\,dx = \frac{1}{4}[-sen^2x\cos x + \frac{3}{2}x - \frac{3}{2}\cdot\frac{1}{2}sen\,2x] + c$$

$$\int sen^4x\,dx = -\frac{1}{4}sen^2x\cos x + \frac{3}{8}x - \frac{3}{16}sen\,2x + c$$

$$\int sen^4x\,dx = -\frac{1}{4}sen^2x\cos x + \frac{3}{8}x - \frac{3}{16}\cdot 2\,senx\cos x + c$$

$$\int sen^4x\,dx = -\frac{1}{4}sen^2x\cos x + \frac{3}{8}x - \frac{3}{8}senx\cos x + c$$

38.

$\int \cos^5x\,dx$

$\int \cos^5x\,dx = \int \cos^4x\cos x\,dx$

integrando por partes:

$u = \cos^4x \qquad dv = \cos x\,dx$

$du = 4\cos^3x(-senx)\,dx$

$du = -4\cos^3x\,senx\,dx \qquad v = senx$

$$\int u\,dv = uv - \int v\,du$$

$$\int \cos^5x\,dx = \cos^4x\,senx - \int senx(-4\cos^3x\,senx\,dx)$$

$$\int \cos^5x\,dx = senx\cos^4x + 4\int sen^2x\cos^3x\,dx$$

$$\int \cos^5x\,dx = senx\cos^4x + 4\int(1-\cos^2x)\cos^3x\,dx$$

$$\int \cos^5x\,dx = senx\cos^4x + 4\int\cos^3x\,dx - 4\int\cos^5x\,dx$$

$$\int \cos^5 x\,dx + 4\int \cos^5 x\,dx = \text{sen}x\cos^4 x + 4\int \cos^3 x\,dx$$

$$5\int \cos^5 x\,dx = \text{sen}x\cos^4 x + 4\int \cos^3 x\,dx$$

$$\int \cos^5 x\,dx = \frac{1}{5}\,\text{sen}x\cos^4 x + \frac{4}{5}\int \cos^3 x\,dx \;\ldots\ldots\ldots\; (I)$$

resolveremos la integral $\int \cos^3 x\,dx$ por partes:

$$\int \cos^3 x\,dx = \int \cos^2 x\cos x\,dx$$

sea:

$u = \cos^2 x \qquad dv = \cos x\,dx$

$du = 2\cos x\,(-\text{sen}x)\,dx$

$du = -2\cos x\,\text{sen}x\,dx \qquad v = \text{sen}x$

$$\int u\,dv = uv - \int v\,du$$

$$\int \cos^3 x\,dx = \cos^2 x\,\text{sen}x - \int \text{sen}x\,(-2\cos x\,\text{sen}x\,dx)$$

$$\int \cos^3 x\,dx = \text{sen}x\cos^2 x + 2\int \text{sen}^2 x\cos x\,dx$$

$$\int \cos^3 x\,dx = \text{sen}x\cos^2 x + 2\int (1-\cos^2 x)\cos x\,dx$$

$$\int \cos^3 x\,dx = \text{sen}x\cos^2 x + 2\int \cos x\,dx - 2\int \cos^3 x\,dx$$

$$\int \cos^3 x\,dx + 2\int \cos^3 x\,dx = \text{sen}x\cos^2 x + 2\int \cos x\,dx$$

$$3\int \cos^3 x\,dx = \text{sen}x\cos^2 x + 2\,\text{sen}x$$

$$\int \cos^3 x\,dx = \frac{1}{3}\,\text{sen}x\cos^2 x + \frac{2}{3}\,\text{sen}x \;\ldots\ldots\ldots\; (II)$$

reemplazando (II) en (I)

$$\int \cos^5 x\,dx = \frac{1}{5}\,\text{sen}x\cos^4 x + \frac{4}{5}\left[\frac{1}{3}\,\text{sen}x\cos^2 x + \frac{2}{3}\,\text{sen}x\right] + c$$

$$= \frac{1}{5}\,\text{sen}x\cos^4 x + \frac{4}{15}\,\text{sen}x\cos^2 x + \frac{8}{15}\,\text{sen}x + c$$

$$= \text{sen}x\left(\frac{1}{5}\cos^4 x + \frac{4}{15}\cos^2 x + \frac{8}{15}\right) + c$$

$$= \text{sen}x\left(\frac{3\cos^4 x + 4\cos^2 x + 8}{15}\right) + c$$

$$= \frac{1}{15}\,\text{sen}x\,(3\cos^4 x + 4\cos^2 x + 8) + c$$

39.

$$\int \text{sen}^3 x\cos^2 x\,dx$$

$$\int \text{sen}^3 x\cos^2 x\,dx = \int \text{sen}^2 x\,\text{sen}x\cos^2 x\,dx = \int \text{sen}^2 x\cos^2 x\,\text{sen}x\,dx$$

resolveremos la integral $\int \text{sen}^2 x\cos^2 x\,\text{sen}x\,dx$ por partes

sea

$u = \text{sen}^2 x \;\rightarrow\; du = 2\,\text{sen}x\cos x\,dx$

$dv = \cos^2 x\,\text{sen}x\,dx \;\rightarrow\; \int dv = \int \cos^2 x\,\text{sen}x\,dx \;\rightarrow\; v = \int (\cos x)^2\,\text{sen}x\,dx$

hacemos el cambio de variable: $z = \cos x$; diferenciando: $dz = -\text{sen}x\,dx$;
despejando $\text{sen}x\,dx$: $-dz = \text{sen}x\,dx$: entonces:

$$v = \int z^2(-dz) \;\rightarrow\; v = -\int z^2\,dz \;\rightarrow\; v = -\frac{z^3}{3} \;\rightarrow\; v = -\frac{1}{3}z^3$$

$$v = -\frac{1}{3}(\cos x)^3 \;\rightarrow\; v = -\frac{1}{3}\cos^3 x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \text{sen}^3 x\cos^2 x\,dx = \text{sen}^2 x\left(-\frac{1}{3}\cos^3 x\right) - \int -\frac{1}{3}\cos^3 x\cdot 2\,\text{sen}x\cos x\,dx$$

$$\int \text{sen}^3 x\cos^2 x\,dx = -\frac{1}{3}\,\text{sen}^2 x\cos^3 x + \frac{2}{3}\int \text{sen}x\cos^4 x\,dx$$

$$\int \text{sen}^3 x\cos^2 x\,dx = -\frac{1}{3}\,\text{sen}^2 x\cos^3 x + \frac{2}{3}\int \text{sen}x\cos^2 x\cos^2 x\,dx$$

$$\int \text{sen}^3 x\cos^2 x\,dx = -\frac{1}{3}\,\text{sen}^2 x\cos^3 x + \frac{2}{3}\int \text{sen}x\,(1-\text{sen}^2 x)\cos^2 x\,dx$$

$$\int \text{sen}^3 x\cos^2 x\,dx = -\frac{1}{3}\,\text{sen}^2 x\cos^3 x + \frac{2}{3}\int (\text{sen}x\cos^2 x - \text{sen}^3 x\cos^2 x)\,dx$$

$$\int sen^3x\cos^2x\,dx = -\frac{1}{3}sen^2x\cos^3x + \frac{2}{3}\int senx\cos^2x\,dx - \frac{2}{3}\int sen^3x\cos^2x\,dx$$

$$\int sen^3x\cos^2x\,dx + \frac{2}{3}\int sen^3\cos^2x\,dx = -\frac{1}{3}sen^2x\cos^3x + \frac{2}{3}\int \cos^2x\,senx\,dx$$

$$(1+\frac{2}{3})\int sen^3x\cos^2x\,dx = -\frac{1}{3}sen^2x\cos^3x + \frac{2}{3}\int \cos^2x\,senx\,dx$$

$$\frac{5}{3}\int sen^3x\cos^2x\,dx = -\frac{1}{3}sen^2x\cos^3x + \frac{2}{3}\int \cos^2x\,senx\,dx$$

$$\int sen^3x\cos^2x\,dx = \frac{3}{5}\left[-\frac{1}{3}sen^2x\cos^3x + \frac{2}{3}\int \cos^2x\,senx\,dx\right]$$

$$\int sen^3x\cos^3x\,dx = -\frac{1}{5}sen^2x\cos^3x + \frac{2}{5}\int \cos^2x\,senx\,dx \quad \ldots\ldots\ldots\ldots \quad (I)$$

resolviendo la integral $\int \cos^2x\,senx\,dx$ por sustitución

$$\int \cos^2x\,senx\,dx = \int (\cos x)^2\,senx\,dx$$

hacemos el cambio: $t = \cos x$; diferenciando: $dt = -senx\,dx$;
despejando $senx\,dx$: $-dt = senx\,dx$; entonces:

$$\int \cos^2x\,senx\,dx = \int t^2(-dt) = -\int t^2dt = -\frac{t^3}{3} = -\frac{1}{3}t^3$$

$$\int \cos^2x\,senx\,dx = -\frac{1}{3}(\cos x)^3 = -\frac{1}{3}\cos^3x \quad \ldots\ldots\ldots\ldots \quad (II)$$

reemplazando (II) en (I)

$$\int sen^3x\cos^2x\,dx = -\frac{1}{2}sen^2x\cos^3x + \frac{2}{5}(-\frac{1}{3}\cos^3x) + c$$

$$\int sen^3x\cos^2x\,dx = -\frac{1}{5}sen^2x\cos^3x - \frac{2}{15}\cos^3x + c$$

$$\int sen^3x\cos^2x\,dx = -\frac{1}{5}\cos^3x\,(sen^2x + \frac{2}{3}) + c$$

40.

$$\int sen^4x\cos^5x\,dx$$

$$\int sen^4x\cos^5x\,dx = \int sen^4x\cos^4x\cos x\,dx = \int sen^4x\,(\cos^2x)^2\cos x\,dx$$

$$= \int sen^4x\,[1-sen^2x]^2\cos x\,dx = \int (senx)^4[1-(senx)^2]^2\cos x\,dx$$

hacemos el cambio: $t = senx$; diferenciando: $dt = \cos x\,dx$; entonces:

$$\int sen^4x\cos^5x\,dx = \int t^4[1-t^2]^2dt = \int t^4[1-2t^2+t^4]dt = \int t^4dt - 2\int t^6dt + \int t^8dt$$

$$= \frac{t^5}{5} - 2\cdot\frac{t^7}{7} + \frac{t^9}{9} + c = \frac{1}{5}t^5 - \frac{2}{7}t^7 + \frac{1}{9}t^9 + c = t^5\left[\frac{1}{5} - \frac{2}{7}t^2 + \frac{1}{9}t^4\right] + c$$

$$= (senx)^5\left[\frac{1}{5} - \frac{2}{7}(senx)^2 + \frac{1}{9}(senx)^4\right] + c$$

$$= sen^5x\left[\frac{1}{5} - \frac{2}{7}sen^2x + \frac{1}{9}sen^4x\right] + c = sen^5x\left[\frac{1}{5} - \frac{2}{7}sen^2x + \frac{1}{9}(sen^2x)^2\right] + c$$

$$= sen^5x\left[\frac{1}{5} - \frac{2}{7}(1\cos^2x) + \frac{1}{9}(1-\cos^2x)^2\right] + c$$

$$= sen^5x\left[\frac{1}{5} - \frac{2}{7}(1-\cos^2x) + \frac{1}{9}(1-2\cos^2x+\cos^4x)\right] + c$$

$$= sen^5x\left[\frac{1}{5} - \frac{2}{7} + \frac{2}{7}\cos^2x + \frac{1}{9} - \frac{2}{9}\cos^2x + \frac{1}{9}\cos^4x\right] + c$$

$$= sen^5x\left[(\frac{1}{5} - \frac{2}{7} + \frac{1}{9}) + (\frac{2}{7} - \frac{2}{9})\cos^2x + \frac{1}{9}\cos^4x\right] + c$$

$$= sen^5x\left[\frac{8}{315} + \frac{4}{63}\cos^2x + \frac{1}{9}\cos^4x\right] + c$$

$$= \frac{1}{9}sen^5x\left[\frac{8}{35} + \frac{4}{7}\cos^2x + \cos^4x\right] + c$$

41.

$$\int e^{2x}\cos 3x\,dx$$

Integrando por partes
sea:

$u = e^{2x}$	$dv = \cos 3x\,dx$
$du = e^{2x}\cdot 2\cdot dx$	$\int dv = \int \cos 3x\,dx$
$du = 2e^{2x}dx$	$v = \frac{1}{3}sen\,3x$

$$\int u\,dv = v\,du - \int v\,du$$

$$\int e^{2x}\cos 3x\,dx = e^{2x}\cdot\frac{1}{3}\,\text{sen}\,3x - \int\frac{1}{3}\,\text{sen}\,3x\cdot 2\,e^{2x}\,dx$$
$$\int e^{2x}\cos 3x\,dx = \frac{1}{3}\,e^{2x}\,\text{sen}\,3x - \frac{2}{3}\int e^{2x}\,\text{sen}\,3x\,dx \;\ldots\ldots\; (I)$$

resolveremos la integral $\int e^{2x}\,\text{sen}\,3x\,dx$ por partes

sea:

$$u = e^{2x} \qquad dv = \text{sen}\,3x\,dx$$
$$du = e^{2x}\cdot 2\cdot dx \qquad \int dv = \int\text{sen}\,3x\,dx$$
$$du = 2\,e^{2x}\,dx \qquad v = -\frac{1}{3}\cos 3x$$

$$\int u\,dv = v\,du - \int v\,du$$

$$\int e^{2x}\,\text{sen}\,3x\,dx = e^{2x}\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x\cdot 2\,e^{2x}\,dx$$
$$\int e^{2x}\,\text{sen}\,3x\,dx = -\frac{1}{3}\,e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x\,dx \;\ldots\ldots\; (II)$$

reemplazando (II) en (I)

$$\int e^{2x}\cos 3x\,dx = \frac{1}{3}\,e^{2x}\,\text{sen}\,3x - \frac{2}{3}\left[-\frac{1}{3}\,e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x\,dx\right]$$
$$\int e^{2x}\cos 3x\,dx = \frac{1}{3}\,e^{2x}\,\text{sen}\,3x + \frac{2}{9}\,e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x\,dx$$
$$\int e^{2x}\cos 3x\,dx + \frac{4}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}\,e^{2x}\,\text{sen}\,3x + \frac{2}{9}\,e^{2x}\cos 3x + c$$
$$\left(1 + \frac{4}{9}\right)\int e^{2x}\cos 3x\,dx = e^{2x}\left(\frac{1}{3}\,\text{sen}\,3x + \frac{2}{9}\cos 3x\right) + c$$
$$\frac{13}{9}\int e^{2x}\cos 3x\,dx = e^{2x}\left(\frac{3\,\text{sen}\,3x + 2\cos 3x}{9}\right) + c$$
$$\int e^{2x}\cos 3x\,dx = \frac{1}{13}\,e^{2x}(3\,\text{sen}\,3x + 2\cos 3x) + c$$

42.

$$\int e^{3x}(2\,\text{sen}\,4x - 5\cos 4x)\,dx$$
$$\int e^{3x}(2\,\text{sen}\,4x - 5\cos 4x)\,dx = 2\int e^{3x}\,\text{sen}\,4x\,dx - 5\int e^{3x}\cos 4x\,dx \;\ldots\ldots\ldots\ldots\; (I)$$

resolviendo la integral $\int e^{3x}\,\text{sen}\,4x\,dx$ por partes

sea:

$$u = e^{3x} \qquad dv = \text{sen}\,4x\,dx$$
$$du = e^{3x}\cdot 3\cdot dx \qquad \int dv = \int\text{sen}\,4x\,dx$$
$$du = 3\,e^{3x}\,dx \qquad v = -\frac{1}{4}\cos 4x$$

$$\int u\,dv = u\,v - \int v\,du$$

$$\int e^{3x}\,\text{sen}\,4x\,dx = e^{3x}\left(-\frac{1}{4}\cos 4x\right) - \int -\frac{1}{4}\cos 4x\cdot 3\,e^{3x}\,dx$$
$$\int e^{3x}\,\text{sen}\,4x\,dx = -\frac{1}{4}\,e^{3x}\cos 4x + \frac{3}{4}\int e^{3x}\cos 4x\,dx \;\ldots\ldots\; (II)$$

resolviendo la integral $\int e^{3x}\cos 4x\,dx$ nuevamente por partes

sea:

$$u = e^{3x} \qquad dv = \cos 4x\,dx$$
$$du = e^{3x}\cdot 3\cdot dx \qquad \int dv = \int\cos 4x\,dx$$
$$du = 3\,e^{3x}\,dx \qquad v = \frac{1}{4}\,\text{sen}\,4x$$

$$\int u\,dv = u\,v - \int v\,du$$

$$\int e^{3x}\cos 4x\,dx = \frac{1}{4}\,e^{3x}\cdot\text{sen}\,4x - \int\frac{1}{4}\,\text{sen}\,4x\cdot 3\,e^{3x}\,dx$$
$$\int e^{3x}\cos 4x\,dx = \frac{1}{4}\,e^{3x}\,\text{sen}\,4x - \frac{3}{4}\int e^{3x}\,\text{sen}\,4x\,dx \;\ldots\ldots\; (III)$$

43.

$\int sen3x\cos 2x\,dx$

Itegrando por partes

sea

$$u = sen3x \qquad dv = \cos 2x\,dx$$
$$du = \cos 3x \cdot 3 \cdot dx \qquad \int dv = \int \cos 2x\,dx$$
$$du = 3\cos 3x\,dx \qquad v = \frac{1}{2}sen2x$$
$$\int u\,dv = uv - \int v\,du$$

$$\int sen3x\cos 2x\,dx = sen3x \cdot \frac{1}{2}\cos 2x - \int \frac{1}{2}sen2x \cdot 3\cos 3x\,dx$$
$$= \frac{1}{2}sen3x\cos 2x - \frac{3}{2}\int sen2x\cos 3x\,dx \;\ldots\ldots\; (I)$$

resolvermos la integral $\int sen2x\cos 3x\,dx$ por partes

$$u = \cos 3x \qquad dv = sen2x\,dx$$
$$du = -sen3x \cdot 3 \cdot dx \qquad \int dv = \int sen2x\,dx$$
$$du = -3\,sen3x\,dx \qquad v = -\frac{1}{2}\cos 2x$$
$$\int u\,dv = uv - \int v\,du$$

$$\int sen3x\cos 2x\,dx = \cos 3x\left(-\frac{1}{2}\cos 2x\right) - \int\left(-\frac{1}{2}\cos 2x\right)(-3\,sen3x\,dx)$$
$$= -\frac{1}{2}\cos 3x\cos 2x - \frac{3}{2}\int sen3x\cos 2x\,dx \;\ldots\ldots\ldots\; (II)$$

reemplazando (II) en (I)

$$\int sen3x\cos 2x\,dx = \frac{1}{2}sen3x\cos 2x - \frac{3}{2}\left[-\frac{1}{2}\cos 3x\cos 2x - \frac{3}{2}\int sen3x\cos 2x\,dx\right]$$
$$\int sen3x\cos 2x\,dx = \frac{1}{2}sen3x\cos 2x + \frac{3}{4}\cos 3x\cos 2x + \frac{9}{4}\int sen3x\cos 2x\,dx$$
$$\int sen3x\cos 2x\,dx - \frac{9}{4}\int sen3x\cos 2x\,dx = \frac{1}{2}sen3x\cos 2x + \frac{3}{4}\cos 3c\cos 2x + c$$
$$\left(1 - \frac{9}{4}\right)\int sen3x\cos 2x\,dx = \frac{2\,sen3x\cos 2x + 3\cos 3x\cos 2x}{4} + c$$
$$-\frac{5}{4}\int sen3x\cos 2x\,dx = \frac{1}{4}\left(2\,sen3x\cos 2x + 3\cos 3x\cos 2x\right) + c$$
$$\int sen3x\cos 2x\,dx = -\frac{1}{5}\left(2\,sen3x\cos 2x + 3\cos 3x\cos 2x\right) + c$$