Límites

Límites de funciones racionales. Si la función $f(x)$ es una función racional de la forma $f(x) = \dfrac{P(x)}{Q(x)}$, donde $P(x)$ y $Q(x)$ son polinomios y si el límite de $f(x)$, cuando $x \to x_0$, evaluando el límite, toma la forma:

$$\lim_{x\to x_0} \frac{P(x)}{Q(x)} = \frac{P(x_0)}{Q(x_0)} = \frac{0}{0}$$

es preciso transformarla para "levantar la indeterminación"; es decir; simplificar el factor que hace indeterminada la fraccion, tan solo factorizando los polinomios $P(x)$ y $Q(x)$. Es de esperar que si $x \to x_0$, implica que $(x - x_0) \to 0$, en este caso habría que encontrar el factor $(x - x_0)$, seguidamente simplificar el factor $(x - x_0)$ y evaluar la expresión resultante.

Para ello se utilizan criterios de factorización y racionalización, según requiera el caso, para encontrar el factor $(x - x_0)$. Si el factor $(x - x_0)$ hace que $P(x_0) = Q(x_0) = 0$, entonces $(x - x_0)$ es un factor tanto para $P(x)$ y $Q(x)$:

$$\lim_{x\to x_0} \frac{P(x)}{Q(x)} = \lim_{x\to x_0} \frac{(x - x_0)P_1(x)}{(x - x_0)Q_1(x)} = \lim_{x\to x_0} \frac{\cancel{(x - x_0)}P_1(x)}{\cancel{(x - x_0)}Q_1(x)} = \lim_{x\to x_0} \frac{P_1(x)}{Q_1(x)} = \frac{P_1(x_0)}{Q_1(x_0)}$$

Hallar los límites de las funciones:

1. $\displaystyle\lim_{x\to 0} \frac{x^7 + 5x^6 + 4x^3}{x^7 + 2x^3}$

Solución:

Al evaluar el límite se presenta la indeterminación de la forma $\dfrac{0}{0}$, es decir:

$$\lim_{x\to 0} \frac{x^7 + 5x^6 + 4x^3}{x^7 + 2x^3} = \frac{(0)^7 + 5(0)^6 + 4(0)^3}{(0)^7 + 2(0)^3} = \frac{0 + 5(0) + 4(0)}{0 + 2(0)} = \frac{0 + 0 + 0}{0 + 0} = \frac{0}{0}$$

Como $x \to 0$, para levantar la indeterminación buscaremos el factor x tantas veces se pueda, para lo cual se factoriza el numerador y denominador:

factorizando el numerador: $x^7 + 5x^6 + 4x^3 = x^3(x^4 + 5x^3 + 4)$

factorizando el denominador: $x^7 + 2x^3 = x^3(x^4 + 2)$

reemplazando y simplificando:

$$\lim_{x\to 0} \frac{x^7 + 5x^6 + 4x^3}{x^7 + 2x^3} = \lim_{x\to 0} \frac{x^3(x^4 + 5x^3 + 4)}{x^3(x^4 + 2)} = \lim_{x\to 0} \frac{\cancel{x^3}(x^4 + 5x^3 + 4)}{\cancel{x^3}(x^4 + 2)} = \lim_{x\to 0} \frac{x^4 + 5x^3 + 4}{x^4 + 2}$$

evaluando el límite resultante:

$$\lim_{x\to 0} \frac{x^4 + 5x^3 + 4}{x^4 + 2} = \frac{(0)^4 + 5(0)^3 + 4}{(0)^4 + 2} = \frac{0 + 5(0) + 4}{0 + 2} = \frac{0 + 4}{4} = \frac{4}{2} = 2$$

finalmente:

$$\lim_{x\to 0} \frac{x^7 + 5x^6 + 4x^3}{x^7 + 2x^3} = 2$$

3. $\displaystyle\lim_{x\to 1} \frac{x^4 - x^3 + x^2 - 3x + 2}{x^3 - x^2 - x + 1}$

Solución:

Al evaluar el límite, se presenta la indeterminación $\dfrac{0}{0}$

$$\lim_{x\to 1} \frac{x^4 - x^3 + x^2 - 3x + 2}{x^3 - x^2 - x + 1} = \frac{(1)^4 - (1)^3 + (1)^2 - 3(1) + 2}{(1)^3 - (1)^2 - 1 + 1} = \frac{1 + 1 - 1 - 3 + 2}{1 - 1 - 1 + 1} = \frac{4 - 4}{2 - 2} = \frac{0}{0}$$

Como $x \to 1$, entonces $(x - 1) \to 0$, buscaremos el factor $(x - 1)$ tanto en el numerador y denominador, factorizando por la regla de Ruffini:

Numerador

$$\begin{array}{c|ccccc} & 1 & -1 & 1 & -3 & 2 \\ 1 & & 1 & 0 & 1 & -2 \\ \hline & 1 & 0 & 1 & -2 & 0 \\ 1 & & 1 & 1 & 2 & \\ \hline & 1 & 1 & 2 & 0 & \end{array}$$

$$x^4 - x^3 + x^2 - 3x + 2 = (x-1)(x-1)(x^2+x+2) = (x-1)^2(x^2+x+2)$$

Denominador

$$\begin{array}{c|cccc} & 1 & -1 & -1 & 1 \\ 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \\ 1 & & 1 & 1 & \\ \hline & 1 & 1 & 0 & \end{array}$$

$$x^3 - x^2 - x + 1 = (x-1)(x-1)(x+1) = (x-1)^2(x+1)$$

reemplazando y simplificando:

$$\lim_{x\to 1} \frac{x^4 - x^3 + x^2 - 3x + 2}{x^3 - x^2 - x + 1} = \lim_{x\to 1} \frac{(x-1)^2(x^2+x+2)}{(x-1)^2(x+1)} = \lim_{x\to 1} \frac{\cancel{(x-1)^2}(x^2+x+2)}{\cancel{(x-1)^2}(x+1)} = \lim_{x\to 1} \frac{x^2+x+2}{x+1}$$

evaluando en el límite resultante:

$$\lim_{x\to 1} \frac{x^2+x+2}{x+1} = \frac{(1)^2+1+2}{1+1} = \frac{1+3}{2} = \frac{4}{2} = 2$$

finalmente:

$$\lim_{x\to 1} \frac{x^4 - x^3 + x^2 - 3x + 2}{x^3 - x^2 - x + 1} = 2$$

4. $\lim\limits_{x\to 1} \dfrac{x^4 - 2x + 1}{x^8 - 2x + 1}$

Solución:
Evaluando el límite:

$$\lim_{x\to 1} \frac{x^4 - 2x + 1}{x^8 - 2x + 1} = \frac{(1)^4 - 2(1) + 1}{(1)^8 - 2()1 + 1} = \frac{1 - 2 + 1}{1 - 2 + 1} = \frac{2-2}{2-2} = \frac{0}{0}$$

indeterminación.
Como $x \to 1$, entonces $(x-1) \to 0$, luego buscaremos el factor $(x-1)$, factorizando el numerador y denominador

$$x^4 - 2x + 1 = x^4 - x - x + 1 = x(x^3 - 1) - (x-1) = x(x-1)(x^2+x+1) - (x-1) = (x-1)[x(x^2+x+1) - 1]$$
$$= (x-1)(x^3 + x^2 + x - 1)$$
$$x^8 - 2x + 1 = x^8 - x - x + 1 = x(x^7 - 1) - (x-1) = x(x-1)(x^6+x^5+x^4+x^3+x^2+x+1) - (x-1)$$
$$= (x-1)[x(x^6+x^5+x^4+x^3+x^2+x+1) - 1]$$
$$= (x-1)(x^7+x^6+x^5+x^4+x^3+x^2+x-1]$$

reemplazando y simplificando:

$$\lim_{x\to 1} \frac{x^4 - 2x + 1}{x^8 - 2x + 1} = \lim_{x\to 1} \frac{(x-1)(x^3+x^2+x-1)}{(x-1)(x^7+x^6+x^5+x^4+x^3+x^2+x-1]}$$
$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^3+x^2+x-1)}{\cancel{(x-1)}(x^7+x^6+x^5+x^4+x^3+x^2+x-1]}$$
$$= \lim_{x\to 1} \frac{x^3+x^2+x-1}{x^7+x^6+x^5+x^4+x^3+x^2+x-1}$$

evaluando el límite resultante

$$\lim_{x\to 1} \frac{x^3+x^2+x-1}{x^7+x^6+x^5+x^4+x^3+x^2+x-1} = \frac{(1)^3+(1)^2+1-1}{(1)^7+(1)^6+(1)^5+(1)^4+(1)^3+(1)^2+1-1}$$

$$= \frac{1+1+0}{1+1+1+1+1+1+0} = \frac{2}{6} = \frac{1}{3}$$

finalmente:

$$\lim_{x\to 1} \frac{x^4-2x+1}{x^8-2x+1} = \frac{1}{3}$$

Para factorizar el numerador se reescribe el segundo término algebraico: $-2x = -x - x$ y se empleo la formula: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Para factorizar el denominador, se realiza la misma operación en el segudo término algebraico: $-2x = -x-x$, y se empleo la formula: $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + ... + ab^{n-2} + b^{n-1})$

5. $\lim\limits_{x\to 1} \dfrac{x^{101} - 101x + 100}{x^2 - 2x + 1}$

Solución:

Evaluando el límite:

$$\lim_{x\to 1} \frac{x^{101} - 101x + 100}{x^2 - 2x + 1} = \frac{(1)^{101} - 101(1) + 100}{(1)^2 - 2(1) + 1} = \frac{1 - 101 + 100}{1 - 2 + 1} = \frac{101 - 101}{2 - 2} = \frac{0}{0}, \text{ indeterminación.}$$

Como $x \to 1$, entonces $(x-1) \to 0$, buscaremos el factor $(x-1)$, tantas veces se pueda, en el numerado y denominador, para eliminar la indeterminación.

$$\begin{aligned}
x^{101} - 101x + 100 &= x^{101} - 100x - x + 100 = x(x^{100} - 1) - 100(x-1) \\
&= x(x-1)(x^{99} + x^{98} + x^{97} + x^{96} + ... + x + 1) - 100(x-1) \\
&= (x-1)[x(x^{99} + x^{98} + x^{97} + x^{96} + ... + x + 1) - 100] \\
&= (x-1)[x^{100} + x^{99} + x^{98} + x^{97} + ... + x^2 + x - (1+1+1+1+...+1)] \\
&= (x-1)[x^{100} + x^{99} + x^{98} + x^{97} + ... + x^2 + x - 1 - 1 - 1 - 1 - ... - 1] \\
&= (x-1)[x^{100} - 1 + x^{99} - 1 + x^{98} - 1 + x^{97} - 1 + ... + x^2 - 1 + x - 1] \\
&= (x-1)[(x^{100} - 1) + (x^{99} - 1) + (x^{98} - 1) + (x^{97} - 1) + ... + (x^2 - 1) + (x-1)] \\
&= (x-1)[(x-1)(x^{99} + x^{98} + ... + 1) + (x-1)(x^{98} + x^{97} + .. + 1) + ... + (x-1)(x+1) + (x-1)] \\
&= (x-1)(x-1)[(x^{99} + x^{98} + ... + 1) + (x^{98} + x^{97} + .. + 1) + ... + (x+1) + 1] \\
&= (x-1)^2[(x^{99} + x^{98} + ... + 1) + (x^{98} + x^{97} + .. + 1) + ... + (x+1) + 1]
\end{aligned}$$

$x^2 - 2x + 1 = (x-1)^2$

reemplazando y simplificando:

$$\begin{aligned}
\lim_{x\to 1} \frac{x^{101} - 101x + 100}{x^2 - 2x + 1} &= \lim_{x\to 1} \frac{(x-1)^2[(x^{99} + x^{98} + ... + x + 1) + (x^{98} + x^{97} + ... + x + 1) + ... + (x+1) + 1]}{(x-1)^2} \\
&= \lim_{x\to 1} \frac{\cancel{(x-1)^2}[(x^{99} + x^{98} + ... + x + 1) + (x^{98} + x^{97} + ... + x + 1) + ... + (x+1) + 1]}{\cancel{(x-1)^2}} \\
&= \lim_{x\to 1} [(x^{99} + x^{98} + ... + x + 1) + (x^{98} + x^{97} + ... + x + 1) + ... + (x+1) + 1]
\end{aligned}$$

evaluando el límite resultante:

$$\lim_{x\to 1} [(x^{99} + x^{98} + ... + x + 1) + (x^{98} + x^{97} + ... + x + 1) + ... + (x+1) + 1] =$$

$$\begin{aligned}
&= [(1)^{99} + (1)^{98} + ... + 1 + 1) + ((1)^{98} + (1)^{97} + ... + 1 + 1) + ... + (1+1) + 1] \\
&= [(1 + 1 + ... + 1 + 1) + (1 + 1 + ... + 1 + 1) + ... + (1+1) + 1] \\
&= 100 + 99 + ... + 2 + 1
\end{aligned}$$

se presenta una progreción aritmética: $r = 1$, $a_1 = 1$, $a_n = 100$, $n = 100$, $S = \dfrac{n(a_1 + a_n)}{2}$

reemplazando: $S = \dfrac{100(1+100)}{2} = \dfrac{100(101)}{2} = \dfrac{10100}{2} = 5050$

finalmente:

$$\lim_{x\to 1} \frac{x^{101}-101x+100}{x^2-2x+1} = 5050$$

Límites con radicales. Si $f(x) = \dfrac{P(x)}{Q(x)}$ en $P(x)$ y/o $Q(x)$ intervienen radicales de índice par o impar y el límite de $f(x)$, cuando $x \to x_0$ es indeterminado $\dfrac{0}{0}$, si $x \to x_0$, implica que $(x - x_0) \to 0$, en este caso habría que encontrar el factor $(x - x_0)$, seguidamente simplificar el factor $(x - x_0)$ y evaluar la expresión resultante la indeterminación se elimina racionalizando $P(x)$ y/o $Q(x)$, el factor racionalizante se busca a travéz de la identidades:

1. $a^2 - b^2 = (a-b)(a+b)$

$a - b = \sqrt{a^2} - \sqrt{b^2} = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

2. $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a - b = \sqrt[3]{a^3} - \sqrt[3]{b^3} = (\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2})$

3. $a^3 + b^3 = (a+b)(a^2-ab+b^3)$

$a + b = \sqrt[3]{a^3} + \sqrt[3]{b^3} = (\sqrt[3]{a} + \sqrt[3]{b})(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2})$

4. $a^4 - b^4 = (a-b)(a^3+a^2b+ab^2+b^3)$
5. $a^5 - b^5 = (a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$
6. $a^5 + b^5 = (a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$
7. $a^6 + b^6 = (a-b)(a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5)$
8. $a^7 - b^7 = (a-b)(a^6+a^5b+a^4b^2+a^3b^3+a^2b^4+ab^5+b^6)$
9. $a^7 + b^7 = (a+b)(a^6-a^5b+a^4b^2-a^3b^3+a^2b^4-ab^5+b^6)$

en general:

10. $a^n - b^n = (a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+...+ab^{n-1}+b^{n-1})$ para n par o impar.
11. $a^n + b^n = (a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-...-ab^{n-1}+b^{n-1})$ para n par.

En el caso 1, cualquiera de los dos es el factor racionalizante, en los demas casos es el sugundo factor:

Hallar el límite de las siguientes funciones:

6. $\displaystyle\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$

Solución:

La sustitución directa lleva al límite a la indeterminación $\dfrac{0}{0}$

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} = \frac{\sqrt{6-2}-2}{6-6} = \frac{\sqrt{4}-2}{0} = \frac{2-2}{0} = \frac{0}{0}$$

Si $x \to 6$, entonces $(x-6) \to 0$. Para levantar la indeterminación buscaremos el factor $(x-6)$, racionalizando el numerador, para poder anular la raiz cuadrada se requiere tomar en consideración la diferencia de cuadrados, es decir, $a^2 - b^2 = (a-b)(a+b)$, se multiplica y divide por la conjugada de la expresión $\sqrt{x-2}-2$ que es $\sqrt{x-2}+2$ donde $a = \sqrt{x-2}$ y $b = 2$

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} = \lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} \cdot \frac{\sqrt{x-2}+2}{\sqrt{x-2}+2} = \lim_{x\to 6} \frac{\overbrace{(\sqrt{x-2}-2)(\sqrt{x-2}+2)}^{(a-b)(a+b)}}{(x-6)(\sqrt{x-2}+2)}$$

$$= \lim_{x\to 6} \frac{\overbrace{\sqrt{(x-2)^2}-2^2}^{a^2-b^2}}{(x-6)(\sqrt{x-2}+2)} = \lim_{x\to 6} \frac{x-2-4}{(x-6)(\sqrt{x-2}+2)} = \lim_{x\to 6} \frac{x-6}{(x-6)(\sqrt{x-2}+2)}$$

$$= \lim_{x\to 6} \frac{\cancel{x-6}}{\cancel{(x-6)}(\sqrt{x-2}+2)} = \lim_{x\to 6} \frac{1}{\sqrt{x-2}+2}$$

evaluando el límite resultante:

$\lim\limits_{x\to 6} \dfrac{1}{\sqrt{x-2}+2} = \dfrac{1}{\sqrt{6-2}+2} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{2+2} = \dfrac{1}{4}$

finalmente:

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} = \frac{1}{4}$$

7. $\lim\limits_{x\to 0} \dfrac{x}{\sqrt[3]{1+x}-1}$

Solución:

La sustitución directa lleva al límite a la forma indeterminada $\dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{x}{\sqrt[3]{1+x}-1} = \dfrac{0}{\sqrt[3]{1+0}-1} = \dfrac{0}{\sqrt[3]{1}-1} = \dfrac{0}{1-1} = \dfrac{0}{0}$

Como $x \to 0$, buscaremos el factor x, racionalizando el denominador, para poder anular la raiz cúbica se requiere tomar en consideración la diferencia de cubos, es decir, $a^3 - b^3 = (a-b)(a^2+ab+b^2)$, se multiplica y divide por la conjugada de la expresión $\sqrt[3]{1+x}-1$ que es $\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1$ donde $a = \sqrt[3]{1+x}$ y $b = 1$.

$$\lim_{x\to 0} \frac{x}{\sqrt[3]{1+x}-1} = \lim_{x\to 0} \frac{x}{\sqrt[3]{1+x}-1} \cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1}{\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1} = \lim_{x\to 0} \frac{x(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)}{\underbrace{(\sqrt[3]{1+x}-1)(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)}_{(a-b)(a^2+ab+b^2)}}$$

$$= \lim_{x\to 0} \frac{x(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)}{\underbrace{\sqrt[3]{(1+x)^3}-1^3}_{a^3-b^3}} = \lim_{x\to 0} \frac{x(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)}{1+x-1}$$

$$= \lim_{x\to 0} \frac{x(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)}{x} = \lim_{x\to 0} \frac{\cancel{x}(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)}{\cancel{x}}$$

$$= \lim_{x\to 0}(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)$$

evaluando el límite resultante:

$\lim\limits_{x\to 0}(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1) = \sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1 = \sqrt[3]{(1)^2}+\sqrt[3]{1}+1 = \sqrt[3]{1}+1+1 = 1+2 = 3$

finalmente

$$\lim_{x\to 0} \frac{x}{\sqrt[3]{1+x}-1} = 3$$

8. $\lim\limits_{x\to 0} \dfrac{1-\sqrt[3]{x}}{1-\sqrt[5]{x}}$

Solución:

La sustitución directa lleva al límite a la forma indeterminada $\dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{1-\sqrt[3]{x}}{1-\sqrt[5]{x}} = \dfrac{1-\sqrt[3]{1}}{1-\sqrt[5]{1}} = \dfrac{1-1}{1-1} = \dfrac{0}{0}$

reescribiendo el problema

$\lim\limits_{x\to 0} \dfrac{1-\sqrt[3]{x}}{1-\sqrt[5]{x}} = \lim\limits_{x\to 0} \dfrac{-\sqrt[3]{x}+1}{-\sqrt[5]{x}+1} = \lim\limits_{x\to 0} \dfrac{-(\sqrt[3]{x}-1)}{-(\sqrt[5]{x}-1)} = \lim\limits_{x\to 0} \dfrac{\sqrt[3]{x}-1}{\sqrt[5]{x}-1}$

Como $x \to 1$, entonces $(x-1) \to 0$, para levantar la indeterminación, buscaremos el factor $(x-1)$, racionalizando el numerador y denominador, para poder anular la raiz cúbica se requiere tomar en consideración la diferencia de cubos, es decir, $a^3 - b^3 = (a-b)(a^2+ab+b^2)$, se multiplica y divide por la conjugada de la expresión $\sqrt[3]{x}-1$ que es $\sqrt[3]{x^2}+\sqrt[3]{x}+1$ donde $a = \sqrt[3]{x}$ y $b = 1$.

Para anular la raíz quinta, consiramos $a^5 - b^5 = (a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$, multiplicando y dividiendo por la conjugada de la expresión $\sqrt[5]{x}-1$ que es $\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1$ donde $a = \sqrt[5]{x}$ y $b = 1$.

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[5]{x}-1}=\lim_{x\to 0}\frac{\sqrt[3]{x}-1}{\sqrt[5]{x}-1}\cdot\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}\cdot\frac{\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1}{\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1}$$

$$=\lim_{x\to 0}\frac{\overbrace{(\sqrt[3]{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}^{(a-b)(a^2+ab+b^2)}}{\underbrace{(\sqrt[5]{x}-1)(\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1)}_{(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)}}\cdot\frac{\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$=\lim_{x\to 0}\frac{\overbrace{\sqrt[3]{x^3}-1^3}^{a^3-b^3}}{\underbrace{(\sqrt[5]{x^5}-1^5}_{a^5-b^5}}\cdot\frac{\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}=\lim_{x\to 0}\frac{\cancel{x-1}}{\cancel{x-1}}\cdot\frac{\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$=\lim_{x\to 0}\frac{\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

evaluando el límite resultante:

$$\lim_{x\to 0}\frac{\sqrt[5]{x^4}+\sqrt[5]{x^3}+\sqrt[5]{x^2}+\sqrt[5]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}=\frac{\sqrt[5]{(1)^4}+\sqrt[5]{(1)^3}+\sqrt[5]{(1)^2}+\sqrt[5]{1}+1}{\sqrt[3]{(1)^2}+\sqrt[3]{1}+1}=\frac{\sqrt[5]{1}+\sqrt[5]{1}+\sqrt[5]{1}+1+1}{\sqrt[3]{1}+1+1}$$

$$=\frac{1+1+1+2}{1+2}=\frac{5}{3}$$

finalmente:

$$\lim_{x\to 0}\frac{1-\sqrt[3]{x}}{1-\sqrt[3]{x}}=\frac{5}{3}$$

9. $\lim_{x\to 0}\dfrac{2\sqrt{x^2+x+1}-2-x}{x^2}$

Solución:

La evaluación directa nos da la indeterminación $\dfrac{0}{0}$

$$\lim_{x\to 0}\frac{2\sqrt{x^2+x+1}-2-x}{x^2}=\frac{2\sqrt{(0)^2+0+1}-2-0}{(0)^2}=\frac{2\sqrt{0+1}-2}{0}=\frac{2\sqrt{1}-2}{0}=\frac{2(1)-2}{0}=\frac{2(1)-2}{0}=\frac{0}{0}$$

Si $x\to 0$, para eliminar la indeterminación, buscaremos el factor x, racionalizando el numerador.

reescribiendo el límite:

$$\lim_{x\to 0}\frac{2\sqrt{x^2+x+1}-2-x}{x^2}=\lim_{x\to 0}\frac{2\sqrt{x^2+x+1}-(2+x)}{x^2}$$

Para anular la raiz cuadrada consideramos la diferencia de cuadrados, es decir $a^2-b^2=(a-b)(a+b)$ multiplicando y dividiendo por la conjugada de la expresión $2\sqrt{x^2+x+2}-(2+x)$ que es igual a $2\sqrt{x^2+x+2}+(2+x)$ donde $a=2\sqrt{x^2+x+2}$ y $b=2+x$

$$\lim_{x\to 0}\frac{2\sqrt{x^2+x+1}-2-x}{x^2}=\lim_{x\to 0}\frac{2\sqrt{x^2+x+1}-(2+x)}{x^2}\cdot\frac{2\sqrt{x^2+x+1}+(2+x)}{2\sqrt{x^2+x+1}+(2+x)}$$

$$=\lim_{x\to 0}\frac{\overbrace{[2\sqrt{x^2+x+1}-(2+x)][2\sqrt{x^2+x+1}+(2+x)]}^{(a-b)(a+b)}}{x^2[2\sqrt{x^2+x+1}+(2+x)]}$$

$$= \lim_{x\to 0} \frac{\overbrace{4\sqrt{(x^2+x+1)^2}-(2+x)^2}^{a^2-b^2}}{x^2[2\sqrt{x^2+x+1}+(2+x)]} = \lim_{x\to 0} \frac{4(x^2+x+1)-(4+4x+x^2)}{x^2[2\sqrt{x^2+x+1}+(2+x)]}$$

$$= \lim_{x\to 0} \frac{4x^2+4x+4-4-4x-x^2}{x^2[2\sqrt{x^2+x+1}+(2+x)]} = \lim_{x\to 0} \frac{3x^2}{x^2[2\sqrt{x^2+x+1}+(2+x)]}$$

$$= \lim_{x\to 0} \frac{3\cancel{x^2}}{\cancel{x^2}[2\sqrt{x^2+x+1}+(2+x)]} = \lim_{x\to 0} \frac{3}{2\sqrt{x^2+x+1}+(2+x)}$$

evaluando el límite resultante:

$$\lim_{x\to 0} \frac{3}{2\sqrt{x^2+x+1}+(2+x)} = \frac{3}{2\sqrt{(0)^2+0+1}+(2+0)} = \frac{3}{2\sqrt{0+1}+2} = \frac{3}{2\sqrt{1}+2} = \frac{3}{2(1)+2} = \frac{3}{2+2} = \frac{3}{4}$$

finalmente:

$$\lim_{x\to 0} \frac{2\sqrt{x^2+x+1}-2-x}{x^2} = \frac{3}{4}$$

10. $\lim_{x\to 5} \frac{\sqrt{6-x}-1}{3-\sqrt{4+x}}$

Solución:

La sustitución directa lleva al límite a la forma indeterminada $\frac{0}{0}$

$$\lim_{x\to 0} \frac{\sqrt{6-x}-1}{3-\sqrt{4+x}} = \frac{\sqrt{6-5}-1}{3-\sqrt{4+5}} = \frac{\sqrt{1}-1}{3-\sqrt{9}} = \frac{1-1}{3-3} = \frac{0}{0}$$

Como $x \to 5$, entonces $(x-5) \to 0$, se eliminara al indeterminación racionalizando el numerador y denominador, y encontrar el factor $(x-5)$.
Para anular la raiz cuadrada consideramos la diferencia de cuadrados, es decir $a^2-b^2=(a-b)(a+b)$ multiplicando y dividiendo tanto en el numerador y denominador, por la conjugada de la expresión $\sqrt{6-x}-1$ que es igual a $\sqrt{6-x}+1$ donde $a=\sqrt{6-x}$ y $b=1$ en numerador y por la conjugada de la expresión $3-\sqrt{4+x}$ que es igual $3+\sqrt{4+x}$ donde $a=3$ y $\sqrt{4+x}$ en el denominador.

$$\lim_{x\to 5} \frac{\sqrt{6-x}-1}{3-\sqrt{4+x}} = \lim_{x\to 5} \frac{\sqrt{6-x}-1}{3-\sqrt{4+x}} \cdot \frac{\sqrt{6-x}+1}{\sqrt{6-x}+1} \cdot \frac{3+\sqrt{4+x}}{3+\sqrt{4+x}}$$

$$= \lim_{x\to 5} \frac{\overbrace{(\sqrt{6-x}-1)(\sqrt{6-x}+1)}^{(a-b)(a+b)}}{\underbrace{(3-\sqrt{4+x})(3+\sqrt{4+x})}_{(a-b)(a+b)}} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1}$$

$$= \lim_{x\to 5} \frac{\overbrace{\sqrt{(6-x)^2}-1^2}^{a^2-b^2}}{\underbrace{(3^2-\sqrt{(4+x)^2}}_{a^2-b^2}} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1} = \lim_{x\to 5} \frac{(6-x)-1}{9-(4+x)} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1}$$

$$= \lim_{x\to 5} \frac{6-x-1}{9-4-x} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1} = \lim_{x\to 5} \frac{-x+5}{-x+5} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1}$$

$$= \lim_{x\to 5} \frac{-(x-5)}{-(x-5)} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1} = \lim_{x\to 5} \frac{x-5}{x-5} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1}$$

$$= \lim_{x\to 5} \frac{\cancel{x-5}}{\cancel{x-5}} \cdot \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1} = \lim_{x\to 5} \frac{3+\sqrt{4+x}}{\sqrt{6-x}+1}$$

evaluando en el límite resultante:

$$\lim_{x\to 5}\frac{3+\sqrt{4+x}}{\sqrt{6-x}+1}=\frac{3+\sqrt{4+5}}{\sqrt{6-5}+1}=\frac{3+\sqrt{9}}{\sqrt{1}+1}=\frac{3+3}{1+1}=\frac{6}{2}=3$$

entonces:

$$\lim_{x\to 5}\frac{\sqrt{6-x}-1}{3-\sqrt{4+x}}=3$$

11. $\displaystyle\lim_{x\to -2}\frac{\sqrt[3]{x-6}+2}{x^3+8}$

Solución:

$$\lim_{x\to -2}\frac{\sqrt[3]{x-6}+2}{x^3+8}=\frac{\sqrt[3]{-2-6}+2}{(-2)^3+8}=\frac{\sqrt[3]{-8}+2}{-8+8}=\frac{-2+2}{0}=\frac{0}{0}$$

Aqui tiene lugar una indeterminación del tipo $\frac{0}{0}$. Como $x\to -2$, entonces $(x+2)\to 0$, para levantar la indeterminación, buscaremos el factor $(x+2)$, racionalizando el numerador, para poder anular la raiz cúbica se requiere tomar en consideración la suma de cubos, es decir, $a^3+b^3=(a+b)(a^2-ab+b^2)$, se multiplica y divide por la conjugada de la expresión $\sqrt[3]{x-6}+2$ que es $\sqrt[3]{(x-6)^2}-\sqrt[3]{x-6}\cdot 2+2^2$ donde $a=\sqrt[3]{x-6}$ y $b=2$.

$$\begin{aligned}
\lim_{x\to -2}\frac{\sqrt[3]{x-6}+2}{x^3+8} &= \lim_{x\to -2}\frac{\sqrt[3]{x-6}+2}{x^3+8}\cdot\frac{\sqrt[3]{(x-6)^2}-\sqrt[3]{x-6}\cdot 2+2^2}{\sqrt[3]{(x-6)^2}-\sqrt[3]{x-6}\cdot 2+2^2}\\
&= \lim_{x\to -2}\frac{\overbrace{(\sqrt[3]{x-6}+2)}^{a+b}\overbrace{(\sqrt[3]{(x-6)^2}-\sqrt[3]{x-6}\cdot 2+2^2)}^{a^2-ab+b^2}}{(x^3+8)(\sqrt[3]{(x-6)^2}-\sqrt[3]{x-6}\cdot 2+2^2)}\\
&= \lim_{x\to -2}\frac{\overbrace{\sqrt[3]{(x-6)^3}+2^3}^{a^3+b^3}}{(x^3+2^3)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)}\\
&= \lim_{x\to -2}\frac{x-6+8}{(x+2)(x^2-x\cdot 2+2^2)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)}\\
&= \lim_{x\to -2}\frac{x+2}{(x+2)(x^2-2x+4)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)}\\
&= \lim_{x\to -2}\frac{\cancel{x+2}}{\cancel{(x+2)}(x^2-2x+4)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)}\\
&= \lim_{x\to -2}\frac{1}{(x^2-2x+4)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)}
\end{aligned}$$

evaluando el límite resultante:

$$\begin{aligned}
\lim_{x\to -2}\frac{1}{(x^2-2x+4)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)} &= \frac{1}{\big((-2)^2-2(-2)+4\big)(\sqrt[3]{(-2-6)^2}-2\sqrt[3]{-2-6}+4)}\\
&= \frac{1}{(4+4+4)(\sqrt[3]{(-8)^2}-2\sqrt[3]{-8}+4)}\\
&= \frac{1}{12(\sqrt[3]{64}-2(-2)+4)}=\frac{1}{12(4+4+4)}\\
&= \frac{1}{12(12)}=\frac{1}{144}
\end{aligned}$$

finalmente:

$$\lim_{x\to -2}\frac{\sqrt[3]{x-6}+2}{x^3+8}=\frac{1}{144}$$

12. $\lim\limits_{x\to 3} \dfrac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$

Solución:

$$\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} = \frac{\sqrt{(3)^2-2(3)+6}-\sqrt{(3)^2+2(3)+6}}{(3)^2-4(3)+3} = \frac{\sqrt{9-6+6}-\sqrt{9+6-6}}{9-12+3}$$

$$= \frac{\sqrt{9}-\sqrt{9}}{12-12} = \frac{3-3}{0} = \frac{0}{0}$$

Aqui tiene lugar una indeterminación del tipo $\dfrac{0}{0}$. Si $x \to 3$, entonces $(x-3) \to 0$. Para levantar la indeterminación buscaremos el factor $(x-6)$, racionalizando el numerador, para poder anular la raiz cuadrada se requiere tomar en consideración la diferencia de cuadrados, es decir, $a^2-b^2=(a-b)(a+b)$, se multiplica y divide por la conjugada de la expresión $\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}$ que es $\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}$ donde $a=\sqrt{x^2-2x+6}$ y $b=\sqrt{x^2+2x-6}$

$$\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} = \lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} \cdot \frac{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$= \lim_{x\to 3} \frac{\overbrace{(\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6})}^{a-b}\overbrace{(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}^{a+b}}{(x^2-4x+3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x\to 3} \frac{\overbrace{\sqrt{(x^2-2x+6)^2}-\sqrt{(x^2+2x-6)^2}}^{a^2-b^2}}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x\to 3} \frac{(x^2-2x+6)-(x^2+2x-6)}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x\to 3} \frac{x^2-2x+6-x^2-2x+6}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x\to 3} \frac{-4x+12}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x\to 3} \frac{-4(x-3)}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x\to 3} \frac{-4\cancel{(x-3)}}{\cancel{(x-3)}(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$= \lim_{x\to 3} \frac{-4}{(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

evaluando el límite resultante:

$$\lim_{x\to 3} \frac{-4}{(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})} = \frac{-4}{(3-1)(\sqrt{(3)^2-2(3)+6}+\sqrt{(3)^2+2(3)-6})}$$

$$= \frac{-4}{2(\sqrt{9-6+6}+\sqrt{9+6-6})}$$

$$= \frac{-2}{\sqrt{9}+\sqrt{9}} = -\frac{2}{3+3} = -\frac{2}{6} = -\frac{1}{3}$$

finalmente

$$\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} = -\frac{1}{3}$$

13. $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

Solución:

$$\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}=\frac{\sqrt[3]{7+(1)^3}-\sqrt{3+(1)^2}}{1-1}=\frac{\sqrt[3]{7+1}-\sqrt{3+1}}{0}=\frac{\sqrt[3]{8}-\sqrt{4}}{0}=\frac{2-2}{0}=\frac{0}{0}$$

La sutitución directa da la indeterminación $\frac{0}{0}$. Observamos que los términos $\sqrt[3]{7+x^3}$ y $\sqrt{3+x^2}$ tienden a 2 cuando $(x\to 1)$, de modo que restamos y sumamos 2 al numerador y se tiene:

$$\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}=\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-2-\sqrt{3+x^2}+2}{x-1}=\lim_{x\to 1}\frac{(\sqrt[3]{7+x^3}-2)-(\sqrt{3+x^2}-2)}{x-1}$$

$$=\lim_{x\to 1}\left(\frac{\sqrt[3]{7+x^3}-2}{x-1}-\frac{\sqrt{3+x^2}-2}{x-1}\right)$$

Como $x\to 1$ entonces $(x-1)\to 0$, para levantar la indeterminación, buscaremos el factor $(x-1)$.
En la primera fracción, para poder anular la raiz cúbica se requiere tomar en consideración la diferencia de cubos, es decir, $a^3-b^3=(a-b)(a^2+ab+b^2)$, se multiplica y divide por la conjugada de la expresión $\sqrt[3]{7+x^3}-2$ que es $\sqrt[3]{(7+x^3)^2}+\sqrt[3]{7+x^3}\cdot 2+2^2$ donde $a=\sqrt[3]{7+x^3}$ y $b=2$.

En la segunda fracción, para anular la raiz cuadrada consideramos la diferencia de cuadrados, es decir $a^2-b^2=(a-b)(a+b)$ multiplicando y dividiendo tanto en el numerador y denominador, por la conjugada de la expresión $\sqrt{3+x^2}-2$ que es igual a $\sqrt{3+x^2}+2$ donde $a=\sqrt{3+x^2}$ y $b=2$.

$$\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}=\lim_{x\to 1}\left(\frac{\sqrt[3]{7+x^3}-2}{x-1}\cdot\frac{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\right.$$
$$\left.-\frac{\sqrt{3+x^2}-2}{x-1}\cdot\frac{\sqrt{3+x^2}+2}{\sqrt{3+x^2}+2}\right)$$

$$=\lim_{x\to 1}\left(\frac{\overbrace{(\sqrt[3]{7+x^3}-2)(\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)}^{(a-b)(a^2+ab+b^2)}}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\right.$$
$$\left.-\frac{\overbrace{(\sqrt{3+x^2}+2)(\sqrt{3+x^2}-2)}^{(a-b)(a+b)}}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\right)$$

$$=\lim_{x\to 1}\left(\frac{\overbrace{(\sqrt[3]{(7+x^3)^3}-2^3}^{a^3-b^3}}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\right.$$
$$\left.-\frac{\overbrace{\sqrt{(3+x^2)^2}-2^2}^{a^2-b^2}}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\right)$$

$$=\lim_{x\to 1}\left(\frac{7+x^3-8}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\right.$$
$$\left.-\frac{3+x^2-4}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\right)$$

$$=\lim_{x\to 1}\left(\frac{x^3-1}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\right.$$
$$\left.-\frac{x^2-1}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\right)$$

$$=\lim_{x\to 1}\left(\frac{(x-1)(x^2+x+1)}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\right.$$

$$-\frac{(x-1)(x+1)}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\left(\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\right.$$

$$\left.-\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\cdot\frac{1}{\sqrt{3+x^2}+2}\right)$$

$$=\lim_{x\to 1}\left(\frac{x^2+x+1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\frac{x+1}{\sqrt{3+x^2}+2}\right)$$

evaluando el límite resultante:

$$\lim_{x\to 1}\left(\frac{x^2+x+1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\frac{x+1}{\sqrt{3+x^2}+2}\right)=\frac{1^2+1+1}{\sqrt[3]{(7+1^3)^2}+2\sqrt[3]{7+1^3}+4}-\frac{1+1}{\sqrt{3+1^2}+2}$$

$$=\frac{3}{4+4+4}-\frac{2}{2+2}=\frac{3}{12}-\frac{2}{4}$$

$$=\frac{1}{4}-\frac{1}{2}=\frac{1-2}{4}=\frac{-1}{4}=-\frac{1}{4}$$

finalmente:

$$\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}=-\frac{1}{4}$$

Problemas resueltos

1. $\displaystyle\lim_{x\to -1}\frac{x^2-2x-3}{x^3-5x^2+3x+9}$

Solución:

$$\lim_{x\to -1}\frac{x^2-2x-3}{x^3-5x^2+3x+9}=\lim_{x\to -1}\frac{(-1)^2-2(-1)-3}{(-1)^3-5(-1)^2+3(-1)+9}=\frac{0}{0}\text{ (Indeterminación)}$$

Para eliminar la indeterminación debemos factorizar el numerador y el denominados para encontrar el factor $(x+1)$ entonces: factorizando el numerador:

$x^2-2x-3=(x-3)(x+1)$

factorizando del denominador por el método de Ruffini para $x=-1$

	1	−5	3	9
−1		−1	6	−9
	1	−6	9	0

$$x^3-5x^2+3x+9=(x-1)(x^2-6x+9)=(x+1)(x-3)^2$$

reemplazando y simplificando, tenemos:

$$\lim_{x\to -1}\frac{x^2-2x-3}{x^3-5x^2+3x+9}=\lim_{x\to -1}\frac{(x-3)(x+1)}{(x+1)(x-3)^2}$$

$$=\lim_{x\to -1}\frac{\cancel{(x-3)}\cancel{(x+1)}}{\cancel{(x+1)}(x-3)^{\cancel{2}}}$$

$$=\lim_{x\to -1}\frac{1}{x-3}=\frac{1}{-1-3}$$

$$=\frac{1}{-4}=-\frac{1}{4}$$

2. $\displaystyle\lim_{x\to -2}\frac{x^4+4x^3+5x^2+4x+4}{x^3+5x^2+8x+4}$

Solución

$$\lim_{x\to -2}\frac{x^4+4x^3+5x^2+4x+4}{x^3+5x^2+8x+4}=\frac{(-2)^4+4(-2)^3+5(-2)^2+4(-2)+4}{(-2)^3+5(-2)^2+8(-2)+4}=\frac{0}{0}\qquad\text{(Indeterminación)}$$

Para cancelar la indeterminación debemos encontrar el factor $(x+2)$ tanto en numerador y en denominador, entonces: factorizando el numerador y el denominador por la regla de Ruffini dos veces para $x=-2$

$$\begin{array}{r|rrrrr} & 1 & 4 & 5 & 4 & 4 \\ -2 & & -2 & -4 & -2 & -4 \\ \hline & 1 & 2 & 1 & 2 & 0 \\ -2 & & -2 & 0 & -2 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$$x^4+4x^3+5x^2+4x+4=(x+2)(x+2)(x^2+1)=(x+2)^2(x^2+1)$$

$$\begin{array}{r|rrrr} & 1 & 5 & 8 & 4 \\ -2 & & -2 & -6 & -4 \\ \hline & 1 & 3 & 2 & 0 \\ -2 & & -2 & -2 & \\ \hline & 1 & 1 & 0 & \end{array}$$

$$x^3+5x^2+8x+4=(x+2)(x+2)(x+1)=(x+2)^2(x+1)$$

reemplazando y simplificando, tenemos:

$$\begin{aligned}\lim_{x\to-2}\frac{x^4+4x^3+5x^2+4x+4}{x^3+5x^2+8x+4} &= \lim_{x\to-2}\frac{(x+2)^2(x^2+1)}{(x+2)^2(x+1)}\\ &= \lim_{x\to-2}\frac{\cancel{(x+2)^2}(x^2+1)}{\cancel{(x+2)^2}(x+1)}\\ &= \lim_{x\to-2}\frac{x^2+1}{x+1}=\frac{(-2)^2+1}{-2+1}\\ &= \frac{4+1}{-1}=\frac{5}{-1}=-5\end{aligned}$$

3. $\displaystyle\lim_{x\to-\frac{3}{2}}\frac{4x^4+12x^3-7x^2-48x-36}{4x^4+12x^3+21x^2+36x+27}$

Solución:

$$\lim_{x\to-\frac{3}{2}}\frac{4x^4+12x^3-7x^2-48x-36}{4x^4+12x^3+21x^2+36x+27}=\frac{4\left(-\frac{3}{2}\right)^4+12\left(-\frac{3}{2}\right)^3-7\left(-\frac{3}{2}\right)^2-48\left(-\frac{3}{2}\right)-36}{4\left(-\frac{3}{2}\right)^4+12\left(-\frac{3}{2}\right)^3+21\left(-\frac{3}{2}\right)^2+36\left(-\frac{3}{2}\right)+27}=\frac{0}{0}\qquad\text{(Ind.)}$$

$$\begin{array}{r|rrrrr} & 4 & 12 & -7 & -48 & -36 \\ -\frac{3}{2} & & -6 & -9 & 24 & 36 \\ \hline & 4 & 6 & -16 & -24 & 0 \\ -\frac{3}{2} & & -6 & 0 & 24 & \\ \hline & 4 & 0 & -16 & 0 & \end{array}$$

$$4x^4+12x^3-7x^2-48x-36=\left(x+\frac{3}{2}\right)\left(x+\frac{3}{2}\right)(4x^2-16)=\left(x+\frac{3}{2}\right)^2(4x^2-16)$$

$$\begin{array}{r|rrrrr} & 4 & 12 & 21 & 36 & 27 \\ -\frac{3}{2} & & -6 & -9 & -18 & -27 \\ \hline & 4 & 6 & 12 & 18 & 0 \\ -\frac{3}{2} & & -6 & 0 & -18 & \\ \hline & 4 & 0 & 12 & 0 & \end{array}$$

$$4x^4+12x^3+21x^2+36x+27=\left(x+\frac{3}{2}\right)\left(x+\frac{3}{2}\right)(4x^2-12)=\left(x+\frac{3}{2}\right)^2(4x^2+12)$$

reemplazando y simplificando:

$$\lim_{x\to-\frac{3}{2}}\frac{4x^4+12x^3-7x^2-48x-36}{4x^4+12x^3+21x^2+36x+27}=\lim_{x\to-\frac{3}{2}}\frac{\left(x+\frac{3}{2}\right)^2(4x^2-16)}{\left(x+\frac{3}{2}\right)^2(4x^2+12)}$$

$$= \lim_{x\to -\frac{3}{2}} \frac{\cancel{\left(x+\frac{3}{2}\right)^2}(4x^2-16)}{\cancel{\left(x+\frac{3}{2}\right)^2}(4x^2+12)}$$

$$= \lim_{x\to -\frac{3}{2}} \frac{4x^2-16}{4x^2+12} = \frac{4\left(\frac{3}{2}\right)^2-16}{4\left(\frac{3}{2}\right)^2+12}$$

$$= \frac{4\left(\frac{9}{4}\right)-16}{4\left(\frac{9}{4}\right)+12} = \frac{9-16}{9+12} = \frac{-7}{21} = -\frac{1}{3}$$

4. $\displaystyle\lim_{x\to -1} \frac{2x^3+7x^2+8x+3}{8x^3+5x^2+x-1}$

Solución:

$$\lim_{x\to -1} \frac{2x^3+7x^2+8x+3}{8x^3+5x^2+x-1} = \frac{2(-1)^3+7(-1)^2+8(-1)+3}{8(-1)^3+5(-1)^2+(-1)-1} = \frac{0}{0} \qquad \text{(Ind.)}$$

$$\begin{array}{r|rrrr} & 2 & 7 & 8 & 3 \\ -1 & & -2 & -5 & -3 \\ \hline & 2 & 5 & 3 & 0 \\ -1 & & -2 & -3 & \\ \hline & 2 & 3 & 0 & \end{array}$$

$$2x^3+7x^2+8x+3 = (x+1)(x+1)(2x+3) = (x+1)^2(2x+3)$$

$$\begin{array}{r|rrrr} & 3 & 5 & 1 & -1 \\ -1 & & -3 & -2 & 1 \\ \hline & 3 & 2 & -1 & 0 \\ -1 & & -3 & 1 & \\ \hline & 3 & -1 & 0 & \end{array}$$

$$3x^3+5x^2+x-1 = (x+1)(x+1)(3x-1) = (x+1)^2(3x-1)$$

reemplazando y simplificando:

$$\lim_{x\to -1} \frac{2x^3+7x^2+8x+3}{8x^3+5x^2+x-1} = \lim_{x\to -1} \frac{(x+1)^2(2x+3)}{(x+1)^2(3x-1)} = \lim_{x\to -1} \frac{\cancel{(x+1)^2}(2x+3)}{\cancel{(x+1)^2}(3x-1)}$$

$$= \lim_{x\to -1} \frac{2x+3}{3x-1} = \frac{2(-1)+3}{3(-1)-1} = \frac{-2+3}{-3-1} = \frac{1}{-4} = -\frac{1}{4}$$

5. $\displaystyle\lim_{x\to 1} \frac{x^{2n}-3+2x^{-2n}}{x^{2n}-4+3x^{-2n}}$

Solución:

$$\lim_{x\to 1} \frac{x^{2n}-3+2x^{-2n}}{x^{2n}-4+3x^{-2n}} = \frac{(1)^{2n}-3+2(1)^{-2n}}{(1)^{2n}-4+3(1)^{-2n}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1} \frac{x^{2n}-3+2x^{-2n}}{x^{2n}-4+3x^{-2n}} = \lim_{x\to 1} \frac{x^{2n}-3+\frac{2}{x^{2n}}}{x^{2n}-4+\frac{3}{x^{2n}}}$$

$$= \lim_{x\to 1} \frac{\frac{(x^{2n})^2-3x^{2n}+2}{x^{2n}}}{\frac{(x^{2n})^2-4x^{2n}+3}{x^{2n}}}$$

$$= \lim_{x\to 1} \frac{((x^{2n})^2-3x^{2n}+2)x^{2n}}{x^{2n}((x^{2n})^2-4x^{2n}+3)}$$

$$= \lim_{x\to 1} \frac{((x^{2n})^2-3x^{2n}+2)\cancel{x^{2n}}}{\cancel{x^{2n}}((x^{2n})^2-4x^{2n}+3)}$$

$$= \lim_{x\to 1} \frac{(x^{2n})^2 - 3x^{2n} + 2}{(x^{2n})^2 - 4x^{2n} + 3}$$
$$= \lim_{x\to 1} \frac{(x^{2n}-2)(x^{2n}-1)}{(x^{2n}-3)(x^{2n}-1)}$$
$$= \lim_{x\to 1} \frac{(x^{2n}-2)\cancel{(x^{2n}-1)}}{(x^{2n}-3)\cancel{(x^{2n}-1)}}$$
$$= \lim_{x\to 1} \frac{x^{2n}-2}{x^{2n}-3} = \frac{(1)^{2n}-2}{(1)^{2n}-3}$$
$$= \frac{1-2}{1-3} = \frac{-1}{-2} = \frac{1}{2}$$

7. $\lim\limits_{x\to \frac{1}{2}} \left(\dfrac{1}{1-2x} - \dfrac{3}{1-8x^3} \right)$

Solución:

$$\lim_{x\to \frac{1}{2}} \left(\frac{1}{1-2x} - \frac{3}{1-8x^3} \right) = \frac{1}{1-2\left(\frac{1}{2}\right)} - \frac{3}{1-8\left(\frac{1}{2}\right)^3} = \infty - \infty \qquad \text{(Indeterminación)}$$

$$\lim_{x\to \frac{1}{2}} \left(\frac{1}{1-2x} - \frac{3}{1-8x^3} \right) = \lim_{x\to \frac{1}{2}} \left(\frac{1}{1-2x} - \frac{3}{1-(2x)^3} \right)$$
$$= \lim_{x\to \frac{1}{2}} \left(\frac{1}{1-2x} - \frac{3}{(1-2x)(1+2x+4x^2)} \right)$$
$$= \lim_{x\to \frac{1}{2}} \frac{1+2x+4x^2-3}{(1-2x)(1+2x+4x^2)}$$
$$= \lim_{x\to \frac{1}{2}} \frac{4x^2+2x-2}{(-2x+1)(1+2x+4x^2)}$$
$$= \lim_{x\to \frac{1}{2}} \frac{2(2x^2+x-1)}{-(2x-1)(1+2x+4x^2)}$$
$$= \lim_{x\to \frac{1}{2}} -\frac{2(x+1)(2x-1)}{(2x-1)(1+2x+4x^2)}$$
$$= \lim_{x\to \frac{1}{2}} -\frac{2(x+1)\cancel{(2x-1)}}{\cancel{(2x-1)}(1+2x+4x^2)}$$
$$= \lim_{x\to \frac{1}{2}} -\frac{2(x+1)}{1+2x+4x^2}$$
$$= \lim_{x\to \frac{1}{2}} -\frac{2x+2}{1+2x+4x^2}$$
$$= -\frac{2\left(\frac{1}{2}\right)+2}{1+2\left(\frac{1}{2}\right)+4\left(\frac{1}{2}\right)^2}$$
$$= -\frac{1+2}{1+1+1} = -\frac{3}{3} = -1$$

8. $\lim\limits_{x\to 3} \left(\dfrac{2x-5}{x^2-5x+6} - \dfrac{2x-1}{x^2-x-6} \right)$

Solución:

$$\lim_{x\to 3} \left(\frac{2x-5}{x^2-5x+6} - \frac{2x-1}{x^2-x-6} \right) = \frac{2\cdot 3-5}{3^2-5\cdot 3+6} - \frac{2\cdot 3-1}{3^2-3-6} = \infty - \infty \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 3} \left(\frac{2x-5}{x^2-5x+6} - \frac{2x-1}{x^2-x-6} \right) = \lim_{x\to 3} \left(\frac{2x-5}{(x-3)(x-2)} - \frac{2x-1}{(x-3)(x+2)} \right)$$
$$= \lim_{x\to 3} \frac{(x+2)(2x-5)-(2x-1)(x-2)}{(x-3)(x-2)(x+2)}$$

$$
\begin{aligned}
&= \lim_{x\to 3} \frac{(2x^2-5x+4x-10)-(2x^2-4x-x+2)}{(x-3)(x^2-4)}\\
&= \lim_{x\to 3} \frac{(2x^2-x-10)-(2x^2-5x+2)}{(x-3)(x^2-4)}\\
&= \lim_{x\to 3} \frac{2x^2-x-10-2x^2+5x-2}{(x-3)(x^2-4)}\\
&= \lim_{x\to 3} \frac{4x-12}{(x-3)(x^2-4)}\\
&= \lim_{x\to 3} \frac{4(x-3)}{(x-3)(x^2-4)}\\
&= \lim_{x\to 3} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}(x^2-4)}\\
&= \lim_{x\to 3} \frac{4}{x^2-4} = \frac{4}{3^2-4}\\
&= \frac{4}{9-4} = \frac{4}{5}
\end{aligned}
$$

9. $\displaystyle\lim_{x\to -\frac{1}{2}} \left(\frac{x+1}{2x^2-3x-2} - \frac{x}{2x^2+7x+3}\right)$

Solución:

$$
\begin{aligned}
\lim_{x\to -\frac{1}{2}} \left(\frac{x+1}{2x^2-3x-2} - \frac{x}{2x^2+7x+3}\right) &= \frac{-\frac{1}{2}+1}{2\left(-\frac{1}{2}\right)^2-3\left(-\frac{1}{2}\right)-2} - \frac{-\frac{1}{2}}{2\left(-\frac{1}{2}\right)^2+7\left(-\frac{1}{2}\right)+3}\\
&= \infty - \infty \qquad \text{(Indeterminación)}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{x\to -\frac{1}{2}} \left(\frac{x+1}{2x^2-3x-2} - \frac{x}{2x^2+7x+3}\right) &= \lim_{x\to -\frac{1}{2}} \left(\frac{x+1}{(x-2)(2x+1)} - \frac{x}{(x+3)(2x+1)}\right)\\
&= \lim_{x\to -\frac{1}{2}} \frac{(x+1)(x+3)-x(x-2)}{(x-2)(2x+1)(x+3)}\\
&= \lim_{x\to -\frac{1}{2}} \frac{x^2+3x+x+3-x^2+2x}{(x-2)(2x+1)(x+3)}\\
&= \lim_{x\to -\frac{1}{2}} \frac{6x+3}{(x-2)(2x+1)(x+3)}\\
&= \lim_{x\to -\frac{1}{2}} \frac{3(2x+1)}{(x-2)(2x+1)(x+3)}\\
&= \lim_{x\to -\frac{1}{2}} \frac{3\cancel{(2x+1)}}{(x-2)\cancel{(2x+1)}(x+3)}\\
&= \lim_{x\to -\frac{1}{2}} \frac{3}{(x-2)(x+3)}\\
&= \frac{3}{\left(-\frac{1}{2}-2\right)\left(-\frac{1}{2}+3\right)}\\
&= \frac{3}{\left(\frac{-1-4}{2}\right)\left(\frac{-1+6}{2}\right)}\\
&= \frac{3}{\left(\frac{-5}{2}\right)\left(\frac{5}{2}\right)} = \frac{3}{-\frac{25}{4}}\\
&= \frac{12}{-25} = -\frac{12}{25}
\end{aligned}
$$

10. $\lim\limits_{x\to 2}\left(\dfrac{x+4}{x^2-4}-\dfrac{x+1}{x^2-2x}\right)^{\frac{1}{3}}$

Solución

$$\lim_{x\to 2}\left(\frac{x+4}{x^2-4}-\frac{x+1}{x^2-2x}\right)^{\frac{1}{3}}=\left(\frac{2+4}{2^2-4}-\frac{2+1}{2^2-2\cdot 2}\right)^{\frac{1}{3}}=\infty-\infty \qquad \text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to 2}\left(\frac{x+4}{x^2-4}-\frac{x+1}{x^2-2x}\right)^{\frac{1}{3}} &= \lim_{x\to 2}\left(\frac{x+4}{(x-2)(x+2)}-\frac{x+1}{x(x-2)}\right)^{\frac{1}{3}}\\
&= \lim_{x\to 2}\left(\frac{x(x+4)-(x+2)(x+1)}{x(x-2)(x+2)}\right)^{\frac{1}{3}}\\
&= \lim_{x\to 2}\left(\frac{x^2+4x-(x^2+x+2x+1)}{x(x-2)(x+2)}\right)^{\frac{1}{3}}\\
&= \lim_{x\to 2}\left(\frac{x^2+4x-(x^2+3x+2)}{x(x-2)(x+2)}\right)^{\frac{1}{3}}\\
&= \lim_{x\to 2}\left(\frac{x^2+4x-x^2-3x-2}{x(x-2)(x+2)}\right)^{\frac{1}{3}}\\
&= \lim_{x\to 2}\left(\frac{x-2}{x(x-2)(x+2)}\right)^{\frac{1}{3}}\\
&= \lim_{x\to 2}\left(\frac{\cancel{x-2}}{x\cancel{(x-2)}(x+2)}\right)^{\frac{1}{3}}\\
&= \lim_{x\to 2}\left(\frac{1}{x(x+2)}\right)^{\frac{1}{3}}=\left(\frac{1}{2(2+2)}\right)^{\frac{1}{3}}\\
&= \left(\frac{1}{2(4)}\right)^{\frac{1}{3}}=\left(\frac{1}{8}\right)^{\frac{1}{3}}=\sqrt[3]{\frac{1}{8}}=\frac{1}{2}
\end{aligned}$$

11. $\lim\limits_{x\to 1}\dfrac{x^m-1}{x^n-1}, \quad m,\, n\in Z^{+}$

Solución:

$$\lim_{x\to 1}\frac{x^m-1}{x^n-1}=\frac{1^m-1}{1^n-1}=\frac{0}{0}\text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to 1}\frac{x^m-1}{x^n-1} &= \lim_{x\to 1}\frac{(x-1)(x^{m-1}+x^{m-2}+x^{m-2}+\ldots+1)}{(x-1)(x^{n-1}+x^{n-2}+x^{n-2}+\ldots+1)}\\
&= \lim_{x\to 1}\frac{\cancel{(x-1)}(x^{m-1}+x^{m-2}+x^{m-2}+\ldots+1)}{\cancel{(x-1)}(x^{n-1}+x^{n-2}+x^{n-2}+\ldots+1)}\\
&= \lim_{x\to 1}\frac{x^{m-1}+x^{m-2}+x^{m-2}+\ldots+1}{x^{n-1}+x^{n-2}+x^{n-2}+\ldots+1}\\
&= \frac{1^{m-1}+1^{m-2}+1^{m-2}+\ldots+1}{1^{n-1}+1^{1-2}+1^{n-2}+\ldots+1}=\frac{m\cdot 1}{n\cdot 1}=\frac{m}{n}
\end{aligned}$$

12. $\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}$

Solución:

$$\lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)-1}{x}=\frac{(1+0)(1+2\cdot 0)(1+3\cdot 0)-1}{0}=\frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)-1}{x} &= \lim_{x\to 0}\frac{(1+3x+2x^2)(1+3x)-1}{x}\\
&= \lim_{x\to 0}\frac{1+3x+3x+9x^2+2x^2+6x^3-1}{x}\\
&= \lim_{x\to 0}\frac{6x^3+11x^2+6x}{x}\\
&= \lim_{x\to 0}\frac{x(6x^2+11x+6)}{x}
\end{aligned}$$

$$
\begin{aligned}
&= \lim_{x\to 0} \frac{\cancel{x}(6x^2+11x+6)}{\cancel{x}} \\
&= \lim_{x\to 0} (6x^2+11x+6) \\
&= 6\cdot 0^2 + 11\cdot 0 + 6 = 0+0+6 = 6
\end{aligned}
$$

13. $\lim\limits_{x\to 0} \dfrac{(1+x)^5-(1+5x)}{x^2+x^5}$

Solución:

$$\lim_{x\to 0} \frac{(1+x)^5-(1+5x)}{x^2+x^5} = \frac{(1+0)^5-(1+5\cdot 0)}{0^2+0^5} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$
\begin{aligned}
\lim_{x\to 0} \frac{(1+x)^5-(1+5x)}{x^2+x^5} &= \lim_{x\to 0} \frac{1+5x+10x^2+10x^3+5x^4+x^5-1-5x}{x^2(1+x^3)} \\
&= \lim_{x\to 0} \frac{10x^2+10x^3+5x^4+x^5}{x^2(1+x^3)} \\
&= \lim_{x\to 0} \frac{x^2(10+10x+5x^2+x^3)}{x^2(1+x^3)} \\
&= \lim_{x\to 0} \frac{\cancel{x^2}(10+10x+5x^2+x^3)}{\cancel{x^2}(1+x^3)} \\
&= \lim_{x\to 0} \frac{10+10x+5x^2+x^3}{1+x^3} \\
&= \frac{10+10\cdot 0+5\cdot 0^2+0^3}{1+0^3} \\
&= \frac{10+0+0+0}{1+0} = \frac{10}{1} = 10
\end{aligned}
$$

15. $\lim\limits_{x\to 2} \dfrac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}}$

Solución.

$$\lim_{x\to 2} \frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}} = \frac{((2)^2-2-2)^{20}}{((2)^3-12(2)+16)^{10}} = \frac{(4^2-2-2)^{20}}{(8-24+16)^{10}} = \frac{(0)^{20}}{(0)^{10}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

factorizando el numerador: $x^2-x-2=(x-2)(x+1)$

factorizando el denominador por el método de Ruffini para $x=2$

$$
\begin{array}{c|cccc}
 & 1 & 0 & -12 & 16 \\
2 & & 2 & 4 & -16 \\
\hline
 & 1 & 2 & -8 & 0 \\
2 & & 2 & 8 & \\
\hline
 & 1 & 4 & 0 &
\end{array}
$$

$$x^3-12x+16 = (x-2)(x-2)(x+4) = (x-2)^2(x+4)$$

reemplazando y simplificando:

$$
\begin{aligned}
\lim_{x\to 2} \frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}} &= \lim_{x\to 2} \frac{((x-2)(x+1))^{20}}{((x-2)^2(x+4))^{10}} \\
&= \lim_{x\to 2} \frac{(x-2)^{20}(x+1)^{20}}{(x-2)^{20}(x+4)^{10}} \\
&= \lim_{x\to 2} \frac{\cancel{(x-2)^{20}}(x+1)^{20}}{\cancel{(x-2)^{20}}(x+4)^{10}} \\
&= \lim_{x\to 2} \frac{(x+1)^{20}}{(x+4)^{10}} = \frac{(2+1)^{20}}{(2+4)^{10}} \\
&= \frac{(3)^{20}}{(6)^{10}} = \frac{(3^{10})^2}{(3\cdot 2)^{10}} = \frac{(3^{10})^2}{3^{10}\cdot 2^{10}} \\
&= \frac{3^{10}}{2^{10}} = \left(\frac{3}{2}\right)^{10}
\end{aligned}
$$

25. $\lim\limits_{x\to a} \dfrac{\sqrt{a^2-x^2}+\sqrt{(a-x)^3}}{\sqrt{a^3-x^3}+\sqrt{a-x}}$

Solución:

$$\lim_{x\to a} \frac{\sqrt{a^2-x^2}+\sqrt{(a-x)^3}}{\sqrt{a^3-x^3}+\sqrt{a-x}} = \frac{\sqrt{a^2-a^2}+\sqrt{(a-a)^3}}{\sqrt{a^3-a^3}+\sqrt{a-a}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt{a^2-x^2}+\sqrt{(a-x)^3}}{\sqrt{a^3-x^3}+\sqrt{a-x}} &= \lim_{x\to a} \frac{\sqrt{(a-x)(a+x)}+\sqrt{(a-x)^2(a-x)}}{\sqrt{(a-x)(a^2+ax+x^2)}+\sqrt{a-x}} \\
&= \lim_{x\to a} \frac{\sqrt{a-x}\sqrt{a+x}+\sqrt{(a-x)^2}\sqrt{a-x}}{\sqrt{a-x}\sqrt{a^2+ax+x^2}+\sqrt{a-x}} \\
&= \lim_{x\to a} \frac{\sqrt{a-x}\sqrt{a+x}+(a-x)\sqrt{a-x}}{\sqrt{a-x}\sqrt{a^2+ax+x^2}+\sqrt{a-x}} \\
&= \lim_{x\to a} \frac{\sqrt{a-x}(\sqrt{a+x}+a-x)}{\sqrt{a-x}(\sqrt{a^2+ax+x^2}+1)} \\
&= \lim_{x\to a} \frac{\cancel{\sqrt{a-x}}(\sqrt{a+x}+a-x)}{\cancel{\sqrt{a-x}}(\sqrt{a^2+ax+x^2}+1)} \\
&= \lim_{x\to a} \frac{\sqrt{a+x}+a-x}{\sqrt{a^2+ax+x^2}+1} \\
&= \frac{\sqrt{a+a}+a-a}{\sqrt{a^2+a\cdot a+a^2}+1} \\
&= \frac{\sqrt{2a}}{\sqrt{a^2+a^2+a^2}+1} \\
&= \frac{\sqrt{2a}}{\sqrt{3a^2}+1} = \frac{\sqrt{2a}}{\sqrt{3}a+1}
\end{aligned}$$

26. $\lim\limits_{x\to a} \dfrac{a\sqrt{ax}-x^2}{a-\sqrt{ax}}$

Solución:

$$\lim_{x\to a} \frac{a\sqrt{ax}-x^2}{a-\sqrt{ax}} = \frac{a\sqrt{a\cdot a}-a^2}{a-\sqrt{a\cdot a}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to a} \frac{a\sqrt{ax}-x^2}{a-\sqrt{ax}} &= \lim_{x\to a} \frac{\sqrt{a^2}\cdot\sqrt{ax}-\sqrt{(x^2)^2}}{\sqrt{a^2}-\sqrt{ax}} = \lim_{x\to a} \frac{\sqrt{a^3x}-\sqrt{x^4}}{\sqrt{a^2}-\sqrt{ax}} \\
&= \lim_{x\to a} \frac{\sqrt{a^3}\sqrt{x}-\sqrt{x\cdot x^3}}{\sqrt{a}\sqrt{a}-\sqrt{a}\sqrt{x}} = \lim_{x\to a} \frac{\sqrt{a^3}\sqrt{x}-\sqrt{x}\sqrt{x^3}}{\sqrt{a}\sqrt{a}-\sqrt{a}\sqrt{x}} \\
&= \lim_{x\to a} \frac{\sqrt{x}(\sqrt{a^3}-\sqrt{x^3})}{\sqrt{a}\sqrt{a}-\sqrt{x})} = \lim_{x\to a} \frac{\sqrt{x}(\sqrt{a}-\sqrt{x})(\sqrt{a^2}+\sqrt{a}\cdot\sqrt{x}+\sqrt{x^2})}{\sqrt{a}(\sqrt{a}-\sqrt{x})} \\
&= \lim_{x\to a} \frac{\sqrt{x}\cancel{(\sqrt{a}-\sqrt{x})}(\sqrt{a^2}+\sqrt{a}\cdot\sqrt{x}+\sqrt{x^2})}{\sqrt{a}\cancel{(\sqrt{a}-\sqrt{x})}} = \lim_{x\to a} \frac{\sqrt{x}(a+\sqrt{ax}+x)}{\sqrt{a}} \\
&= \frac{\sqrt{a}(a+\sqrt{a\cdot a}+a)}{\sqrt{a}} = \frac{\cancel{\sqrt{a}}(a+\sqrt{a^2}+a)}{\cancel{\sqrt{a}}} = a+a+a = 3a
\end{aligned}$$

27. $\lim\limits_{x\to 3a} \dfrac{3a\sqrt{x+a}-2\sqrt{a}x}{2\sqrt{a}-\sqrt{x+a}}$

Solución:

$$\lim_{x\to 3a} \frac{3a\sqrt{x+a}-2\sqrt{a}x}{2\sqrt{a}-\sqrt{x+a}} = \frac{3a\sqrt{3a+a}-2\sqrt{a}\cdot 3a}{2\sqrt{a}-\sqrt{3a+a}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 3a} \frac{3a\sqrt{x+a}-2\sqrt{a}x}{2\sqrt{a}-\sqrt{x+a}} = \lim_{x\to 3a} \frac{3a\sqrt{x+a}-2\sqrt{a}x}{2\sqrt{a}-\sqrt{x+a}} \cdot \frac{3a\sqrt{x+a}+2\sqrt{a}x}{3a\sqrt{x+a}+2\sqrt{a}x} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{2\sqrt{a}+\sqrt{x+a}}$$

$$= \lim_{x\to 3a} \frac{\overbrace{(3a\sqrt{x+a}-2\sqrt{a}x)(3a\sqrt{x+a}+2\sqrt{a}x)}^{(a-b)(a+b)}}{\underbrace{(2\sqrt{a}-\sqrt{x+a}\,)(2\sqrt{a}+\sqrt{x+a}\,)}_{(a-b)(a+b)}} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{\overbrace{(3a\sqrt{x+a}\,)^2-(2\sqrt{a}x)^2}^{a^2-b^2}}{\underbrace{(2\sqrt{a}\,)^2-(\sqrt{x+a}\,)^2}_{a^2-b^2}} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{9a^2(x+a)-4ax^2}{4a-(x+a)} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{9a^2x+9a^3-4ax^2}{4a-x-a} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{-4ax^2+9a^2x+9a^3}{-x+3a} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{-a(4x^2-9ax-9a^2)}{-x+3a} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{-a(x-3a)(4x+3a)}{-(x-3a)} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{a\cancel{(x-3a)}(4x+3a)}{\cancel{x-3a}} \cdot \frac{2\sqrt{a}+\sqrt{x+a}}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \lim_{x\to 3a} \frac{a(4x+3a)(2\sqrt{a}+\sqrt{x+a}\,)}{3a\sqrt{x+a}+2\sqrt{a}x}$$

$$= \frac{a(4\cdot 3a+3a)(2\sqrt{a}+\sqrt{3a+a}\,)}{3a\sqrt{3a+a}+2\sqrt{a}\cdot 3a}$$

$$= \frac{a(12a+3a)(2\sqrt{a}+\sqrt{4a}\,)}{3a\sqrt{4a}+6a\sqrt{a}}$$

$$= \frac{a(15a)(2\sqrt{a}+2\sqrt{a}\,)}{3a\cdot 2\sqrt{a}+6a\sqrt{a}}$$

$$= \frac{15a^2(4\sqrt{a}\,)}{6a\sqrt{a}+6a\sqrt{a}} = \frac{60a^2\sqrt{a}}{12a\sqrt{a}} = 5a$$

28. $\displaystyle\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$

Solición:

$$\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} = \frac{\sqrt{3^2-2\cdot 3+6}-\sqrt{3^2+2\cdot 3-6}}{3^2-4\cdot 3+3} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} = \lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3} \cdot \frac{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$= \lim_{x\to 3} \frac{\overbrace{(\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}\,)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\,)}^{(a-b)(a+b)}}{(x^2-4x+3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\,)}$$

$$= \lim_{x\to 3} \frac{\overbrace{(\sqrt{x^2-2x+6}\,)^2-(\sqrt{x^2+2x-6}\,)^2}^{a^2-b^2}}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\,)}$$

$$= \lim_{x\to 3} \frac{(x^2-2x+6)-(x^2+2x-6)}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\,)}$$

$$= \lim_{x\to 3} \frac{x^2-2x+6-x^2-2x+6}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\,)}$$

$$= \lim_{x\to 3} \frac{-4x+12}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$
$$= \lim_{x\to 3} \frac{-4(x-3)}{(x-3)(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$
$$= \lim_{x\to 3} \frac{-4\cancel{(x-3)}}{\cancel{(x-3)}(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$
$$= \lim_{x\to 3} \frac{-4}{(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$
$$= \frac{-4}{(3-1)(\sqrt{3^2-2\cdot 3+6}+\sqrt{3^2+2\cdot 3-6})}$$
$$= \frac{-4}{(2)(\sqrt{9-6+6}+\sqrt{9+6-6})} = \frac{-4}{(2)(\sqrt{9}+\sqrt{9})}$$
$$= \frac{-4}{2(3+3)} = \frac{-4}{2(6)} = \frac{-4}{12} = -\frac{1}{3}$$

29. $\lim_{x\to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

Solución:

$$\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} = \frac{\sqrt{a+2a}-\sqrt{3a}}{\sqrt{3a+a}-2\sqrt{a}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} = \lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \cdot \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \cdot \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}$$

$$= \lim_{x\to a} \frac{\overbrace{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{a+2x}+\sqrt{3x})}^{(a-b)(a+b)}}{\underbrace{(\sqrt{3a+x}-2\sqrt{x})(\sqrt{3a+x}+2\sqrt{x})}_{(a-b)(a+b)}} \cdot \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{\overbrace{(\sqrt{a+2x})^2-(\sqrt{3x})^2}^{a^2-b^2}}{\underbrace{(\sqrt{3a+x})^2-(2\sqrt{x})^2}_{a^2-b^2}} \cdot \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}} = \lim_{x\to a} \frac{a+2x-3x}{3a+x-4x} \cdot \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{-x+a}{-3x+3a} \cdot \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}} = \lim_{x\to a} \frac{-(x-a)}{-3(x-a)} \cdot \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{\cancel{x-a}}{3\cancel{(x-a)}} \cdot \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}} = \lim_{x\to a} \frac{\sqrt{3a+x}+2\sqrt{x}}{3(\sqrt{a+2x}+\sqrt{3x})} = \frac{\sqrt{3a+a}+2\sqrt{a}}{3(\sqrt{a+2a}+\sqrt{3a})}$$

$$= \frac{\sqrt{4a}+2\sqrt{a}}{3(\sqrt{3a}+\sqrt{3a})} = \frac{2\sqrt{a}+2\sqrt{a}}{3(2\sqrt{3a})} = \frac{4\sqrt{a}}{6\sqrt{3a}} = \frac{2\sqrt{a}}{3\sqrt{3}\cdot\sqrt{a}} = \frac{2\cancel{\sqrt{a}}}{3\sqrt{3}\cancel{\sqrt{a}}} = \frac{2}{3\sqrt{3}} = \frac{2}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3\sqrt{3^2}} = \frac{2\sqrt{3}}{3\cdot 3} = \frac{2\sqrt{3}}{9}$$

30. $\lim_{x\to 0} \dfrac{\sqrt{a^2+ax+x^2}-\sqrt{a^2-ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$

Solución:

$$\lim_{x\to 0} \frac{\sqrt{a^2+ax+x^2}-\sqrt{a^2-ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}} = \frac{\sqrt{a^2+a\cdot 0+0^2}-\sqrt{a^2-a\cdot 0+0^2}}{\sqrt{a+0}-\sqrt{a-0}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0} \frac{\sqrt{a^2+ax+x^2}-\sqrt{a^2-ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}} = \lim_{x\to 0} \frac{\sqrt{a^2+ax+x^2}-\sqrt{a^2-ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}} \cdot \frac{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}} \cdot$$
$$\cdot \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$$

$$= \lim_{x\to 0} \frac{\overbrace{(\sqrt{a^2+ax+x^2}-\sqrt{a^2-ax+x^2})(\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2})}^{(a-b)(a+b)}}{\underbrace{(\sqrt{a+x}-\sqrt{a-x})(\sqrt{a+x}+\sqrt{a-x})}_{(a-b)(a+b)}}$$

$$\cdot \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}$$

$$= \lim_{x\to 0} \frac{\overbrace{(\sqrt{a^2+ax+x^2}\,)^2-(\sqrt{a^2-ax+x^2}\,)^2}^{a^2-b^2}}{\underbrace{(\sqrt{a+x}\,)^2-(\sqrt{a-x}\,)^2}_{a^2-b^2}} \cdot \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}$$

$$= \lim_{x\to 0} \frac{(a^2+ax+x^2)-(a^2-ax+x^2)}{(a+x)-(a-x)} \cdot \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}$$

$$= \lim_{x\to 0} \frac{a^2+ax+x^2-a^2+ax-x^2}{a+x-a+x} \cdot \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}$$

$$= \lim_{x\to 0} \frac{2ax}{2x} \cdot \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}$$

$$= \lim_{x\to 0} \frac{\not{2}a\not{x}}{\not{2}\not{x}} \cdot \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}$$

$$= \lim_{x\to 0} \frac{a(\sqrt{a+x}+\sqrt{a-x}\,)}{\sqrt{a^2+ax+x^2}+\sqrt{a^2-ax+x^2}}$$

$$= \frac{a(\sqrt{a+0}+\sqrt{a-0}\,)}{\sqrt{a^2+a\cdot 0+0^2}+\sqrt{a^2-a\cdot 0+0^2}} = \frac{a(\sqrt{a}+\sqrt{a}\,)}{\sqrt{a^2}+\sqrt{a^2}}$$

$$= \frac{a(2\sqrt{a}\,)}{a+a} = \frac{2a\sqrt{a}}{2a} = \frac{\not{2}\not{a}\sqrt{a}}{\not{2}\not{a}} = \sqrt{a}$$

32. $\lim_{x\to 8} \dfrac{2-\sqrt[3]{5+\sqrt{x+1}}}{3-\sqrt{x+1}}$

Solución:

$$\lim_{x\to 8} \frac{2-\sqrt[3]{5+\sqrt{x+1}}}{3-\sqrt{x+1}} = \frac{2-\sqrt[3]{5+\sqrt{8+1}}}{3-\sqrt{8+1}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 8} \frac{2-\sqrt[3]{5+\sqrt{x+1}}}{3-\sqrt{x+1}} = \lim_{x\to 8} \frac{2-\sqrt[3]{5+\sqrt{x+1}}}{3-\sqrt{x+1}} \cdot \frac{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}}{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}} \cdot \frac{3+\sqrt{x+1}}{3+\sqrt{x+1}}$$

$$= \lim_{x\to 8} \frac{\overbrace{(2-\sqrt[3]{5+\sqrt{x+1}}\,)(4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}\,)}^{(a-b)(a^2+ab+b^2)}}{\underbrace{(3-\sqrt{x+1}\,)(3+\sqrt{x+1}\,)}_{(a-b)(a+b)}} \cdot$$

$$\cdot \frac{3+\sqrt{x+1}}{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}}$$

$$= \lim_{x\to 8} \frac{\overbrace{2^3-\sqrt[3]{(5+\sqrt{x+1}\,)^3}}^{a^3-b^3}}{\underbrace{3^2-\sqrt{(x+1)^2}}_{a^2-b^2}} \cdot \frac{3+\sqrt{x+1}}{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}}$$

$$= \lim_{x\to 8} \frac{8-(5+\sqrt{x+1}\,)}{9-(x+1)} \cdot \frac{3+\sqrt{x+1}}{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}}$$

$$= \lim_{x\to 8} \frac{8-5-\sqrt{x+1}}{9-x-1} \cdot \frac{3+\sqrt{x+1}}{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}}$$

$$= \lim_{x\to 8} \frac{3-\sqrt{x+1}}{-x+8} \cdot \frac{3+\sqrt{x+1}}{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}}$$

$$= \lim_{x\to 8} \frac{(3-\sqrt{x+1}\,)(3+\sqrt{x+1}\,)}{(-x+8)(4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}\,)}$$

$$= \lim_{x\to 8} \frac{3^2-\sqrt{(x+1)^2}}{(-x+8)(4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}\,)}$$

$$= \lim_{x\to 8} \cdot \frac{9-(x+1)}{(-x+8)(4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}\,)}$$

$$= \lim_{x\to 8} \cdot \frac{9-x-1}{(-x+8)(4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}\,)}$$

$$= \lim_{x\to 8} \cdot \frac{-x+8}{(-x+8)(4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}\,)}$$

$$= \lim_{x\to 8} \cdot \frac{\cancel{-x+8}}{\cancel{(-x+8)}(4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}\,)}$$

$$= \lim_{x\to 8} \frac{1}{4+2\sqrt[3]{5+\sqrt{x+1}}+\sqrt[3]{(5+\sqrt{x+1}\,)^2}}$$

$$= \frac{1}{4+2\sqrt[3]{5+\sqrt{8+1}}+\sqrt[3]{(5+\sqrt{8+1}\,)^2}} = \frac{1}{4+2\sqrt[3]{5+\sqrt{9}}+\sqrt[3]{(5+\sqrt{9}\,)^2}}$$

$$= \frac{1}{4+2\sqrt[3]{5+3}+\sqrt[3]{(5+3)^2}} = \frac{1}{4+2\sqrt[3]{8}+\sqrt[3]{(8)^2}} = \frac{1}{4+2\cdot 2+\sqrt[3]{64}}$$

$$= \frac{1}{4+4+4} = \frac{1}{12}$$

33. $\lim\limits_{x\to 2} \dfrac{\sqrt{3x}-\sqrt{8-x}}{3x-2\sqrt{15-3x}}$

Solución:

$$\lim_{x\to 2} \frac{\sqrt{3x}-\sqrt{8-x}}{3x-2\sqrt{15-3x}} = \frac{\sqrt{3\cdot 2}-\sqrt{8-2}}{3\cdot 2-2\sqrt{15-3\cdot 2}} = \frac{0}{0}(\text{Indeterminación})$$

$$\lim_{x\to 2} \frac{\sqrt{3x}-\sqrt{8-x}}{3x-2\sqrt{15-3x}} = \lim_{x\to 2} \frac{\sqrt{3x}-\sqrt{8-x}}{3x-2\sqrt{15-3x}} \cdot \frac{\sqrt{3x}+\sqrt{8-x}}{\sqrt{3x}+\sqrt{8-x}} \cdot \frac{3x+2\sqrt{15-3x}}{3x+2\sqrt{15-3x}}$$

$$= \lim_{x\to 2} \frac{\overbrace{(\sqrt{3x}-\sqrt{8-x}\,)(\sqrt{3x}+\sqrt{8-x}\,)}^{(a-b)(a+b)}}{\underbrace{(3x-2\sqrt{15-3x}\,)(3x+2\sqrt{15-3x}\,)}_{(a-b)(a+b)}} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$

$$= \lim_{x\to 2} \frac{\overbrace{\sqrt{(3x)^2}-\sqrt{(8-x)^2}}^{a^2-b^2}}{\underbrace{(3x)^2-4\sqrt{(15-3x)^2}}_{a^2-b^2}} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$

$$= \lim_{x\to 2} \frac{3x-(8-x)}{9x^2-4(15-3x)} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$

$$= \lim_{x\to 2} \frac{3x-8+x}{9x^2-60+12x} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$
$$= \lim_{x\to 2} \frac{4x-8}{9x^2+12x-60} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$
$$= \lim_{x\to 2} \frac{4(x-2)}{3(3x^2+4x-20)} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$
$$= \lim_{x\to 2} \frac{4(x-2)}{3(3x+10)(x-2)} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$
$$= \lim_{x\to 2} \frac{4\cancel{(x-2)}}{3(3x+10)\cancel{(x-2)}} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$
$$= \lim_{x\to 2} \frac{4}{3(3x+10)} \cdot \frac{3x+2\sqrt{15-3x}}{\sqrt{3x}+\sqrt{8-x}}$$
$$= \lim_{x\to 2} \frac{4(3x+2\sqrt{15-3x}\,)}{3(3x+10)(\sqrt{3x}+\sqrt{8-x}\,)}$$
$$= \frac{4(3\cdot 2+2\sqrt{15-3\cdot 2}\,)}{3(3\cdot 2+10)(\sqrt{3\cdot 2}+\sqrt{8-2}\,)}$$
$$= \frac{4(6+2\sqrt{15-6}\,)}{3(6+10)(\sqrt{6}+\sqrt{6}\,)} = \frac{4(6+2\sqrt{9}\,)}{3(16)(2\sqrt{6}\,)}$$
$$= \frac{6+2\cdot 3}{3(4)(2\sqrt{6}\,)} = \frac{6+6}{24\sqrt{6}} = \frac{12}{24\sqrt{6}} = \frac{1}{2\sqrt{6}}$$
$$= \frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{2\sqrt{6^2}} = \frac{\sqrt{6}}{2\cdot 6} = \frac{\sqrt{6}}{12}$$

34. $\lim\limits_{x\to 2} \dfrac{4\sqrt{x^2-4x+6}-\sqrt{x^2+14x}}{\sqrt{x}-\sqrt{2}}$

Solución:

$$\lim_{x\to 2} \frac{4\sqrt{x^2-4x+6}-\sqrt{x^2+14x}}{\sqrt{x}-\sqrt{2}} = \frac{4\sqrt{2^2-4\cdot 2+6}-\sqrt{2^2+14\cdot 2}}{\sqrt{2}-\sqrt{2}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 2} \frac{4\sqrt{x^2-4x+6}-\sqrt{x^2+14x}}{\sqrt{x}-\sqrt{2}} = \lim_{x\to 2} \frac{4\sqrt{x^2-4x+6}-\sqrt{x^2+14x}}{\sqrt{x}-\sqrt{2}} \cdot \frac{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$$

$$= \lim_{x\to 2} \frac{\overbrace{(4\sqrt{x^2-4x+6}-\sqrt{x^2+14x}\,)(4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}\,)}^{(a-b)(a+b)}}{\underbrace{(\sqrt{x}-\sqrt{2}\,)(\sqrt{x}+\sqrt{2}\,)}_{(a-b)(a+b)}}$$

$$\cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$

$$= \lim_{x\to 2} \frac{\overbrace{(4\sqrt{x^2-4x+6}\,)^2-(\sqrt{x^2+14x}\,)^2}^{a^2-b^2}}{\underbrace{(\sqrt{x}\,)^2-(\sqrt{2}\,)^2}_{a^2-b^2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$

$$= \lim_{x\to 2} \frac{16(x^2-4x+6)-(x^2+14x)}{x-2} \cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$
$$= \lim_{x\to 2} \frac{16x^2-64x+96-x^2-14x}{x-2} \cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$
$$= \lim_{x\to 2} \frac{15x^2-78x+96}{x-2} \cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$

$$= \lim_{x\to 2} \frac{3(5x^2-26x+32)}{x-2} \cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$
$$= \lim_{x\to 2} \frac{3(5x-16)(x-2)}{x-2} \cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$
$$= \lim_{x\to 2} \frac{3(5x-16)\cancel{(x-2)}}{\cancel{x-2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}}$$
$$= \lim_{x\to 2} \frac{3(5x-16)(\sqrt{x}+\sqrt{2}\,)}{4\sqrt{x^2-4x+6}+\sqrt{x^2+14x}} = \frac{3(5\cdot 2-16)(\sqrt{2}+\sqrt{2}\,)}{4\sqrt{2^2-4\cdot 2+6}+\sqrt{2^2+14\cdot 2}}$$
$$= \frac{3(10-16)(2\sqrt{2}\,)}{4\sqrt{4-8+6}+\sqrt{4+28}} = \frac{3(-6)(2\sqrt{2}\,)}{4\sqrt{2}+\sqrt{32}} = \frac{-36\sqrt{2}}{4\sqrt{2}+4\sqrt{2}} = \frac{-36\sqrt{2}}{8\sqrt{2}}$$
$$= \frac{-36\cancel{\sqrt{2}}}{8\cancel{\sqrt{2}}} = -\frac{9}{2}$$

35. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{8+3x}-2}{\sqrt[4]{16+6x}-2}$

Solución:

$$\lim_{x\to 0} \frac{\sqrt[3]{8+3x}-2}{\sqrt[4]{16+6x}-2} = \frac{\sqrt[3]{8+3\cdot 0}-2}{\sqrt[4]{16+6\cdot 0}-2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0} \frac{\sqrt[3]{8+3x}-2}{\sqrt[4]{16+6x}-2} = \lim_{x\to 0} \frac{\sqrt[3]{8+3x}-2}{\sqrt[4]{16+6x}-2} \cdot \frac{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4}{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4} \cdot$$
$$\cdot \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}$$

$$= \lim_{x\to 0} \frac{\overbrace{(\sqrt[3]{8+3x}-2)(\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4)}^{(a-b)(a^2+ab+b^2)}}{\underbrace{(\sqrt[4]{16+6x}-2)(\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8)}_{(a-b)(a^3+a^2b+ab^2+b^3)}} \cdot$$
$$\cdot \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4}$$

$$= \lim_{x\to 0} \frac{\overbrace{\sqrt[3]{(8+3x)^3}-2^3}^{a^3-b^3}}{\underbrace{\sqrt[4]{(16+6x)^4}-2^4}_{a^4-b^4}} \cdot \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4}$$

$$= \lim_{x\to 0} \frac{8+3x-8}{16+6x-16} \cdot \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4}$$
$$= \lim_{x\to 0} \frac{3x}{6x} \cdot \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4}$$
$$= \lim_{x\to 0} \frac{3\cancel{x}}{6\cancel{x}} \cdot \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4}$$
$$= \lim_{x\to 0} \frac{1}{2} \cdot \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4}$$
$$= \lim_{x\to 0} \frac{\sqrt[4]{(16+6x)^3}+2\sqrt[4]{(16+6x)^2}+4\sqrt[4]{16+6x}+8}{2(\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4)}$$
$$= \frac{\sqrt[4]{(16+6\cdot 0)^3}+2\sqrt[4]{(16+6\cdot 0)^2}+4\sqrt[4]{16+6\cdot 0}+8}{2(\sqrt[3]{(8+3\cdot 0)^2}+2\sqrt[3]{8+3\cdot 0}+4)}$$
$$= \frac{8+2\cdot 4+4\cdot 2+8}{2(4+2\cdot 2+4)} = \frac{8+8+8+8}{2(4+4+4)} = \frac{32}{2(12)} = \frac{16}{12} = \frac{4}{3}$$

36. $\lim_{x\to 1} \dfrac{\sqrt[3]{(x^2+1)^2} - 2\sqrt[3]{2x^2+2} + \sqrt[3]{4}}{(x-1)^2}$

Solución:

$$\lim_{x\to 1} \frac{\sqrt[3]{(x^2+1)^2} - 2\sqrt[3]{2x^2+2} + \sqrt[3]{4}}{(x-1)^2} = \frac{\sqrt[3]{(1^2+1)^2} - 2\sqrt[3]{2\cdot 1^2+2} + \sqrt[3]{4}}{(1-1)^2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt[3]{(x^2+1)^2} - 2\sqrt[3]{2x^2+2} + \sqrt[3]{4}}{(x-1)^2} &= \lim_{x\to 1} \frac{\sqrt[3]{(x^2+1)^2} - 2\sqrt[3]{2(x^2+1)} + \sqrt[3]{2^2}}{(x-1)^2} \\
&= \lim_{x\to 1} \frac{\sqrt[3]{(x^2+1)^2} - 2\sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{2^2}}{(x-1)^2} \\
&= \lim_{x\to 1} \frac{(\sqrt[3]{x^2+1} - \sqrt[3]{2})^2}{(x-1)^2} = \lim_{x\to 1} \left(\frac{\sqrt[3]{x^2+1} - \sqrt[3]{2}}{x-1} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{\sqrt[3]{x^2+1} - \sqrt[3]{2}}{x-1} \cdot \frac{\sqrt[3]{(x^2+1)^2} + \sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4}}{\sqrt[3]{(x^2+1)^2} + \sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4}} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{\overbrace{(\sqrt[3]{x^2+1} - \sqrt[3]{2})(\sqrt[3]{(x^2+1)^2} + \sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4})}^{(a-b)(a^2+ab+b^2)}}{(x-1)(\sqrt[3]{(x^2+1)^2} + \sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4})} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{\overbrace{\sqrt[3]{(x^2+1)^3} - \sqrt[3]{2^3}}^{a^3-b^3}}{(x-1)(\sqrt[3]{(x^2+1)^2} + 2\sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4})} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{x^2+1-2}{(x-1)(\sqrt[3]{(x^2+1)^2} + 2\sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4})} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{x^2-1}{(x-1)(\sqrt[3]{(x^2+1)^2} + 2\sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4})} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{(x-1)(x+1)}{(x-1)(\sqrt[3]{(x^2+1)^2} + 2\sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4})} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(\sqrt[3]{(x^2+1)^2} + \sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4})} \right)^2 \\
&= \lim_{x\to 1} \left(\frac{x+1}{\sqrt[3]{(x^2+1)^2} + \sqrt[3]{2}\sqrt[3]{x^2+1} + \sqrt[3]{4}} \right)^2 \\
&= \left(\frac{1+1}{\sqrt[3]{(1^2+1)^2} + \sqrt[3]{2}\sqrt[3]{1^2+1} + \sqrt[3]{4}} \right)^2 \\
&= \left(\frac{2}{\sqrt[3]{4} + \sqrt[3]{2}\sqrt[3]{2} + \sqrt[3]{4}} \right)^2 = \left(\frac{2}{\sqrt[3]{4} + \sqrt[3]{4} + \sqrt[3]{4}} \right)^2 \\
&= \left(\frac{2}{3\sqrt[3]{4}} \right)^2 = \frac{4}{9\sqrt[3]{16}} = \frac{4}{9\cdot 2\sqrt[3]{2}} = \frac{2}{9\sqrt[3]{2}} = \frac{\sqrt[3]{4}}{9}
\end{aligned}$$

37. $\lim_{x\to 25} \dfrac{\sqrt{8} - \sqrt{2+\sqrt{11+x}}}{25-x}$

Solución:

$$\lim_{x\to 25} \frac{\sqrt{8} - \sqrt{2+\sqrt{11+x}}}{25-x} = \frac{\sqrt{8} - \sqrt{2+\sqrt{11+25}}}{25-25} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 25} \frac{\sqrt{8} - \sqrt{2+\sqrt{11+x}}}{25-x} = \lim_{x\to 25} \frac{\sqrt{8} - \sqrt{2+\sqrt{11+x}}}{-x+25} \cdot \frac{\sqrt{8} + \sqrt{2+\sqrt{11+x}}}{\sqrt{8} + \sqrt{2+\sqrt{11+x}}}$$

$$= \lim_{x\to 25} \frac{\overbrace{(\sqrt{8}-\sqrt{2+\sqrt{11+x}}\,)(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}^{(a-b)(a+b)}}{-(x-25)} \cdot \frac{1}{\sqrt{8}+\sqrt{2+\sqrt{11+x}}}$$

$$= \lim_{x\to 25} \frac{\overbrace{(\sqrt{8}\,)^2-(\sqrt{2+\sqrt{11+x}}\,)^2}^{a^2-b^2}}{-(x-25)} \cdot \frac{1}{\sqrt{8}+\sqrt{2+\sqrt{11+x}}}$$

$$= \lim_{x\to 25} \frac{8-(2+\sqrt{11+x}\,)}{-(x-25)} \cdot \frac{1}{\sqrt{8}+\sqrt{2+\sqrt{11+x}}}$$

$$= \lim_{x\to 25} \frac{8-2-\sqrt{11+x}}{-(x-25)} \cdot \frac{1}{\sqrt{8}+\sqrt{2+\sqrt{11+x}}}$$

$$= \lim_{x\to 25} \frac{6-\sqrt{11+x}}{-(x-25)} \cdot \frac{1}{\sqrt{8}+\sqrt{2+\sqrt{11+x}}}$$

$$= \lim_{x\to 25} \frac{6-\sqrt{11+x}}{-(x-25)} \cdot \frac{6+\sqrt{11+x}}{6+\sqrt{11+x}} \cdot \frac{1}{\sqrt{8}+\sqrt{2+\sqrt{11+x}}}$$

$$= \lim_{x\to 25} \frac{\overbrace{(6-\sqrt{11+x}\,)(6+\sqrt{11+x}\,)}^{(a-b)(a+b)}}{-(x-25)} \cdot \frac{1}{(6+\sqrt{11+x}\,)(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \lim_{x\to 25} \frac{\overbrace{6^2-(\sqrt{11+x}\,)^2}^{a^2-b^2}}{-(x-25)} \cdot \frac{1}{(6+\sqrt{11+x})(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \lim_{x\to 25} \frac{36-(11+x)}{-(x-25)} \cdot \frac{1}{(6+\sqrt{11+x})(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \lim_{x\to 25} \frac{36-11-x}{-(x-25)} \cdot \frac{1}{(6+\sqrt{11+x})(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \lim_{x\to 25} \frac{-x+25}{-(x-25)} \cdot \frac{1}{(6+\sqrt{11+x})(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \lim_{x\to 25} \frac{-(x-25)}{-(x-25)} \cdot \frac{1}{(6+\sqrt{11+x})(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \lim_{x\to 25} \frac{\cancel{x-25}}{\cancel{x-25}} \cdot \frac{1}{(6+\sqrt{11+x})(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \lim_{x\to 25} \frac{1}{(6+\sqrt{11+x})(\sqrt{8}+\sqrt{2+\sqrt{11+x}}\,)}$$

$$= \frac{1}{(6+\sqrt{11+25})(\sqrt{8}+\sqrt{2+\sqrt{11+25}}\,)}$$

$$= \frac{1}{(6+\sqrt{36})(\sqrt{8}+\sqrt{2+\sqrt{36}}\,)}$$

$$= \frac{1}{(6+6)(\sqrt{8}+\sqrt{2+6}\,)} = \frac{1}{(12)(\sqrt{8}+\sqrt{8}\,)}$$

$$= \frac{1}{12(2\sqrt{8}\,)} = \frac{1}{12(2\cdot 2\sqrt{2}\,)} = \frac{1}{12(4\sqrt{2}\,)}$$

$$= \frac{1}{48\sqrt{2}} = \frac{1}{48\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{48(\sqrt{2}\,)^2} = \frac{\sqrt{2}}{48\cdot 2} = \frac{\sqrt{2}}{96}$$

38. $\lim_{x\to 8} \dfrac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$

Solución:

$$\lim_{x\to 8}\frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}=\frac{\sqrt{9+2\cdot 8}-5}{\sqrt[3]{8}-2}=\frac{0}{0}\qquad \text{(Indeterminación)}$$

$$\lim_{x\to 8}\frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}=\lim_{x\to 8}\frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}\cdot\frac{\sqrt{9+2x}+5}{\sqrt{9+2x}+5}\cdot\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}$$

$$=\lim_{x\to 8}\frac{\overbrace{(\sqrt{9+2x}-5)(\sqrt{9+2x}+5)}^{(a-b)(a+b)}}{\underbrace{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}_{(a-b)(a^2+ab+b^2)}}\cdot\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt{9+2x}+5}$$

$$=\lim_{x\to 8}\frac{\overbrace{(\sqrt{9+2x})^2-5^2}^{a^2-b^2}}{\underbrace{\sqrt[3]{x^3}-2^3}_{a^3-b^3}}\cdot\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt{9+2x}+5}=\lim_{x\to 8}\frac{9+2x-25}{x-8}\cdot\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt{9+2x}+5}$$

$$=\lim_{x\to 8}\frac{2x-16}{x-8}\cdot\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt{9+2x}+5}=\lim_{x\to 8}\frac{2(x-8)}{x-8}\cdot\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt{9+2x}+5}$$

$$=\lim_{x\to 8}\frac{2\cancel{(x-8)}}{\cancel{x-8}}\cdot\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt{9+2x}+5}=\lim_{x\to 8}\frac{2(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}{\sqrt{9+2x}+5}$$

$$=\frac{2(\sqrt[3]{8^2}+2\sqrt[3]{8}+4)}{\sqrt{9+2\cdot 8}+5}=\frac{2(\sqrt[3]{64}+2\cdot 2+4)}{\sqrt{9+16}+5}$$

$$=\frac{2(4+4+4)}{\sqrt{25}+5}=\frac{2(12)}{5+5}=\frac{24}{10}=\frac{12}{5}$$

39. $\lim_{x\to 0}\frac{\sqrt[3]{8+3x-x^2}-2}{x+x^2}$

Solución:

$$\lim_{x\to 0}\frac{\sqrt[3]{8+3x-x^2}-2}{x+x^2}=\frac{\sqrt[3]{8+3\cdot 0-0^2}-2}{0+0^2}=\frac{0}{0}\qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{8+3x-x^2}-2}{x+x^2}=\lim_{x\to 0}\frac{\sqrt[3]{8+3x-x^2}-2}{x^2+x}\cdot\frac{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$=\lim_{x\to 0}\frac{\overbrace{(\sqrt[3]{8+3x-x^2}-2)(\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4)}^{(a-b)(a^2+ab+b^2)}}{x(x+1)}\cdot$$

$$\cdot\frac{1}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$=\lim_{x\to 0}\frac{\overbrace{\sqrt[3]{(8+3x-x^2)^3}-2^3}^{a^3-b^3}}{x(x+1)}\cdot\frac{1}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$=\lim_{x\to 0}\frac{8+3x-x^2-8}{x(x+1)}\cdot\frac{1}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$=\lim_{x\to 0}\frac{-x^2+3x}{x(x+1)}\cdot\frac{1}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$=\lim_{x\to 0}\frac{-x(x-3)}{x(x+1)}\cdot\frac{1}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$=\lim_{x\to 0}-\frac{\cancel{x}(x-3)}{\cancel{x}(x+1)}\cdot\frac{1}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$=\lim_{x\to 0}-\frac{x-3}{x+1}\cdot\frac{1}{\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4}$$

$$= \lim_{x\to 0} -\frac{x-3}{(x+1)(\sqrt[3]{(8+3x-x^2)^2}+2\sqrt[3]{8+3x-x^2}+4)}$$
$$= -\frac{0-3}{(0+1)(\sqrt[3]{(8+3\cdot 0-0^2)^2}+2\sqrt[3]{8+3\cdot 0-0^2}+4)}$$
$$= -\frac{-3}{(1)(\sqrt[3]{(8+0-0)^2}+2\sqrt[3]{8+0-0}+4)} = \frac{3}{\sqrt[3]{64}+2\sqrt[3]{8}+4}$$
$$= \frac{3}{4+4+4} = \frac{3}{12} = \frac{1}{4}$$

40. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}}$

Solución:

$$\lim_{x\to 0} \frac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}} = \frac{\sqrt[3]{27+0}-\sqrt[3]{27-0}}{0+2\sqrt[3]{0^4}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0} \frac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}} = \lim_{x\to 0} \frac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2x\sqrt[3]{x}} = \lim_{x\to 0} \frac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x(1+2\sqrt[3]{x})}$$
$$= \lim_{x\to 0} \frac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x(1+2\sqrt[3]{x})} \cdot \frac{\sqrt[3]{(27+x)^2}+\sqrt[3]{27+x}\cdot\sqrt[3]{27-x}+\sqrt[3]{(27-x)^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{27+x}\cdot\sqrt[3]{27-x}+\sqrt[3]{(27-x)^2}} \cdot$$
$$\cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}$$
$$= \lim_{x\to 0} \frac{\overbrace{(\sqrt[3]{27+x}-\sqrt[3]{27-x})(\sqrt[3]{(27+x)^2}+\sqrt[3]{27+x}\cdot\sqrt[3]{27-x}+\sqrt[3]{(27-x)^2})}^{(a-b)(a^2+ab+b^2)}}{x\underbrace{(1+2\sqrt[3]{x})(1-2\sqrt[3]{x}+4\sqrt[3]{x^2})}_{(a-b)(a^2+ab+b^2)}}$$
$$\cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{27+x}\cdot\sqrt[3]{27-x}+\sqrt[3]{(27-x)^2}}$$
$$= \lim_{x\to 0} \frac{\overbrace{\sqrt[3]{(27+x)^3}-\sqrt[3]{(27-x)^3}}^{a^3-b^3}}{x\underbrace{(1^3+8\sqrt[3]{x^3})}_{a^3-b^3}} \cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{27+x}\cdot\sqrt[3]{27-x}+\sqrt[3]{(27-x)^2}}$$
$$= \lim_{x\to 0} \frac{(27+x)-(27-x)}{x(1+8x)} \cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{(27+x)(27-x)}+\sqrt[3]{(27-x)^2}}$$
$$= \lim_{x\to 0} \frac{27+x-27+x}{x(1+8x)} \cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{729-x^2}+\sqrt[3]{(27-x)^2}}$$
$$= \lim_{x\to 0} \frac{2x}{x(1+8x)} \cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{729-x^2}+\sqrt[3]{(27-x)^2}}$$
$$= \lim_{x\to 0} \frac{2\not{x}}{\not{x}(1+8x)} \cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{729-x^2}+\sqrt[3]{(27-x)^2}}$$
$$= \lim_{x\to 0} \frac{2}{1+8x} \cdot \frac{1-2\sqrt[3]{x}+4\sqrt[3]{x^2}}{\sqrt[3]{(27+x)^2}+\sqrt[3]{729-x^2}+\sqrt[3]{(27-x)^2}}$$
$$= \lim_{x\to 0} \frac{2(1-2\sqrt[3]{x}+4\sqrt[3]{x^2})}{(1+8x)(\sqrt[3]{(27+x)^2}+\sqrt[3]{729-x^2}+\sqrt[3]{(27-x)^2})}$$
$$= \frac{2(1-2\sqrt[3]{0}+4\sqrt[3]{0^2})}{(1+8\cdot 0)(\sqrt[3]{(27+0)^2}+\sqrt[3]{729-0^2}+\sqrt[3]{(27-0)^2})}$$
$$= \frac{2(1-2\cdot 0+4\cdot 0)}{(1+0)(\sqrt[3]{(27)^2}+\sqrt[3]{729}+\sqrt[3]{(27)^2})} = \frac{2(1-0+0)}{(1)(\sqrt[3]{729}+\sqrt[3]{729}+\sqrt[3]{729})}$$

$$= \frac{2(1)}{9+9+9} = \frac{2}{27}$$

41. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}$

Solución:

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}} = \frac{\sqrt{1+0}-\sqrt{1-0}}{\sqrt[3]{1+0}-\sqrt[3]{1-0}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}} = \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}\cdot\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}}{\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}\cdot\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}}$$

$$= \lim_{x\to 0} \frac{\overbrace{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}^{(a-b)(a+b)}}{\underbrace{(\sqrt[3]{1+x}-\sqrt[3]{1-x})(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}\cdot\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2})}_{(a-b)(a^2+ab+b^2)}} \cdot$$

$$\cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}\cdot\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{\overbrace{\sqrt{(1+x)^2}-\sqrt{(1-x)^2}}^{(a^2-b^2)}}{\underbrace{\sqrt[3]{(1+x)^3}-\sqrt[3]{(1-x)^3}}_{a^3-b^3}} \cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{(1+x)-(1-x)}{(1+x)-(1-x)} \cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{1+x-1+x}{1+x-1+x} \cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{2x}{2x} \cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{\cancel{2x}}{\cancel{2x}} \cdot \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \frac{\sqrt[3]{(1+0)^2}+\sqrt[3]{1-0^2}+\sqrt[3]{(1-0)^2}}{\sqrt{1+0}+\sqrt{1-0}}$$

$$= \frac{\sqrt[3]{(1)^2}+\sqrt[3]{1}+\sqrt[3]{(1)^2}}{\sqrt{1}+\sqrt{1}} = \frac{\sqrt[3]{1}+1+\sqrt[3]{1}}{1+1}$$

$$= \frac{1+1+1}{2} = \frac{3}{2}$$

42. $\lim\limits_{x\to 0} \dfrac{x^2}{\sqrt[3]{1+5x}-(1+x)}$

Solución:

$$\lim_{x\to 0} \frac{x^2}{\sqrt[3]{1+5x}-(1+x)} = \frac{0^2}{\sqrt[3]{1+5\cdot 0}-(1+0)} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0} \frac{x^2}{\sqrt[3]{1+5x}-(1+x)} = \lim_{x\to 0} \frac{x^2}{\sqrt[3]{1+5x}-(1+x)} \cdot \frac{\sqrt[3]{(1+5x)^2}+\sqrt[3]{1+5x}(1+x)+(1+x)^2}{\sqrt[3]{(1+5x)^2}+\sqrt[3]{1+5x}(1+x)+(1+x)^2}$$

$$= \lim_{x\to 0} \frac{x^2(\sqrt[3]{(1+5x)^2}+\sqrt[3]{1+5x}(1+x)+(1+x)^2)}{\underbrace{\left(\sqrt[3]{1+5x}-(1+x)\right)\left(\sqrt[3]{(1+5x)^2}+\sqrt[3]{1+5x}(1+x)+(1+x)^2\right)}_{(a-b)(a^2+ab+b^2)}}$$

$$= \lim_{x\to 0} \frac{x^2(\sqrt[3]{(1+5x)^2} + \sqrt[3]{1+5x}(1+x) + (1+x)^2)}{\underbrace{\sqrt[3]{(1+5x)^3 - (1+x)^3}}_{a^3-b^3}}$$

$$= \lim_{x\to 0} \frac{x^2(\sqrt[3]{(1+5x)^2} + \sqrt[3]{1+5x}(1+x) + (1+x)^2)}{1+5x-(1+3x+3x^2+x^3)}$$

$$= \lim_{x\to 0} \frac{x^2(\sqrt[3]{(1+5x)^2} + \sqrt[3]{1+5x}(1+x) + (1+x)^2)}{1+5x-1-3x-3x^2-x^3}$$

$$= \lim_{x\to 0} \frac{x^2(\sqrt[3]{(1+5x)^2} + \sqrt[3]{1+5x}(1+x) + (1+x)^2)}{-x^3-3x^2+2x}$$

$$= \lim_{x\to 0} \frac{x^2(\sqrt[3]{(1+5x)^2} + \sqrt[3]{1+5x}(1+x) + (1+x)^2)}{-x(x^2+3x-2)}$$

$$= \lim_{x\to 0} -\frac{x^{\not 2}(\sqrt[3]{(1+5x)^2} + \sqrt[3]{1+5x}(1+x) + (1+x)^2)}{\not x(x^2+3x-2)}$$

$$= \lim_{x\to 0} -\frac{x(\sqrt[3]{(1+5x)^2} + \sqrt[3]{1+5x}(1+x) + (1+x)^2)}{x^2+3x-2}$$

$$= -\frac{0\cdot(\sqrt[3]{(1+5\cdot 0)^2} + \sqrt[3]{1+5\cdot 0}(1+0) + (1+0)^2)}{0^2+3\cdot 0-2}$$

$$= -\frac{0\cdot(1+1+1)}{-2} = \frac{0\cdot 3}{2} = \frac{0}{2} = 0$$

44. $\lim\limits_{x\to 5} \dfrac{\sqrt{x+4}-3}{\sqrt{x-4}-\sqrt{3x-14}}$

Solución:

$$\lim_{x\to 5} \frac{\sqrt{x+4}-3}{\sqrt{x-4}-\sqrt{3x-14}} = \frac{\sqrt{5+4}-3}{\sqrt{5-4}-\sqrt{3\cdot 5-14}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 5} \frac{\sqrt{x+4}-3}{\sqrt{x-4}-\sqrt{3x-14}} = \lim_{x\to 5} \frac{\sqrt{x+4}-3}{\sqrt{x-4}-\sqrt{3x-14}} \cdot \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x-4}+\sqrt{3x-14}}$$

$$= \lim_{x\to 5} \frac{\overbrace{(\sqrt{x+4}-3)(\sqrt{x+4}+3)}^{(a-b)(a+b)}}{\underbrace{(\sqrt{x-4}-\sqrt{3x-14})(\sqrt{x-4}+\sqrt{3x-14})}_{(a-b)(a+b)}} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} \frac{\overbrace{\sqrt{(x+4)^2}-3^2}^{a^2-b^2}}{\underbrace{\sqrt{(x-4)^2}-\sqrt{(3x-14)^2}}_{a^2-b^2}} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} \frac{x+4-9}{(x-4)-(3x-14)} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} \frac{x-5}{x-4-3x+14} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} \frac{x-5}{-2x+10} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} \frac{x-5}{-2(x-5)} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} -\frac{\cancel{x-5}}{2\cancel{(x-5)}} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} -\frac{1}{2} \cdot \frac{\sqrt{x-4}+\sqrt{3x-14}}{\sqrt{x+4}+3} = \lim_{x\to 5} -\frac{\sqrt{x-4}+\sqrt{3x-14}}{2(\sqrt{x+4}+3)}$$

$$= -\frac{\sqrt{5-4}+\sqrt{3\cdot 5-14}}{2(\sqrt{5+4}+3)} = -\frac{\sqrt{1}+\sqrt{15-14}}{2(\sqrt{9}+3)}$$

$$= -\frac{1+\sqrt{1}}{2(3+3)} = -\frac{1+1}{2(6)} = -\frac{2}{12} = -\frac{1}{6}$$

45. $\lim\limits_{x\to 5} \dfrac{2-\sqrt{x-1}}{1-\sqrt[3]{3-\sqrt{x-1}}}$

Solución:

$$\lim_{x\to 5} \frac{2-\sqrt{x-1}}{1-\sqrt[3]{3-\sqrt{x-1}}} = \frac{2-\sqrt{x-1}}{1-\sqrt[3]{3-\sqrt{x-1}}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 5} \frac{2-\sqrt{x-1}}{1-\sqrt[3]{3-\sqrt{x-1}}} = \lim_{x\to 5} \frac{2-\sqrt{x-1}}{1-\sqrt[3]{3-\sqrt{x-1}}} \cdot \frac{2+\sqrt{x-1}}{2+\sqrt{x-1}} \cdot \frac{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}$$

$$= \lim_{x\to 5} \frac{\overbrace{(2-\sqrt{x-1}\,)(2+\sqrt{x-1}\,)}^{(a-b)(a+b)}}{\underbrace{(1-\sqrt[3]{3-\sqrt{x-1}}\,)(1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}\,)}_{(a-b)(a^2+ab+b^2)}} \cdot$$

$$\cdot \frac{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}{2+\sqrt{x-1}}$$

$$= \lim_{x\to 5} \frac{\overbrace{2^2-\sqrt{(x-1)^2}}^{a^2-b^2}}{\underbrace{1^3-\sqrt[3]{(3-\sqrt{x-1})^3}}_{a^3-b^3}} \cdot \frac{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}{2+\sqrt{x-1}}$$

$$= \lim_{x\to 5} \frac{4-(x-1)}{1-(3-\sqrt{x-1})} \cdot \frac{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}{2+\sqrt{x-1}}$$

$$= \lim_{x\to 5} \frac{4-x+1}{1-3+\sqrt{x-1}} \cdot \frac{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}{2+\sqrt{x-1}}$$

$$= \lim_{x\to 5} \frac{-x+5}{-2+\sqrt{x-1}} \cdot \frac{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}{2+\sqrt{x-1}}$$

$$= \lim_{x\to 5} \frac{-(x-5)}{\sqrt{x-1}-2} \cdot \frac{1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}}{\sqrt{x-1}+2}$$

$$= \lim_{x\to 5} \frac{-(x-5)(1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}\,)}{\sqrt{(x-1)^2}-2^2}$$

$$= \lim_{x\to 5} \frac{-(x-5)(1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}\,)}{x-1-4}$$

$$= \lim_{x\to 5} \frac{-(x-5)(1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}\,)}{x-5}$$

$$= \lim_{x\to 5} -\frac{\cancel{(x-5)}(1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}\,)}{\cancel{x-5}}$$

$$= \lim_{x\to 5} -(1+\sqrt[3]{3-\sqrt{x-1}}+\sqrt[3]{(3-\sqrt{x-1})^2}\,)$$

$$= -(1+\sqrt[3]{3-\sqrt{5-1}}+\sqrt[3]{(3-\sqrt{5-1})^2}\,)$$

$$= -(1 + \sqrt[3]{3-\sqrt{4}} + \sqrt[3]{(3-\sqrt{4})^2}\,) = -(1 + \sqrt[3]{3-2} + \sqrt[3]{(3-2)^2}\,)$$
$$= -(1 + \sqrt[3]{1} + \sqrt[3]{(1)^2}\,) = -(1 + 1 + \sqrt[3]{1}\,)$$
$$= -(1+1+1) = -3$$

47. $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x+7}-2}{\sqrt{x+7}-2\sqrt{2}}$

Solución:

$$\lim_{x\to 1} \frac{\sqrt[3]{x+7}-2}{\sqrt{x+7}-2\sqrt{2}} = \frac{\sqrt[3]{1+7}-2}{\sqrt{1+7}-2\sqrt{2}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1} \frac{\sqrt[3]{x+7}-2}{\sqrt{x+7}-2\sqrt{2}} = \lim_{x\to 1} \frac{\sqrt[3]{x+7}-2}{\sqrt{x+7}-2\sqrt{2}} \cdot \frac{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4} \cdot \frac{\sqrt{x+7}+2\sqrt{2}}{\sqrt{x+7}+2\sqrt{2}}$$

$$= \lim_{x\to 1} \frac{\overbrace{(\sqrt[3]{x+7}-2)(\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4)}^{(a-b)(a^2+ab+b^2)}}{\underbrace{(\sqrt{x+7}-2\sqrt{2}\,)(\sqrt{x+7}+2\sqrt{2}\,)}_{(a-b)(a+b)}} \cdot \frac{\sqrt{x+7}+2\sqrt{2}}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}$$

$$= \lim_{x\to 1} \frac{\overbrace{\sqrt[3]{(x+7)^3}-2^3}^{a^3-b^3}}{\underbrace{\sqrt{(x+7)^2}-(2\sqrt{2}\,)^2}_{a^2-b^2}} \cdot \frac{\sqrt{x+7}+2\sqrt{2}}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}$$

$$= \lim_{x\to 1} \frac{x+7-8}{x+7-8} \cdot \frac{\sqrt{x+7}+2\sqrt{2}}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}$$

$$= \lim_{x\to 1} \frac{x-1}{x-1} \cdot \frac{\sqrt{x+7}+2\sqrt{2}}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}$$

$$= \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{x-1}} \cdot \frac{\sqrt{x+7}+2\sqrt{2}}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}$$

$$= \lim_{x\to 1} \frac{\sqrt{x+7}+2\sqrt{2}}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}$$

$$= \frac{\sqrt{1+7}+2\sqrt{2}}{\sqrt[3]{(1+7)^2}+2\sqrt[3]{1+7}+4} = \frac{\sqrt{8}+2\sqrt{2}}{\sqrt[3]{(8)^2}+2\sqrt[3]{8}+4}$$

$$= \frac{2\sqrt{2}+2\sqrt{2}}{\sqrt[3]{64}+2\cdot 2+4} = \frac{4\sqrt{2}}{4+4+4} = \frac{4\sqrt{2}}{12} = \frac{\sqrt{2}}{3}$$

48. $\lim\limits_{x\to 20} \dfrac{2\sqrt[4]{x-4}-4}{\sqrt[5]{x+12}-2}$

Solución:

$$\lim_{x\to 20} \frac{2\sqrt[4]{x-4}-4}{\sqrt[5]{x+12}-2} = \frac{2\sqrt[4]{20-4}-4}{\sqrt[5]{20+12}-2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 20} \frac{2\sqrt[4]{x-4}-4}{\sqrt[5]{x+12}-2} = \lim_{x\to 20} \frac{2(\sqrt[4]{x-4}-2)}{\sqrt[5]{x+12}-2} = 2\lim_{x\to 20} \frac{\sqrt[4]{x-4}-2}{\sqrt[5]{x+12}-2}$$

$$= 2\lim_{x\to 20} \frac{\sqrt[4]{x-4}-2}{\sqrt[5]{x+12}-2} \cdot \frac{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8}{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8} \cdot$$
$$\cdot \frac{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}$$

$$= 2\lim_{x\to 20} \frac{\overbrace{(\sqrt[4]{x-4}-2)(\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8)}^{(a-b)(a^3+a^2b+ab^2+b^3)}}{\underbrace{(\sqrt[5]{x+12}-2)(\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16)}_{(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)}} \cdot$$

$$\cdot \frac{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8}$$

$$= 2\lim_{x\to 20} \frac{\overbrace{\sqrt[4]{(x-4)^4}-2^4}^{a^4-b^4}}{\underbrace{\sqrt[5]{(x+12)^5}-2^5}_{a^5-b^5}} \cdot \frac{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8}$$

$$= 2\lim_{x\to 20} \frac{x-4-16}{x+12-32} \cdot \frac{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8}$$

$$= 2\lim_{x\to 20} \frac{x-20}{x-20} \cdot \frac{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8}$$

$$= 2\lim_{x\to 20} \frac{\cancel{x-20}}{\cancel{x-20}} \cdot \frac{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8}$$

$$= 2\lim_{x\to 20} \frac{\sqrt[5]{(x+12)^4}+2\sqrt[5]{(x+12)^3}+4\sqrt[5]{(x+12)^2}+8\sqrt[5]{x+12}+16}{\sqrt[4]{(x-4)^3}+2\sqrt[4]{(x-4)^2}+4\sqrt[4]{x-4}+8}$$

$$= 2 \cdot \frac{\sqrt[5]{(20+12)^4}+2\sqrt[5]{(20+12)^3}+4\sqrt[5]{(20+12)^2}+8\sqrt[5]{20+12}+16}{\sqrt[4]{(20-4)^3}+2\sqrt[4]{(20-4)^2}+4\sqrt[4]{20-4}+8}$$

$$= 2 \cdot \frac{\sqrt[5]{(32)^4}+2\sqrt[5]{(32)^3}+4\sqrt[5]{(32)^2}+8\sqrt[5]{32}+16}{\sqrt[4]{(16)^3}+2\sqrt[4]{(16)^2}+4\sqrt[4]{16}+8}$$

$$= 2 \cdot \frac{2^4+2\cdot 2^3+4\cdot 2^2+8\cdot 2+16}{2^3+2\cdot 2^2+4\cdot 2+8}$$

$$= 2 \cdot \frac{16+16+16+16+16}{8+8+8+8} = 2\cdot\frac{80}{32} = \frac{80}{16} = 5$$

59. $\displaystyle\lim_{x\to 4} \frac{x-\sqrt{x}-2}{x-4}$

Solución:

$$\lim_{x\to 4} \frac{x-\sqrt{x}-2}{x-4} = \frac{4-\sqrt{4}-2}{4-4} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 4} \frac{x-\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{x-2-\sqrt{x}}{x-4} = \lim_{x\to 4} \frac{(x-2)-\sqrt{x}}{x-4}$$

$$= \lim_{x\to 4} \frac{(x-2)-\sqrt{x}}{x-4} \cdot \frac{(x-2)+\sqrt{x}}{(x-2)+\sqrt{x}}$$

$$= \lim_{x\to 4} \frac{\overbrace{((x-2)-\sqrt{x}\,)((x-2)+\sqrt{x}\,)}^{(a-b)(a+b)}}{x-4} \cdot \frac{1}{(x-2)+\sqrt{x}}$$

$$= \lim_{x\to 4} \frac{\overbrace{(x-2)^2-\sqrt{x^2}}^{a^2-b^2}}{x-4} \cdot \frac{1}{(x-2)+\sqrt{x}}$$

$$= \lim_{x\to 4} \frac{x^2-4x+4-x}{x-4} \cdot \frac{1}{(x-2)+\sqrt{x}}$$

$$= \lim_{x\to 4} \frac{x^2-5x+4}{x-4} \cdot \frac{1}{(x-2)+\sqrt{x}}$$

$$= \lim_{x\to 4} \frac{(x-4)(x-1)}{x-4} \cdot \frac{1}{(x-2)+\sqrt{x}}$$
$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(x-1)}{\cancel{x-4}} \cdot \frac{1}{(x-2)+\sqrt{x}}$$
$$= \lim_{x\to 4} (x-1) \cdot \frac{1}{(x-2)+\sqrt{x}}$$
$$= \lim_{x\to 4} \frac{x-1}{(x-2)+\sqrt{x}}$$
$$= \frac{4-1}{(4-2)+\sqrt{4}} = \frac{3}{2+2} = \frac{3}{4}$$

60. $\lim_{x\to 2} \frac{\sqrt{x+14}-2\sqrt{x+2}}{x-2}$

Solución:

$$\lim_{x\to 2} \frac{\sqrt{x+14}-2\sqrt{x+2}}{x-2} = \frac{\sqrt{2+14}-2\sqrt{2+2}}{2-2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$
$$\lim_{x\to 2} \frac{\sqrt{x+14}-2\sqrt{x+2}}{x-2} = \lim_{x\to 2} \frac{\sqrt{x+14}-2\sqrt{x+2}}{x-2} \cdot \frac{\sqrt{x+14}+2\sqrt{x+2}}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} \frac{\overbrace{(\sqrt{x+14}-2\sqrt{x+2})(\sqrt{x+14}+2\sqrt{x+2})}^{(a-b)(a+b)}}{x-2} \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} \frac{\overbrace{\sqrt{(x+14)^2}-4\sqrt{(x+2)^2}}^{a^2-b^2}}{x-2} \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} \frac{x+14-4(x+2)}{x-2} \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} \frac{x+14-4x-8}{x-2} \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} \frac{-3x+6}{x-2} \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} \frac{-3(x-2)}{x-2} \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} \frac{-3\cancel{(x-2)}}{\cancel{x-2}} \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} -3 \cdot \frac{1}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= \lim_{x\to 2} -\frac{3}{\sqrt{x+14}+2\sqrt{x+2}}$$
$$= -\frac{3}{\sqrt{2+14}+2\sqrt{2+2}} = -\frac{3}{4+2\cdot 2}$$
$$= -\frac{3}{4+4} = -\frac{3}{8}$$

61. $\lim_{x\to 3} \frac{x^2-6-\sqrt{x+6}}{\sqrt{x+1}-2}$

Solución:

$$\lim_{x\to 3} \frac{x^2-6-\sqrt{x+6}}{\sqrt{x+1}-2} = \lim_{x\to 3} \frac{3^2-6-\sqrt{3+6}}{\sqrt{3+1}-2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$
$$\lim_{x\to 3} \frac{x^2-6-\sqrt{x+6}}{\sqrt{x+1}-2} = \lim_{x\to 3} \frac{(x^2-6)-\sqrt{x+6}}{\sqrt{x+1}-2} \cdot \frac{(x^2-6)+\sqrt{x+6}}{(x^2-6)+\sqrt{x+6}} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$$

$$= \lim_{x\to 3} \frac{\overbrace{((x^2-6)-\sqrt{x+6}\,)((x^2-6)+\sqrt{x+6}\,)}^{(a-b)(a+b)}}{\underbrace{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}_{(a-b)(a+b)}} \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$= \lim_{x\to 3} \frac{\overbrace{(x^2-6)^2-\sqrt{(x+6)^2}}^{a^2-b^2}}{\underbrace{\sqrt{(x+1)^2}-2^2}_{a^2-b^2}} \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$= \lim_{x\to 3} \frac{(x^4-12x^2+36)-(x+6)}{x+1-4} \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$= \lim_{x\to 3} \frac{x^4-12x^2+36-x-6}{x-3} \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$= \lim_{x\to 3} \frac{x^4-12x^2-x+30}{x-3} \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$\begin{array}{r|rrrrr} & 1 & 0 & -12 & -1 & 30 \\ 3 & & 3 & 9 & -9 & -30 \\ \hline & 1 & 3 & -3 & -10 & 0 \end{array}$$

$$x^4-12x^2-x+30=(x-3)(x^3+3x^2-3x-10)$$

$$= \lim_{x\to 3} \frac{(x-3)(x^3+3x^2-3x-10)}{x-3} \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x^3+3x^2-3x-10)}{\cancel{x-3}} \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$= \lim_{x\to 3}(x^3+3x^2-3x-10) \cdot \frac{\sqrt{x+1}+2}{(x^2-6)+\sqrt{x+6}}$$

$$= \lim_{x\to 3} \frac{(x^3+3x^2-3x-10)(\sqrt{x+1}+2)}{(x^2-6)+\sqrt{x+6}}$$

$$= \frac{(3^3+3\cdot 3^2-3\cdot 3-10)(\sqrt{3+1}+2)}{(3^2-6)+\sqrt{3+6}}$$

$$= \frac{(27+27-9-10)(\sqrt{4}+2)}{(9-6)+\sqrt{9}}$$

$$= \frac{(27+27-9-10)(\sqrt{4}+2)}{(9-6)+\sqrt{9}}$$

$$= \frac{(35)(4)}{3+3} = \frac{(35)(4)}{6} = \frac{70}{3}$$

63. $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

Solución:

$$\lim_{x\to 1} \frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1} = \lim_{x\to 1} \frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$$

$$\lim_{x\to 1} \frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1} = \lim_{x\to 1} \frac{\sqrt[3]{7+x^3}-2-\sqrt{3+x^2}+2}{x-1} = \lim_{x\to 1} \frac{(\sqrt[3]{7+x^3}-2)-(\sqrt{3+x^2}-2)}{x-1}$$

$$= \lim_{x\to 1}\left(\frac{\sqrt[3]{7+x^3}-2}{x-1} - \frac{\sqrt{3+x^2}-2}{x-1} \right)$$

$$= \lim_{x\to 1}\left(\frac{\sqrt[3]{7+x^3}-2}{x-1} \cdot \frac{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4} - \right.$$

$$-\frac{\sqrt{3+x^2}-2}{x-1}\cdot\frac{\sqrt{3+x^2}+2}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\Bigg(\frac{\overbrace{(\sqrt[3]{7+x^3}-2)(\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)}^{(a-b)(a^2+ab+b^2)}}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-$$

$$-\frac{\overbrace{(\sqrt{3+x^2}+2)(\sqrt{3+x^2}-2)}^{(a-b)(a+b)}}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\Bigg(\frac{\overbrace{(\sqrt[3]{(7+x^3)^3}-2^3}^{a^3-b^3}}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-$$

$$-\frac{\overbrace{\sqrt{(3+x^2)^2}-2^2}^{a^2-b^2}}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\Bigg(\frac{7+x^3-8}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-$$

$$-\frac{3+x^2-4}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\Bigg(\frac{x^3-1}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-$$

$$-\frac{x^2-1}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\Bigg(\frac{(x-1)(x^2+x+1)}{x-1}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-$$

$$-\frac{(x-1)(x+1)}{x-1}\cdot\frac{1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\Bigg(\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}\cdot\frac{1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-$$

$$-\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\cdot\frac{1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\lim_{x\to 1}\Bigg(\frac{x^2+x+1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}-\frac{x+1}{\sqrt{3+x^2}+2}\Bigg)$$

$$=\frac{1^2+1+1}{\sqrt[3]{(7+1^3)^2}+2\sqrt[3]{7+1^3}+4}-\frac{1+1}{\sqrt{3+1^2}+2}$$

$$=\frac{3}{4+4+4}-\frac{2}{2+2}=\frac{3}{12}-\frac{2}{4}=\frac{1}{4}-\frac{1}{2}=\frac{1-2}{4}=\frac{-1}{4}=-\frac{1}{4}$$

64. $\displaystyle\lim_{x\to 1}\frac{\sqrt[3]{2x-1}-3\sqrt{2x-1}+3x-1}{x-1}$

Solución:

$$\lim_{x\to 1}\frac{\sqrt[3]{2x-1}-3\sqrt{2x-1}+3x-1}{x-1}=\lim_{x\to 1}\frac{\sqrt[3]{2\cdot 1-1}-3\sqrt{2\cdot 1-1}+3\cdot 1-1}{1-1}=\frac{0}{0}\qquad\text{(Indeterminación)}$$

$$\lim_{x\to 1}\frac{\sqrt[3]{2x-1}-3\sqrt{2x-1}+3x-1}{x-1}=\lim_{x\to 1}\frac{\sqrt[3]{2x-1}-1-3\sqrt{2x-1}+3x}{x-1}$$

$$=\lim_{x\to 1}\frac{(\sqrt[3]{2x-1}-1)-3(\sqrt{2x-1}-x)}{x-1}$$

$$=\lim_{x\to 1}\frac{1}{x-1}\Big((\sqrt[3]{2x-1}-1)-3(\sqrt{2x-1}-x)\Big)$$

$$= \lim_{x\to 1} \frac{1}{x-1}\Bigg((\sqrt[3]{2x-1}-1)\frac{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \\ -3(\sqrt{2x-1}-x)\frac{\sqrt{2x-1}+x}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \frac{1}{x-1}\Bigg(\frac{\overbrace{(\sqrt[3]{2x-1}-1)(\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \\ -\frac{3\,\overbrace{(\sqrt{2x-1}-x)(\sqrt{2x-1}+x)}^{(a-b)(a+b)}}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \frac{1}{x-1}\Bigg(\frac{\overbrace{\sqrt[3]{(2x-1)^3}-1^3}^{a^3-b^3}}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \frac{3\,\overbrace{(\sqrt{(2x-1)^2}-x^2)}^{a^2-b^2}}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \frac{1}{x-1}\Bigg(\frac{2x-1-1}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \frac{3(2x-1-x^2)}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \frac{1}{x-1}\Bigg(\frac{2x-2}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \frac{3(x^2-2x+1)}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \frac{1}{x-1}\Bigg(\frac{2(x-1)}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \frac{3(x-1)^2}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \frac{x-1}{x-1}\Bigg(\frac{2}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \frac{3(x-1)}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{x-1}}\Bigg(\frac{2}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \frac{3(x-1)}{\sqrt{2x-1}+x}\Bigg)$$

$$= \lim_{x\to 1} \Bigg(\frac{2}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{2x-1}+1} - \frac{3(x-1)}{\sqrt{2x-1}+x}\Bigg)$$

$$= \frac{2}{\sqrt[3]{(2\cdot 1-1)^2}+\sqrt[3]{2\cdot 1-1}+1} - \frac{3(1-1)}{\sqrt{2\cdot 1-1}+1}$$

$$= \frac{2}{\sqrt[3]{(2-1)^2}+\sqrt[3]{2-1}+1} - \frac{3(0)}{\sqrt{2-1}+1} = \frac{2}{1+1+1} - \frac{0}{1+1}$$

$$= \frac{2}{3} - \frac{0}{2} = \frac{2}{3} - 0 = \frac{2}{3}$$

65. $\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt[4]{1+x^4}}{x^2}$

Solución:

$$\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt[4]{1+x^4}}{x^2} = \lim_{x\to 0} \frac{\sqrt{1+0^2}-\sqrt[4]{1+0^4}}{0^2} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt[4]{1+x^4}}{x^2} = \lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt[4]{1+x^4}}{x^2}\cdot\frac{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{\overbrace{(\sqrt{1+x^2}-\sqrt[4]{1+x^4})(\sqrt{1+x^2}+\sqrt[4]{1+x^4})}^{(a-b)(a+b)}}{x^2}\cdot\frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{\overbrace{\sqrt{(1+x^2)^2}-\sqrt[4]{(1+x^4)^2}}^{a^2-b^2}}{x^2}\cdot\frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{(1+x^2)-\sqrt{1+x^4}}{x^2}\cdot\frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{(1+x^2)-\sqrt{1+x^4}}{x^2} \cdot \frac{(1+x^2)+\sqrt{1+x^4}}{(1+x^2)+\sqrt{1+x^4}} \cdot \frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{\overbrace{((1+x^2)-\sqrt{1+x^4}\,)((1+x^2)+\sqrt{1+x^4}\,)}^{(a-b)(a+b)}}{x^2} \cdot \frac{1}{(1+x^2)+\sqrt{1+x^4}} \cdot \frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{\overbrace{(1+x^2)^2-\sqrt{(1+x^4)^2}}^{a^2-b^2}}{x^2} \cdot \frac{1}{(1+x^2)+\sqrt{1+x^4}} \cdot \frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{(1+2x^2+x^4)-(1+x^4)}{x^2} \cdot \frac{1}{(1+x^2)+\sqrt{1+x^4}} \cdot \frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{1+2x^2+x^4-1-x^4}{x^2} \cdot \frac{1}{(1+x^2)+\sqrt{1+x^4}} \cdot \frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{2x^2}{x^2} \cdot \frac{1}{(1+x^2)+\sqrt{1+x^4}} \cdot \frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{2\cancel{x^2}}{\cancel{x^2}} \cdot \frac{1}{(1+x^2)+\sqrt{1+x^4}} \cdot \frac{1}{\sqrt{1+x^2}+\sqrt[4]{1+x^4}}$$

$$= \lim_{x\to 0} \frac{2}{((1+x^2)+\sqrt{1+x^4}\,)(\sqrt{1+x^2}+\sqrt[4]{1+x^4}\,)}$$

$$= \frac{2}{((1+0^2)+\sqrt{1+0^4}\,)(\sqrt{1+0^2}+\sqrt[4]{1+0^4}\,)}$$

$$= \frac{2}{((1+0)+\sqrt{1+0}\,)(\sqrt{1+0}+\sqrt[4]{1+0}\,)}$$

$$= \frac{2}{(1+\sqrt{1}\,)(\sqrt{1}+\sqrt[4]{1}\,)} = \frac{2}{(1+1)(1+1)}$$

$$= \frac{2}{(2)(2)} = \frac{2}{4} = \frac{1}{2}$$

67. $\lim\limits_{x\to 2} \dfrac{\sqrt{16x-x^4}-2\sqrt[3]{4x}}{2-\sqrt[4]{2x^3}}$

Solución:

$$\lim_{x\to 2} \frac{\sqrt{16x-x^4}-2\sqrt[3]{4x}}{2-\sqrt[4]{2x^3}} = \frac{\sqrt{16\cdot 2-2^4}-2\sqrt[3]{4\cdot 2}}{2-\sqrt[4]{2\cdot 2^3}} = \frac{4-4}{2-2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 2} \frac{\sqrt{16x-x^4}-2\sqrt[3]{4x}}{2-\sqrt[4]{2x^3}} = \lim_{x\to 2} \frac{\sqrt{16x-x^4}-4-2\sqrt[3]{4x}+4}{2-\sqrt[4]{2x^3}} = \lim_{x\to 2} \frac{(\sqrt{16x-x^4}-4)-2(\sqrt[3]{4x}-2)}{2-\sqrt[4]{2x^3}}$$

$$= \lim_{x\to 2} \frac{1}{2-\sqrt[4]{2x^3}} \left[(\sqrt{16x-x^4}-4)-2(\sqrt[3]{4x}-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{2-\sqrt[4]{2x^3}} \cdot \frac{2+\sqrt[4]{2x^3}}{2+\sqrt[4]{2x^3}} \left[(\sqrt{16x-x^4}-4) \cdot \frac{\sqrt{16x-x^4}+4}{\sqrt{16x-x^4}+4} - 2(\sqrt[3]{4x}-2) \cdot \frac{\sqrt[3]{(4x)^2}+2\sqrt[3]{4x}+4}{\sqrt[3]{(4x)^2}+2\sqrt[3]{4x}+4}\right]$$

$$= \lim_{x\to 2} \frac{2+\sqrt[4]{2x^3}}{\underbrace{(2-\sqrt[4]{2x^3}\,)(2+\sqrt[4]{2x^3}\,)}_{(a-b)(a+b)}} \left[\frac{\overbrace{(\sqrt{16x-x^4}-4)(\sqrt{16x-x^4}+4)}^{(a-b)(a+b)}}{\sqrt{16x-x^4}+4} - \right.$$

$$-\frac{2\overbrace{(\sqrt[3]{4x}-2)(\sqrt[3]{(4x)^2}+2\sqrt[3]{4x}+4)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(4x)^2}+2\sqrt[3]{4x}+4}\Bigg]$$

$$=\lim_{x\to 2}\frac{2+\sqrt[4]{2x^3}}{\underbrace{2^2-\sqrt[4]{(2x^3)^2}}_{a^2-b^2}}\left[\frac{\overbrace{\sqrt{(16x-x^4)^2-4^2}}^{a^2-b^2}}{\sqrt{16x-x^4}+4}-\frac{2\overbrace{(\sqrt[3]{(4x)^3}-2^3)}^{a^3-b^3}}{\sqrt[3]{(4x)^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{2+\sqrt[4]{2x^3}}{4-\sqrt{2x^3}}\left[\frac{16x-x^4-16}{\sqrt{16x-x^4}+4}-\frac{2(4x-8)}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{2+\sqrt[4]{2x^3}}{4-\sqrt{2x^3}}\cdot\frac{4+\sqrt{2x^3}}{4+\sqrt{2x^3}}\left[\frac{-x^4+16x-16}{\sqrt{16x-x^4}+4}-\frac{2\cdot 4(x-2)}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})}{\underbrace{(4-\sqrt{2x^3})(4+\sqrt{2x^3})}_{(a-b)(a+b)}}\left[\frac{-(x^4-16x+16)}{\sqrt{16x-x^4}+4}-\frac{8(x-2)}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{-(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})}{\underbrace{4^2-\sqrt{(2x^3)^2}}_{a^2-b^2}}\left[\frac{(x-2)(x^3+2x^2+4x-8)}{\sqrt{16x-x^4}+4}+\frac{8(x-2)}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{-(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})}{16-2x^3}\left[(x-2)\left(\frac{x^3+2x^2+4x-8}{\sqrt{16x-x^4}+4}+\frac{8}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right)\right]$$

$$=\lim_{x\to 2}\frac{-(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})(x-2)}{-2x^3+16}\left[\frac{x^3+2x^2+4x-8}{\sqrt{16x-x^4}+4}+\frac{8}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{-(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})(x-2)}{-2(x^3-8)}\left[\frac{x^3+2x^2+4x-8}{\sqrt{16x-x^4}+4}+\frac{8}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})(x-2)}{2(x-2)(x^2+2x+4)}\left[\frac{x^3+2x^2+4x-8}{\sqrt{16x-x^4}+4}+\frac{8}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})\cancel{(x-2)}}{2\cancel{(x-2)}(x^2+2x+4)}\left[\frac{x^3+2x^2+4x-8}{\sqrt{16x-x^4}+4}+\frac{8}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\lim_{x\to 2}\frac{(2+\sqrt[4]{2x^3})(4+\sqrt{2x^3})}{2(x^2+2x+4)}\left[\frac{x^3+2x^2+4x-8}{\sqrt{16x-x^4}+4}+\frac{8}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}\right]$$

$$=\frac{(2+\sqrt[4]{2\cdot 2^3})(4+\sqrt{2\cdot 2^3})}{2(2^2+2\cdot 2+4)}\left[\frac{2^3+2\cdot 2^2+4\cdot 2-8}{\sqrt{16\cdot 2-2^4}+4}+\frac{8}{\sqrt[3]{16\cdot 2^2}+2\sqrt[3]{4\cdot 2}+4}\right]$$

$$=\frac{(2+2)(4+4)}{2(4+4+4)}\left[\frac{8+8+8-8}{4+4}+\frac{8}{4+4+4}\right]=\frac{(4)(8)}{2(12)}\left[\frac{16}{8}+\frac{8}{12}\right]=\frac{4}{3}\left[2+\frac{2}{3}\right]$$

$$=\frac{4}{3}\left[\frac{6+2}{3}\right]=\frac{4}{3}\left[\frac{8}{3}\right]=\frac{32}{9}$$

69. $\displaystyle\lim_{x\to 0}\frac{\sqrt[3]{x^3+1}+\sqrt[5]{x^5+1}+x^3-2}{x-x\sqrt{x^2+1}}$

Solución:

$$\lim_{x\to 0}\frac{\sqrt[3]{x^3+1}+\sqrt[5]{x^5+1}+x^3-2}{x-x\sqrt{x^2+1}}=\frac{\sqrt[3]{0^3+1}+\sqrt[5]{0^5+1}+0^3-2}{0-0\cdot\sqrt{0^2+1}}=\frac{0}{0}\qquad\text{(Indeterminación)}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{x^3+1}+\sqrt[5]{x^5+1}+x^3-2}{x-x\sqrt{x^2+1}}=\lim_{x\to 0}\frac{\sqrt[3]{x^3+1}-1+\sqrt[5]{x^5+1}-1+x^3}{x(1-\sqrt{x^2+1})}$$

$$=\lim_{x\to 0}\frac{1}{x(1-\sqrt{x^2+1})}\left[(\sqrt[3]{x^3+1}-1)+(\sqrt[5]{x^5+1}-1)+x^3\right]$$

$$= \lim_{x\to 0} \frac{1}{x(1-\sqrt{x^2+1}\,)} \cdot \frac{1+\sqrt{x^2+1}}{1+\sqrt{x^2+1}} \Bigg[(\sqrt[3]{x^3+1}-1) \cdot \frac{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ (\sqrt[5]{x^5+1}-1) \cdot \frac{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + x^3 \Bigg]$$

$$= \lim_{x\to 0} \frac{1+\sqrt{x^2+1}}{x\underbrace{(1-\sqrt{x^2+1}\,)(1+\sqrt{x^2+1}\,)}_{(a-b)(a+b)}} \Bigg[\frac{\overbrace{(\sqrt[3]{x^3+1}-1)(\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ \frac{\overbrace{(\sqrt[5]{x^5+1}-1)(\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1)}^{(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)}}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + x^3 \Bigg]$$

$$= \lim_{x\to 0} \frac{1+\sqrt{x^2+1}}{x(\underbrace{1^2-\sqrt{(x^2+1)^2}}_{a^2-b^2}\,)} \Bigg[\frac{\overbrace{\sqrt[3]{(x^3+1)^3}-1^3}^{a^3-b^3}}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ \frac{\overbrace{\sqrt[5]{(x^5+1)^5}-1^5}^{a^5-b^5}}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + x^3 \Bigg]$$

$$= \lim_{x\to 0} \frac{1+\sqrt{x^2+1}}{x(1-x^2-1)} \Bigg[\frac{x^3+1-1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$= \lim_{x\to 0} \frac{1+\sqrt{x^2+1}}{x(1-x^2-1)} \Bigg[\frac{x^3+1-1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$= \lim_{x\to 0} \frac{1+\sqrt{x^2+1}}{x(1-x^2-1)} \Bigg[\frac{x^3+1-1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ \frac{x^5+1-1}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + x^3 \Bigg]$$

$$= \lim_{x\to 0} \frac{1+\sqrt{x^2+1}}{-x^3} \Bigg[\frac{x^3}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ \frac{x^5}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + x^3 \Bigg]$$

$$= \lim_{x\to 0} -\frac{1+\sqrt{x^2+1}}{x^3} \Bigg[x^3 \Bigg(\frac{1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ \frac{x^2}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + 1 \Bigg) \Bigg]$$

$$= \lim_{x\to 0} -\frac{(1+\sqrt{x^2+1}\,)x^3}{x^3} \Bigg[\frac{1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ \frac{x^2}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + 1 \Bigg]$$

$$= \lim_{x\to 0} -\frac{(1+\sqrt{x^2+1}\,)\cancel{x^3}}{\cancel{x^3}} \Bigg[\frac{1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1} +$$

$$+ \frac{x^2}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1} + 1 \Bigg]$$

$$= \lim_{x\to 0} -(1+\sqrt{x^2+1})\left[\frac{1}{\sqrt[3]{(x^3+1)^2}+\sqrt[3]{x^3+1}+1}+\right.$$
$$\left.+\frac{x^2}{\sqrt[5]{(x^5+1)^4}+\sqrt[5]{(x^5+1)^3}+\sqrt[5]{(x^5+1)^2}+\sqrt[5]{x^5+1}+1}+1\right]$$
$$= -(1+\sqrt{0^2+1})\left[\frac{1}{\sqrt[3]{(0^3+1)^2}+\sqrt[3]{0^3+1}+1}+\right.$$
$$\left.+\frac{0^2}{\sqrt[5]{(0^5+1)^4}+\sqrt[5]{(0^5+1)^3}+\sqrt[5]{(0^5+1)^2}+\sqrt[5]{0^5+1}+1}+1\right]$$
$$= -(1+1)\left[\frac{1}{1+1+1}+\frac{0}{1+1+1+1}+1\right] = -2\left[\frac{1}{3}+\frac{0}{4}+1\right]$$
$$= -2\left[\frac{1}{3}+0+1\right] = -2\left[\frac{1}{3}+1\right] = -2\left[\frac{1+3}{3}\right] = -2\left[\frac{4}{3}\right] = -\frac{8}{3}$$

70. $\lim_{x\to 1}\dfrac{\sqrt[3]{5x+3}-\sqrt{3x+1}}{\sqrt{x}-3x+2}$

Solición:

$$\lim_{x\to 1}\frac{\sqrt[3]{5x+3}-\sqrt{3x+1}}{\sqrt{x}-3x+2} = \frac{\sqrt[3]{5\cdot 1+3}-\sqrt{3\cdot 1+1}}{\sqrt{1}-3\cdot 1+2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1}\frac{\sqrt[3]{5x+3}-\sqrt{3x+1}}{\sqrt{x}-3x+2} = \lim_{x\to 1}\frac{\sqrt[3]{5x+3}-2-\sqrt{3x+1}+2}{\sqrt{x}-(3x-2)}$$
$$= \lim_{x\to 1}\frac{1}{\sqrt{x}-(3x-2)}\left[(\sqrt[3]{5x+3}-2)-(\sqrt{3x+1}-2)\right]$$
$$= \lim_{x\to 1}\frac{1}{\sqrt{x}-(3x-2)}\cdot\frac{\sqrt{x}+(3x-2)}{\sqrt{x}+(3x-2)}\left[(\sqrt[3]{5x+3}-2)\cdot\frac{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\right.$$
$$\left.-(\sqrt{3x+1}-2)\cdot\frac{\sqrt{3x+1}+2}{\sqrt{3x+1}+2}\right]$$

$$= \lim_{x\to 1}\frac{\sqrt{x}+(3x-2)}{\underbrace{(\sqrt{x}-(3x-2))(\sqrt{x}+(3x-2))}_{(a-b)(a+b)}}\left[\frac{\overbrace{(\sqrt[3]{5x+3}-2)(\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\right.$$
$$\left.-\frac{\overbrace{(\sqrt{3x+1}-2)(\sqrt{3x+1}+2)}^{(a-b)(a+b)}}{\sqrt{3x+1}+2}\right]$$

$$= \lim_{x\to 1}\frac{\sqrt{x}+(3x-2)}{\underbrace{\sqrt{x^2}-(3x-2)^2}_{a^2-b^2}}\left[\frac{\overbrace{\sqrt[3]{(5x+3)^3}-2^3}^{a^3-b^3}}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{\overbrace{\sqrt{(3x+1)^2}-2^2}^{a^2-b^2}}{\sqrt{3x+1}+2}\right]$$
$$= \lim_{x\to 1}\frac{\sqrt{x}+(3x-2)}{x-(9x^2-12x+4)}\left[\frac{5x+3-8}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3x+1-4}{\sqrt{3x+1}+2}\right]$$
$$= \lim_{x\to 1}\frac{\sqrt{x}+(3x-2)}{x-9x^2+12x-4}\left[\frac{5x-5}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3x-3}{\sqrt{3x+1}+2}\right]$$
$$= \lim_{x\to 1}\frac{\sqrt{x}+(3x-2)}{-9x^2+13x-4}\left[\frac{5(x-1)}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3(x-1)}{\sqrt{3x+1}+2}\right]$$
$$= \lim_{x\to 1}\frac{\sqrt{x}+(3x-2)}{-(9x^2-13x+4)}\left[\frac{5(x-1)}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3(x-1)}{\sqrt{3x+1}+2}\right]$$
$$= \lim_{x\to 1}-\frac{\sqrt{x}+(3x-2)}{(x-1)(9x-4)}\left[\frac{5(x-1)}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3(x-1)}{\sqrt{3x+1}+2}\right]$$

$$= \lim_{x\to 1} -\frac{\sqrt{x}+(3x-2)}{(x-1)(9x-4)}\left[(x-1)\left(\frac{5}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3}{\sqrt{3x+1}+2}\right)\right]$$
$$= \lim_{x\to 1} -\frac{(\sqrt{x}+(3x-2))(x-1)}{(x-1)(9x-4)}\left[\frac{5}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3}{\sqrt{3x+1}+2}\right]$$
$$= \lim_{x\to 1} -\frac{(\sqrt{x}+(3x-2))\cancel{(x-1)}}{\cancel{(x-1)}(9x-4)}\left[\frac{5}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3}{\sqrt{3x+1}+2}\right]$$
$$= \lim_{x\to 1} -\frac{\sqrt{x}+(3x-2)}{9x-4}\left[\frac{5}{\sqrt[3]{(5x+3)^2}+2\sqrt[3]{5x+3}+4}-\frac{3}{\sqrt{3x+1}+2}\right]$$
$$= -\frac{\sqrt{1}+(3\cdot 1-2)}{9\cdot 1-4}\left[\frac{5}{\sqrt[3]{(5\cdot 1+3)^2}+2\sqrt[3]{5\cdot 1+3}+4}-\frac{3}{\sqrt{3\cdot 1+1}+2}\right]$$
$$= -\frac{1+1}{9-4}\left[\frac{5}{4+4+4}-\frac{3}{2+2}\right] = -\frac{2}{5}\left[\frac{5}{12}-\frac{3}{4}\right] = -\frac{2}{5}\left[\frac{5-9}{12}\right] = -\frac{2}{5}\left[\frac{-4}{12}\right]$$
$$= -\frac{2}{5}\left[\frac{-1}{3}\right] = \frac{2}{15}$$

71. $\lim_{x\to 1} \dfrac{\sqrt[3]{(1-x)^2}-\sqrt{x+3}-2}{\sqrt[3]{x^3-3x+2}}$

Solución:

$$\lim_{x\to 1} \frac{\sqrt[3]{(1-x)^2}-\sqrt{x+3}-2}{\sqrt[3]{x^3-3x+2}} = \frac{\sqrt[3]{(1-1)^2}-\sqrt{1+3}-2}{\sqrt[3]{1^3-3\cdot 1+2}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1} \frac{\sqrt[3]{(1-x)^2}+\sqrt{x+3}-2}{\sqrt[3]{x^3-3x+2}} = \lim_{x\to 1}\left[\frac{\sqrt[3]{(1-x)^2}}{\sqrt[3]{x^3-3x+2}}+\frac{\sqrt{x+3}-2}{\sqrt[3]{x^3-3x+2}}\right]$$
$$= \lim_{x\to 1}\left[\frac{\sqrt[3]{(1-x)^2}}{\sqrt[3]{(x-1)^2(x+2)}}+\frac{\sqrt{x+3}-2}{\sqrt[3]{(x-1)^2(x+2)}}\right]$$
$$= \lim_{x\to 1}\left[\frac{\sqrt[3]{(1-x)^2}}{\sqrt[3]{(1-x)^2}\cdot\sqrt[3]{x+2}}+\frac{\sqrt{x+3}-2}{\sqrt[3]{(x-1)^2(x+2)}}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\cdot\frac{\sqrt[3]{((x-1)^2(x+2))^2}}{\sqrt[3]{((x-1)^2(x+2))^2}}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{\overbrace{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}^{(a-b)(a+b)}}{\sqrt[3]{(x-1)^2(x+2)}\cdot\sqrt[3]{((x-1)^2(x+2))^2}}\cdot\frac{\sqrt[3]{((x-1)^2(x+2))^2}}{\sqrt{x+3}+2}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{\overbrace{\sqrt{(x+3)^2}-2^2}^{a^2-b^2}}{\sqrt[3]{((x-1)^2(x+2))^3}}\cdot\frac{\sqrt[3]{(x-1)^4(x+2)^2}}{\sqrt{x+3}+2}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{x+3-4}{(x-1)^2(x+2)}\cdot\frac{(x-1)\sqrt[3]{(x-1)(x+2)^2}}{\sqrt{x+3}+2}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{x-1}{(x-1)^2(x+2)}\cdot\frac{(x-1)\sqrt[3]{(x-1)(x+2)^2}}{\sqrt{x+3}+2}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{(x-1)^2}{(x-1)^2(x+2)}\cdot\frac{\sqrt[3]{(x-1)(x+2)^2}}{\sqrt{x+3}+2}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{\cancel{(x-1)^2}}{\cancel{(x-1)^2}(x+2)}\cdot\frac{\sqrt[3]{(x-1)(x+2)^2}}{\sqrt{x+3}+2}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{1}{x+2}\cdot\frac{\sqrt[3]{(x-1)(x+2)^2}}{\sqrt{x+3}+2}\right]$$
$$= \lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x+2}}+\frac{\sqrt[3]{(x-1)(x+2)^2}}{(x+2)(\sqrt{x+3}+2)}\right]$$
$$= \frac{1}{\sqrt[3]{1+2}}+\frac{\sqrt[3]{(1-1)(1+2)^2}}{(1+2)(\sqrt{1+3}+2)} = \frac{1}{\sqrt[3]{3}}+0 = \frac{1}{\sqrt[3]{3}}$$

73. $\lim_{x\to 2}\dfrac{\sqrt{3x^2-8}-x\sqrt[3]{x+6}+x^2-2}{x^3-2x^2+x-2}$

Solución:

$$\lim_{x\to 2}\frac{\sqrt{3x^2-8}-x\sqrt[3]{x+6}+x^2-2}{x^3-2x^2+x-2}=\frac{\sqrt{3\cdot 2^2-8}-2\sqrt[3]{2+6}+2^2-2}{2^3-2\cdot 2^2+2-2}=\frac{0}{0}\qquad\text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to 2}\frac{\sqrt{3x^2-8}-x\sqrt[3]{x+6}+x^2-2}{x^3-2x^2+x-2}&=\lim_{x\to 2}\frac{\sqrt{3x^2-8}-2-x\sqrt[3]{x+6}+x^2}{(x-2)(x^2+1)}\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[(\sqrt{3x^2-8}-2)-x(\sqrt[3]{x+6}+x)\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[(\sqrt{3x^2-8}-2)\cdot\frac{\sqrt{3x^2-8}+2}{\sqrt{3x^2-8}+2}-\right.\\
&\qquad\left.-x(\sqrt[3]{x+6}-x)\cdot\frac{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[\frac{\overbrace{(\sqrt{3x^2-8}-2)(\sqrt{3x^2-8}+2)}^{(a-b)(a-b)}}{\sqrt{3x^2-8}+2}-\right.\\
&\qquad\left.-\frac{x\ \overbrace{(\sqrt[3]{x+6}-x)(\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[\frac{\overbrace{\sqrt{(3x^2-8)^2}-2^2}^{a^2-b^2}}{\sqrt{3x^2-8}+2}-\frac{x(\overbrace{\sqrt[3]{(x+6)^3}-x^3}^{a^3-b^3})}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[\frac{3x^2-8-4}{\sqrt{3x^2-8}+2}-\frac{x(x+6-x^3)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[\frac{3x^2-12}{\sqrt{3x^2-8}+2}-\frac{x(-x^3+x+6)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[\frac{3(x^2-4)}{\sqrt{3x^2-8}+2}-\frac{-x(x^3-x-6)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[\frac{3(x^2-4)}{\sqrt{3x^2-8}+2}-\frac{-x(x^3-x-6)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[\frac{3(x-2)(x+2)}{\sqrt{3x^2-8}+2}+\frac{x(x-2)(x^2+2x-3)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{(x-2)(x^2+1)}\left[(x-2)\left(\frac{3(x+2)}{\sqrt{3x^2-8}+2}+\frac{x(x^2+2x-3)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right)\right]\\
&=\lim_{x\to 2}\frac{x-2}{(x-2)(x^2+1)}\left[\frac{3(x+2)}{\sqrt{3x^2-8}+2}+\frac{x(x^2+2x-3)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{\cancel{x-2}}{\cancel{(x-2)}(x^2+1)}\left[\frac{3(x+2)}{\sqrt{3x^2-8}+2}+\frac{x(x^2+2x-3)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\lim_{x\to 2}\frac{1}{x^2+1}\left[\frac{3(x+2)}{\sqrt{3x^2-8}+2}+\frac{x(x^2+2x-3)}{\sqrt[3]{(x+6)^2}+x\sqrt[3]{x+6}+x^2}\right]\\
&=\frac{1}{2^2+1}\left[\frac{3(2+2)}{\sqrt{3\cdot 2^2-8}+2}+\frac{2(2^2+2\cdot 2-3)}{\sqrt[3]{(2+6)^2}+2\sqrt[3]{2+6}+2^2}\right]\\
&=\frac{1}{5}\left[\frac{12}{4}+\frac{10}{12}\right]=\frac{1}{5}\left[3+\frac{5}{6}\right]=\frac{1}{5}\left[\frac{18+5}{6}\right]=\frac{1}{5}\left[\frac{23}{6}\right]=\frac{23}{30}
\end{aligned}$$

74. $\lim_{x\to 3}\dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}$

Solución:

$$\lim_{x\to 3}\frac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}=\frac{\sqrt{3+13}-2\sqrt{3+1}}{3^2-9}=\frac{0}{0}\qquad \text{(Indeterminación)}$$

$$\lim_{x\to 3}\frac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}=\lim_{x\to 3}\frac{\sqrt{x+13}-2\sqrt{x+1}}{(x-3)(x+3)}\cdot\frac{\sqrt{x+13}+2\sqrt{x+1}}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}\frac{\overbrace{(\sqrt{x+13}-2\sqrt{x+1})(\sqrt{x+13}+2\sqrt{x+1})}^{(a-b)(a+b)}}{(x-3)(x+3)}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}\frac{\overbrace{\sqrt{(x+13)^2}-4\sqrt{(x+1)^2}}^{a^2-b^2}}{(x-3)(x+3)}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}\frac{x+13-4(x+1)}{(x-3)(x+3)}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}\frac{x+13-4x-4}{(x-3)(x+3)}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}\frac{-3x+9}{(x-3)(x+3)}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}\frac{-3(x-3)}{(x-3)(x+3)}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}\frac{-3\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}-\frac{3}{x+3}\cdot\frac{1}{\sqrt{x+13}+2\sqrt{x+1}}$$

$$=\lim_{x\to 3}-\frac{3}{(x+3)(\sqrt{x+13}+2\sqrt{x+1})}$$

$$=-\frac{3}{(3+3)(\sqrt{3+13}+2\sqrt{3+1})}$$

$$=-\frac{3}{6(4+4)}=-\frac{1}{2(8)}=-\frac{1}{16}$$

75. $\displaystyle\lim_{x\to 3}\frac{\sqrt[5]{x^2+7x+2}-\sqrt{x+1}}{\sqrt[5]{2x-5}-1}$

Solución:

$$\lim_{x\to 3}\frac{\sqrt[5]{x^2+7x+2}-\sqrt{x+1}}{\sqrt[5]{2x-5}-1}=\frac{\sqrt[5]{3^2+7\cdot 3+2}-\sqrt{3+1}}{\sqrt[5]{2\cdot 3-5}-1}=\frac{0}{0}\qquad \text{(Indeterminación)}$$

$$\lim_{x\to 3}\frac{\sqrt[5]{x^2+7x+2}-\sqrt{x+1}}{\sqrt[5]{2x-5}-1}=\lim_{x\to 3}\frac{\sqrt[5]{x^2+7x+2}-2-\sqrt{x+1}+2}{\sqrt[5]{2x-5}-1}$$

$$=\lim_{x\to 3}\frac{1}{\sqrt[5]{2x-5}-1}\left[(\sqrt[5]{x^2+7x+2}-2)-(\sqrt{x+1}-2)\right]$$

$$=\lim_{x\to 3}\frac{1}{\sqrt[5]{2x-5}-1}\cdot\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}\cdot$$

$$\cdot\left[(\sqrt[5]{x^2+7x+2}-2)\cdot\frac{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\right.$$

$$\left.-(\sqrt{x+1}-2)\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{\underbrace{(\sqrt[5]{2x-5}-1)(\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1)}_{(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)}}\times$$

$$\times\left[\frac{\overbrace{(\sqrt[5]{x^2+7x+2}-2)(\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16)}^{(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)}}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\right.$$

$$\left.-\frac{\overbrace{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}^{(a-b)(a+b)}}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{\underbrace{\sqrt[5]{(2x-5)^5}-1^5}_{a^5-b^5}}\times$$

$$\times\left[\frac{\overbrace{\sqrt[5]{(x^2+7x+2)^5}-2^5}^{a^5-b^5}}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\frac{\overbrace{\sqrt{(x+1)^2}-2^2}^{a^2-b^2}}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{2x-5-1}\cdot$$

$$\cdot\left[\frac{x^2+7x+2-32}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\frac{x+1-4}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{2x-6}\cdot$$

$$\cdot\left[\frac{x^2+7x-30}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\frac{x-3}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{2(x-3)}\cdot$$

$$\cdot\left[\frac{(x+10)(x-3)}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\frac{x-3}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{2(x-3)}\cdot$$

$$\cdot(x-3)\left[\frac{x+10}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\frac{1}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{2\cancel{(x-3)}}\cdot$$

$$\cdot\cancel{(x-3)}\left[\frac{x+10}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\frac{1}{\sqrt{x+1}+2}\right]$$

$$=\lim_{x\to 3}\frac{\sqrt[5]{(2x-5)^4}+\sqrt[5]{(2x-5)^3}+\sqrt[5]{(2x-5)^2}+\sqrt[5]{2x-5}+1}{2}\cdot$$

$$\cdot\left[\frac{x+10}{\sqrt[5]{(x^2+7x+2)^4}+2\sqrt[5]{(x^2+7x+2)^3}+4\sqrt[5]{(x^2+7x+2)^2}+8\sqrt[5]{x^2+7x+2}+16}-\frac{1}{\sqrt{x+1}+2}\right]$$

$$=\frac{\sqrt[5]{(2\cdot 3-5)^4}+\sqrt[5]{(2\cdot 3-5)^3}+\sqrt[5]{(2\cdot 3-5)^2}+\sqrt[5]{2\cdot 3-5}+1}{2}\cdot$$

$$\cdot\left[\frac{3+10}{\sqrt[5]{(3^2+7\cdot 3+2)^4}+2\sqrt[5]{(3^2+7\cdot 3+2)^3}+4\sqrt[5]{(3^2+7\cdot 3+2)^2}+8\sqrt[5]{3^2+7\cdot 3+2}+16}-\frac{1}{\sqrt{3+1}+2}\right]$$

$$=\frac{1+1+1+1+1}{2}\left[\frac{13}{16+16+16+16+16}-\frac{1}{2+2}\right]=\frac{5}{2}\left[\frac{13}{80}-\frac{1}{4}\right]$$

$$=\frac{5}{2}\left[\frac{13-20}{80}\right]=\frac{5}{2}\left[\frac{-7}{80}\right]=\frac{1}{2}\left[\frac{-7}{16}\right]=-\frac{7}{32}$$

76. $\lim_{x\to 2}\frac{\sqrt{x-1}-x+\sqrt{x^2-3}}{\sqrt{3x+10}-4}$

Solución:

$$\lim_{x\to 2}\frac{\sqrt{x-1}-x+\sqrt{x^2-3}}{\sqrt{3x+10}-4}=\frac{\sqrt{2-1}-2+\sqrt{2^2-3}}{\sqrt{3\cdot 2+10}-4}=\frac{0}{0}\qquad\text{(Indeterminación)}$$

$$\begin{aligned}
\lim_{x\to 2}\frac{\sqrt{x-1}-x+\sqrt{x^2-3}}{\sqrt{3x+10}-4}&=\lim_{x\to 2}\frac{\sqrt{x-1}-1+\sqrt{x^2-3}-x+1}{\sqrt{3x+10}-4}\\
&=\lim_{x\to 2}\frac{1}{\sqrt{3x+10}-4}\left[(\sqrt{x-1}-1)+(\sqrt{x^2-3}-(x-1))\right]\\
&=\lim_{x\to 2}\frac{1}{\sqrt{3x+10}-4}\cdot\frac{\sqrt{3x+10}+4}{\sqrt{3x+10}+4}\left[(\sqrt{x-1}-1)\cdot\frac{\sqrt{x-1}+1}{\sqrt{x-1}+1}+\right.\\
&\qquad\left.+(\sqrt{x^2-3}-(x-1))\cdot\frac{\sqrt{x^2-3}+(x-1)}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{\underbrace{(\sqrt{3x+10}-4)(\sqrt{3x+10}+4)}_{(a-b)(a+b)}}\left[\frac{\overbrace{(\sqrt{x-1}-1)(\sqrt{x-1}+1)}^{(a-b)(a+b)}}{\sqrt{x-1}+1}+\right.\\
&\qquad\left.+\frac{\overbrace{(\sqrt{x^2-3}-(x-1))(\sqrt{x^2-3}+(x-1))}^{(a-b)(a+b)}}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{\underbrace{\sqrt{(3x+10)^2-4^2}}_{(a^2-b^2)}}\left[\frac{\overbrace{\sqrt{(x-1)^2-1^2}}^{a^2-b^2}}{\sqrt{x-1}+1}+\frac{\overbrace{\sqrt{(x^2-3)^2-(x-1)^2}}^{a^2-b^2}}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{3x+10-16}\left[\frac{x-1-1}{\sqrt{x-1}+1}+\frac{x^2-3-(x^2-2x+1)}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{3x-6}\left[\frac{x-2}{\sqrt{x-1}+1}+\frac{x^2-3-x^2+2x-1}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{3(x-2)}\left[\frac{x-2}{\sqrt{x-1}+1}+\frac{2x-4}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{3(x-2)}\left[\frac{x-2}{\sqrt{x-1}+1}+\frac{2(x-2)}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{3(x-2)}\left[(x-2)\left(\frac{1}{\sqrt{x-1}+1}+\frac{2}{\sqrt{x^2-3}+(x-1)}\right)\right]\\
&=\lim_{x\to 2}\frac{(\sqrt{3x+10}+4)(x-2)}{3(x-2)}\left[\frac{1}{\sqrt{x-1}+1}+\frac{2}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{(\sqrt{3x+10}+4)\cancel{(x-2)}}{3\cancel{(x-2)}}\left[\frac{1}{\sqrt{x-1}+1}+\frac{2}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\lim_{x\to 2}\frac{\sqrt{3x+10}+4}{3}\left[\frac{1}{\sqrt{x-1}+1}+\frac{2}{\sqrt{x^2-3}+(x-1)}\right]\\
&=\frac{\sqrt{3\cdot 2+10}+4}{3}\left[\frac{1}{\sqrt{2-1}+1}+\frac{2}{\sqrt{2^2-3}+(2-1)}\right]\\
&=\frac{4+4}{3}\left[\frac{1}{1+1}+\frac{2}{1+1}\right]=\frac{8}{3}\left[\frac{1}{2}+\frac{2}{2}\right]=\frac{8}{3}\left[\frac{1}{2}+1\right]=\frac{8}{3}\left[\frac{1+2}{2}\right]\\
&=\frac{8}{3}\left[\frac{3}{2}\right]=\frac{24}{6}=4
\end{aligned}$$

77. $\lim_{x\to 2\sqrt{2}}\frac{\sqrt[3]{x^4}-\sqrt[3]{x^2}-2}{x^2-8}$

Solución:

$$\lim_{x\to 2\sqrt{2}} \frac{\sqrt[3]{x^4}-\sqrt[3]{x^2}-2}{x^2-8} = \frac{\sqrt[3]{(2\sqrt{2}\,)^4}-\sqrt[3]{(2\sqrt{2}\,)^2}-2}{(2\sqrt{2}\,)^2-8} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 2\sqrt{2}} \frac{\sqrt[3]{x^4}-\sqrt[3]{x^2}-2}{x^2-8} = \lim_{x\to 2\sqrt{2}} \frac{\sqrt[3]{(x^2)^2}-\sqrt[3]{x^2}-2}{x^2-8} = \lim_{x\to 2\sqrt{2}} \frac{(\sqrt[3]{x^2}-2)(\sqrt[3]{x^2}+1)}{x^2-8}$$

$$= \lim_{x\to 2\sqrt{2}} \frac{(\sqrt[3]{x^2}-2)(\sqrt[3]{x^2}+1)}{x^2-8} \times \frac{\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4}{\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4}$$

$$= \lim_{x\to 2\sqrt{2}} \frac{\overbrace{(\sqrt[3]{x^2}-2)(\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4)}^{(a-b)(a^2+ab+b^2)}(\sqrt[3]{x^2}+1)}{(x^2-8)(\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4)}$$

$$= \lim_{x\to 2\sqrt{2}} \frac{(\overbrace{\sqrt[3]{(x^2)^3}-2^3}^{a^3-b^3})(\sqrt[3]{x^2}+1)}{(x^2-8)(\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4)}$$

$$= \lim_{x\to 2\sqrt{2}} \frac{(x^2-8)(\sqrt[3]{x^2}+1)}{(x^2-8)(\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4)}$$

$$= \lim_{x\to 2\sqrt{2}} \frac{\cancel{(x^2-8)}(\sqrt[3]{x^2}+1)}{\cancel{(x^2-8)}(\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4)}$$

$$= \lim_{x\to 2\sqrt{2}} \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{(x^2)^2}+2\sqrt[3]{x^2}+4}$$

$$= \lim_{x\to 2\sqrt{2}} \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x^4}+2\sqrt[3]{x^2}+4}$$

$$= \frac{\sqrt[3]{(2\sqrt{2})^2}+1}{\sqrt[3]{(2\sqrt{2})^4}+2\sqrt[3]{(2\sqrt{2})^2}+4}$$

$$= \frac{\sqrt[3]{8}+1}{\sqrt[3]{64}+2\sqrt[3]{8}+4} = \frac{2+1}{4+2\cdot 2+4}$$

$$= \frac{3}{4+4+4} = \frac{3}{12} = \frac{1}{4}$$

78. $\displaystyle\lim_{x\to 1} \frac{\sqrt[3]{x^2+7}-\sqrt{x^2+3}}{x-1}$

Solución:

$$\lim_{x\to 1} \frac{\sqrt[3]{x^2+7}-\sqrt{x^2+3}}{x-1} = \lim_{x\to 1} \frac{\sqrt[3]{1^2+7}-\sqrt{1^2+3}}{1-1} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1} \frac{\sqrt[3]{x^2+7}-\sqrt{x^2+3}}{x-1} = \lim_{x\to 1} \frac{\sqrt[3]{x^2+7}-2-\sqrt{x^2+3}+2}{x-1}$$

$$= \lim_{x\to 1} \frac{1}{x-1}\left[(\sqrt[3]{x^2+7}-2)-(\sqrt{x^2+3}-2)\right]$$

$$= \lim_{x\to 1} \frac{1}{x-1}\left[(\sqrt[3]{x^2+7}-2)\cdot\frac{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4} - \right.$$

$$\left. -(\sqrt{x^2+3}-2)\cdot\frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}\right]$$

$$= \lim_{x\to 1} \frac{1}{x-1}\left[\frac{\overbrace{(\sqrt[3]{x^2+7}-2)(\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4} - \right.$$

$$-\frac{\overbrace{(\sqrt{x^2+3}-2)(\sqrt{x^2+3}+2)}^{(a-b)(a+b)}}{\sqrt{x^2+3}+2}\Bigg]$$

$$=\lim_{x\to 1}\frac{1}{x-1}\left[\frac{\overbrace{\sqrt[3]{(x^2+7)^3}-2^3}^{a^3-b^3}}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{\overbrace{\sqrt{(x^2+3)^2}-2^2}^{a^2-b^2}}{\sqrt{x^2+3}+2}\right]$$

$$=\lim_{x\to 1}\frac{1}{x-1}\left[\frac{x^2+7-8}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{x^2+3-4}{\sqrt{x^2+3}+2}\right]$$

$$=\lim_{x\to 1}\frac{1}{x-1}\left[\frac{x^2-1}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{x^2-1}{\sqrt{x^2+3}+2}\right]$$

$$=\lim_{x\to 1}\frac{1}{x-1}\left[(x^2-1)\left(\frac{1}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{1}{\sqrt{x^2+3}+2}\right)\right]$$

$$=\lim_{x\to 1}\frac{x^2-1}{x-1}\left[\frac{1}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{1}{\sqrt{x^2+3}+2}\right]$$

$$=\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}\left[\frac{1}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{1}{\sqrt{x^2+3}+2}\right]$$

$$=\lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\left[\frac{1}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{1}{\sqrt{x^2+3}+2}\right]$$

$$=\lim_{x\to 1}(x+1)\left[\frac{1}{\sqrt[3]{(x^2+7)^2}+2\sqrt[3]{x^2+7}+4}-\frac{1}{\sqrt{x^2+3}+2}\right]$$

$$=(1+1)\left[\frac{1}{\sqrt[3]{(1^2+7)^2}+2\sqrt[3]{1^2+7}+4}-\frac{1}{\sqrt{1^2+3}+2}\right]$$

$$=2\left[\frac{1}{4+4+4}-\frac{1}{2+2}\right]=2\left[\frac{1}{12}-\frac{1}{4}\right]=2\left[\frac{1-3}{12}\right]=2\left[\frac{-2}{12}\right]=\frac{-4}{12}=-\frac{1}{3}$$

80. $\lim_{x\to 27}\dfrac{\sqrt{\dfrac{2x}{9}+3}-\sqrt[3]{x}}{3-\sqrt{\dfrac{x}{3}}}$

Solución:

$$\lim_{x\to 27}\frac{\sqrt{\frac{2x}{9}+3}-\sqrt[3]{x}}{3-\sqrt{\frac{x}{3}}}=\frac{\sqrt{\frac{2\cdot 27}{9}+3}-\sqrt[3]{27}}{3-\sqrt{\frac{27}{3}}}=\frac{0}{0}\qquad\text{(Indeterminación)}$$

$$\lim_{x\to 27}\frac{\sqrt{\frac{2x}{9}+3}-\sqrt[3]{x}}{3-\sqrt{\frac{x}{3}}}=\lim_{x\to 27}\frac{\sqrt{\frac{2x}{9}+3}-3-\sqrt[3]{x}+3}{3-\sqrt{\frac{x}{3}}}=\lim_{x\to 27}\frac{1}{3-\sqrt{\frac{x}{3}}}\left[\left(\sqrt{\frac{2x}{9}+3}-3\right)-(\sqrt[3]{x}-3)\right]$$

$$=\lim_{x\to 27}\frac{1}{3-\sqrt{\frac{x}{3}}}\cdot\frac{3+\sqrt{\frac{x}{3}}}{3+\sqrt{\frac{x}{3}}}\left[\left(\sqrt{\frac{2x}{9}+3}-3\right)\cdot\frac{\sqrt{\frac{2x}{9}+3}+3}{\sqrt{\frac{2x}{9}+3}+3}-(\sqrt[3]{x}-3)\cdot\frac{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$=\lim_{x\to 27}\frac{3+\sqrt{\frac{x}{3}}}{\underbrace{\left(3-\sqrt{\frac{x}{3}}\right)\left(3+\sqrt{\frac{x}{3}}\right)}_{(a-b)(a+b)}}\left[\frac{\overbrace{\left(\sqrt{\frac{2x}{9}+3}-3\right)\left(\sqrt{\frac{2x}{9}+3}+3\right)}^{(a-b)(a+b)}}{\sqrt{\frac{2x}{9}+3}+3}-\frac{\overbrace{(\sqrt[3]{x}-3)(\sqrt[3]{x^2}+3\sqrt[3]{x}+9)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} \frac{3+\sqrt{\frac{x}{3}}}{\underbrace{3^2-\sqrt{\left(\frac{x}{3}\right)^2}}_{a^2-b^2}}\left[\frac{\overbrace{\sqrt{\left(\frac{2x}{9}+3\right)^2}-3^2}^{a^2-b^2}}{\sqrt{\frac{2x}{9}+3}+3}-\frac{\overbrace{\sqrt[3]{x^3}-3^3}^{a^3-b^3}}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} \frac{3+\sqrt{\frac{x}{3}}}{9-\frac{x}{3}}\left[\frac{\frac{2x}{9}+3-9}{\sqrt{\frac{2x}{9}+3}+3}-\frac{x-27}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} \frac{3+\sqrt{\frac{x}{3}}}{\frac{27-x}{3}}\left[\frac{\frac{2x}{9}-6}{\sqrt{\frac{2x}{9}+3}+3}-\frac{x-27}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} \frac{3+\sqrt{\frac{x}{3}}}{\frac{-x+27}{3}}\left[\frac{2\left(\frac{x}{9}-3\right)}{\sqrt{\frac{2x}{9}+3}+3}-\frac{x-27}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} \frac{3+\sqrt{\frac{x}{3}}}{\frac{-(x-27)}{3}}\left[\frac{2\left(\frac{x-27}{9}\right)}{\sqrt{\frac{2x}{9}+3}+3}-\frac{x-27}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} -\frac{3+\sqrt{\frac{x}{3}}}{\frac{x-27}{3}}\left[\frac{\frac{2(x-27)}{9}}{\sqrt{\frac{2x}{9}+3}+3}-\frac{x-27}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} -\frac{3+\sqrt{\frac{x}{3}}}{\frac{x-27}{3}}\left[(x-27)\left(\frac{\frac{2}{9}}{\sqrt{\frac{2x}{9}+3}+3}-\frac{1}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right)\right]$$

$$= \lim_{x\to 27} -\frac{\left(3+\sqrt{\frac{x}{3}}\right)(x-27)}{\frac{x-27}{3}}\left[\frac{\frac{2}{9}}{\sqrt{\frac{2x}{9}+3}+3}-\frac{1}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} -\frac{\left(3+\sqrt{\frac{x}{3}}\right)\cancel{(x-27)}}{\frac{\cancel{x-27}}{3}}\left[\frac{\frac{2}{9}}{\sqrt{\frac{2x}{9}+3}+3}-\frac{1}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= \lim_{x\to 27} -\frac{3+\sqrt{\frac{x}{3}}}{\frac{1}{3}}\left[\frac{\frac{2}{9}}{\sqrt{\frac{2x}{9}+3}+3}-\frac{1}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}\right]$$

$$= -\frac{3+\sqrt{\frac{27}{3}}}{\frac{1}{3}}\left[\frac{\frac{2}{9}}{\sqrt{\frac{2\cdot 27}{9}+3}+3}-\frac{1}{\sqrt[3]{27^2}+3\sqrt[3]{27}+9}\right]$$

$$= -\frac{3+3}{\frac{1}{3}}\left[\frac{\frac{2}{9}}{3+3} - \frac{1}{9+9+9}\right] = -\frac{6}{\frac{1}{3}}\left[\frac{\frac{2}{9}}{6} - \frac{1}{27}\right] = -\frac{18}{1}\left[\frac{2}{54} - \frac{1}{27}\right]$$

$$= -18\left[\frac{1}{27} - \frac{1}{27}\right] = -18[\,0\,] = 0$$

82. $\lim\limits_{x\to 3} \dfrac{\sqrt[3]{x^2+18} - \sqrt{2x+3}}{x^2-9}$

Solución:

$$\lim_{x\to 3} \frac{\sqrt[3]{x^2+18} - \sqrt{2x+3}}{x^2-9} = \lim_{x\to 3} \frac{\sqrt[3]{3^2+18} - \sqrt{2\cdot 3+3}}{3^2-9} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 3} \frac{\sqrt[3]{x^2+18} - \sqrt{2x+3}}{x^2-9} = \lim_{x\to 3} \frac{\sqrt[3]{x^2+18} - 3 - \sqrt{2x+3} + 3}{x^2-9}$$

$$= \lim_{x\to 3} \frac{1}{x^2-9}\left[(\sqrt[3]{x^2+18} - 3) - (\sqrt{2x+3} - 3)\right]$$

$$= \lim_{x\to 3} \frac{1}{x^2-9}\left[(\sqrt[3]{x^2+18} - 3)\cdot \frac{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \right.$$

$$\left. -(\sqrt{2x+3} - 3)\cdot \frac{\sqrt{2x+3}+3}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{1}{x^2-9}\left[\frac{\overbrace{(\sqrt[3]{x^2+18} - 3)(\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \right.$$

$$\left. - \frac{\overbrace{(\sqrt{2x+3} - 3)(\sqrt{2x+3}+3)}^{(a-b)(a+b)}}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{1}{x^2-9}\left[\frac{\overbrace{\sqrt[3]{(x^2+18)^3} - 3^3}^{a^3-b^3}}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{\overbrace{\sqrt{(2x+3)^2 - 3^2}}^{a^2-b^2}}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{1}{x^2-9}\left[\frac{x^2+18-27}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{2x+3-9}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{1}{x^2-9}\left[\frac{x^2-9}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{2x-6}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{1}{(x-3)(x+3)}\left[\frac{(x-3)(x+3)}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{2(x-3)}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{1}{(x-3)(x+3)}\left[(x-3)\left(\frac{x+3}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{2}{\sqrt{2x+3}+3}\right)\right]$$

$$= \lim_{x\to 3} \frac{x-3}{(x-3)(x+3)}\left[\frac{x+3}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{2}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{\cancel{x-3}}{(x-3)\cancel{(x+3)}}\left[\frac{x+3}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{2}{\sqrt{2x+3}+3}\right]$$

$$= \lim_{x\to 3} \frac{1}{x+3}\left[\frac{x+3}{\sqrt[3]{(x^2+18)^2} + 3\sqrt[3]{x^2+18} + 9} - \frac{2}{\sqrt{2x+3}+3}\right]$$

$$= \frac{1}{3+3}\left[\frac{3+3}{\sqrt[3]{(3^2+18)^2} + 3\sqrt[3]{3^2+18} + 9} - \frac{2}{\sqrt{2\cdot 3+3}+3}\right]$$

$$= \frac{1}{6}\left[\frac{6}{9+9+9} - \frac{2}{3+3}\right] = \frac{1}{6}\left[\frac{6}{27} - \frac{2}{6}\right] = \frac{1}{6}\left[\frac{2}{9} - \frac{1}{3}\right] = \frac{1}{6}\left[\frac{2-3}{9}\right]$$

$$= \frac{1}{6}\left[\frac{-1}{9}\right] = -\frac{1}{54}$$

86. $\lim_{x\to 2} \frac{\sqrt[3]{x-1} - x + \sqrt{x^2-3}}{\sqrt{3x+10}-4}$

Solución:

$$\lim_{x\to 2} \frac{\sqrt[3]{x-1} - x + \sqrt{x^2-3}}{\sqrt{3x+10}-4} = \frac{\sqrt[3]{2-1} - 2 + \sqrt{2^2-3}}{\sqrt{3\cdot 2+10}-4} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 2} \frac{\sqrt[3]{x-1} - x + \sqrt{x^2-3}}{\sqrt{3x+10}-4} = \lim_{x\to 2} \frac{\sqrt[3]{x-1} - 1 - x + 1 + \sqrt{x^2-3}}{\sqrt{3x+10}-4}$$

$$= \lim_{x\to 2} \frac{1}{\sqrt{3x+10}-4}\left[(\sqrt[3]{x-1}-1) - \Big((x-1) - \sqrt{x^2-3}\Big)\right]$$

$$= \lim_{x\to 2} \frac{1}{\sqrt{3x+10}-4} \cdot \frac{\sqrt{3x+10}+4}{\sqrt{3x+10}+4}\left[(\sqrt[3]{x-1}-1)\cdot\frac{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} - \Big((x-1)-\sqrt{x^2-3}\Big)\cdot\frac{(x-1)+\sqrt{x^2-3}}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{\underbrace{(\sqrt{3x+10}-4)(\sqrt{3x+10}+4)}_{(a-b)(a+b)}}\left[\frac{\overbrace{(\sqrt[3]{x-1}-1)(\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} - \frac{\overbrace{\Big((x-1)-\sqrt{x^2-3}\Big)\Big((x-1)+\sqrt{x^2-3}\Big)}^{(a-b)(a+b)}}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{\underbrace{\sqrt{(3x+10)^2}-4^2}_{a^2-b^2}}\left[\frac{\overbrace{\sqrt[3]{(x-1)^3}-1^3}^{a^3-b^3}}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} - \frac{\overbrace{(x-1)^2-\sqrt{(x^2-3)^2}}^{a^2-b^2}}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{3x+10-16}\left[\frac{x-1-1}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} - \frac{(x^2-2x+1)-(x^2-3)}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{3x-6}\left[\frac{x-2}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} - \frac{x^2-2x+1-x^2+3}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{3(x-2)}\left[\frac{x-2}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} - \frac{-2x+4}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{3(x-2)}\left[\frac{x-2}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} - \frac{-2(x-2)}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{3(x-2)}\left[\frac{x-2}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} + \frac{2(x-2)}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{3(x-2)}\left[(x-2)\left(\frac{1}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} + \frac{2}{(x-1)+\sqrt{x^2-3}}\right)\right]$$

$$= \lim_{x\to 2} \frac{(\sqrt{3x+10}+4)(x-2)}{3(x-2)}\left[\frac{1}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} + \frac{2}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{(\sqrt{3x+10}+4)\cancel{(x-2)}}{3\cancel{(x-2)}}\left[\frac{1}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} + \frac{2}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \lim_{x\to 2} \frac{\sqrt{3x+10}+4}{3}\left[\frac{1}{\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}+1} + \frac{2}{(x-1)+\sqrt{x^2-3}}\right]$$

$$= \frac{\sqrt{3\cdot 2+10}+4}{3}\left[\frac{1}{\sqrt[3]{(2-1)^2}+\sqrt[3]{2-1}+1}+\frac{2}{(2-1)+\sqrt{2^2-3}}\right]$$

$$= \frac{4+4}{3}\left[\frac{1}{1+1+1}+\frac{2}{1+1}\right] = \frac{8}{3}\left[\frac{1}{3}+\frac{2}{2}\right] = \frac{8}{3}\left[\frac{1}{3}+1\right] = \frac{8}{3}\left[\frac{1+3}{3}\right]$$

$$= \frac{8}{3}\left[\frac{4}{3}\right] = \frac{32}{9}$$

87. $\lim\limits_{x\to 0}\dfrac{3\sqrt[5]{x+1}-2\sqrt{x+1}+4x-1}{x^2+2x}$

Solución:

$$\lim_{x\to 0}\frac{3\sqrt[5]{x+1}-2\sqrt{x+1}+4x-1}{x^2+2x} = \frac{3\sqrt[5]{0+1}-2\sqrt{0+1}+4\cdot 0-1}{0^2+2\cdot 0} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0}\frac{3\sqrt[5]{x+1}-2\sqrt{x+1}+4x-1}{x^2+2x} = \lim_{x\to 0}\frac{3\sqrt[5]{x+1}-3-2\sqrt{x+1}+3-1+4x}{x^2+2x}$$

$$= \lim_{x\to 0}\frac{3(\sqrt[5]{x+1}-1)-2\sqrt{x+1}+2+4x}{x^2+2x}$$

$$= \lim_{x\to 0}\frac{3(\sqrt[5]{x+1}-1)-2(\sqrt{x+1}-1)+4x}{x^2+2x}$$

$$= \lim_{x\to 0}\frac{1}{x^2+2x}\left[3(\sqrt[5]{x+1}-1)-2(\sqrt{x+1}-1)+4x\right]$$

$$= \lim_{x\to 0}\frac{1}{x^2+2x}\left[3(\sqrt[5]{x+1}-1)\cdot\frac{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - 2(\sqrt{x+1}-1)\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}+4x\right]$$

$$= \lim_{x\to 0}\frac{1}{x^2+2x}\left[\frac{3\overbrace{(\sqrt[5]{x+1}-1)(\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1)}^{(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)}}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \frac{2\overbrace{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}^{(a-b)(a+b)}}{\sqrt{x+1}+1}+4x\right]$$

$$= \lim_{x\to 0}\frac{1}{x^2+2x}\left[\frac{3\overbrace{(\sqrt[5]{(x+1)^5}-1^5)}^{a^5-b^5}}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \frac{2\overbrace{(\sqrt{(x+1)^2}-1^2)}^{a^2-b^2}}{\sqrt{x+1}+1}+4x\right]$$

$$= \lim_{x\to 0}\frac{1}{x^2+2x}\left[\frac{3(x+1-1)}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \frac{2(x+1-1)}{\sqrt{x+1}+1}+4x\right]$$

$$= \lim_{x\to 0}\frac{1}{x(x+2)}\left[\frac{3x}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \frac{2x}{\sqrt{x+1}+1}+4x\right]$$

$$= \lim_{x\to 0}\frac{1}{x(x+2)}\left[x\left(\frac{3}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \frac{2}{\sqrt{x+1}+1}+4\right)\right]$$

$$= \lim_{x\to 0} \frac{x}{x(x+2)}\left[\frac{3}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \right.$$
$$\left. -\frac{2}{\sqrt{x+1}+1}+4\right]$$
$$= \lim_{x\to 0} \frac{\not x}{\not x(x+2)}\left[\frac{3}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \right.$$
$$\left. -\frac{2}{\sqrt{x+1}+1}+4\right]$$
$$= \lim_{x\to 0} \frac{1}{x+2}\left[\frac{3}{\sqrt[5]{(x+1)^4}+\sqrt[5]{(x+1)^3}+\sqrt[5]{(x+1)^2}+\sqrt[5]{x+1}+1} - \right.$$
$$\left. -\frac{2}{\sqrt{x+1}+1}+4\right]$$
$$= \frac{1}{0+2}\left[\frac{3}{\sqrt[5]{(0+1)^4}+\sqrt[5]{(0+1)^3}+\sqrt[5]{(0+1)^2}+\sqrt[5]{0+1}+1} - \frac{2}{\sqrt{0+1}+1}+4\right]$$
$$= \frac{1}{2}\left[\frac{3}{1+1+1+1+1} - \frac{2}{1+1}+4\right] = \frac{1}{2}\left[\frac{3}{5}-\frac{2}{2}+4\right] = \frac{1}{2}\left[\frac{3}{5}-1+4\right] = \frac{1}{2}\left[\frac{3}{5}+3\right]$$
$$= \frac{1}{2}\left[\frac{3+15}{5}\right] = \frac{1}{2}\left[\frac{18}{5}\right] = \frac{18}{10} = \frac{9}{5}$$

88. $\lim_{x\to 7} \frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}$

Solución:

$$\lim_{x\to 7} \frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2} = \frac{\sqrt{7+2}-\sqrt[3]{7+20}}{\sqrt[4]{7+9}-2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 7} \frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2} = \lim_{x\to 7} \frac{\sqrt{x+2}-3-\sqrt[3]{x+20}+3}{\sqrt[4]{x+9}-2} = \lim_{x\to 7} \frac{(\sqrt{x+2}-3)-(\sqrt[3]{x+20}-3)}{\sqrt[4]{x+9}-2}$$
$$= \lim_{x\to 7} \frac{1}{\sqrt[4]{x+9}-2}\left[(\sqrt{x+2}-3)-(\sqrt[3]{x+20}-3)\right]$$
$$= \lim_{x\to 7} \frac{1}{\sqrt[4]{x+9}-2}\cdot\frac{\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8}{\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8}\times$$
$$\times\left[(\sqrt{x+2}-3)\cdot\frac{\sqrt{x+2}+3}{\sqrt{x+2}+3}-(\sqrt[3]{x+20}-3)\cdot\frac{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$
$$= \lim_{x\to 7} \frac{\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8}{\underbrace{(\sqrt[4]{x+9}-2)(\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8)}_{(a-b)(a^3+a^2b+ab^2+b^3)}}\times$$
$$\times\left[\frac{\overbrace{(\sqrt{x+2}-3)(\sqrt{x+2}+3)}^{(a-b)(a+b)}}{\sqrt{x+2}+3}-\frac{\overbrace{(\sqrt[3]{x+20}-3)(\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$
$$= \lim_{x\to 7} \frac{\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8}{\underbrace{\overset{a^4-b^4}{\sqrt[4]{(x+9)^4}-2^4}}}\times$$
$$\times\left[\frac{\overbrace{\sqrt{(x+2)^2}-3^2}^{a^2-b^2}}{\sqrt{x+2}+3}-\frac{\overbrace{\sqrt[3]{(x+20)^3}-3^3}^{a^3-b^3}}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$
$$= \lim_{x\to 7} \frac{\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8}{x+9-16}\times$$

$$\times\left[\frac{x+2-9}{\sqrt{x+2}+3}-\frac{x+20-27}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$

$$=\lim_{x\to 7}\frac{\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8}{x-7}\times$$

$$\times\left[\frac{x-7}{\sqrt{x+2}+3}-\frac{x-7}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$

$$=\lim_{x\to 7}\frac{\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8}{x-7}\times$$

$$\times\left[(x-7)\left(\frac{1}{\sqrt{x+2}+3}-\frac{1}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right)\right]$$

$$=\lim_{x\to 7}\frac{(\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8)(x-7)}{x-7}\times$$

$$\times\left[\frac{1}{\sqrt{x+2}+3}-\frac{1}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$

$$=\lim_{x\to 7}\frac{(\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8)\cancel{(x-7)}}{\cancel{x-7}}\times$$

$$\times\left[\frac{1}{\sqrt{x+2}+3}-\frac{1}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$

$$=\lim_{x\to 7}(\sqrt[4]{(x+9)^3}+2\sqrt[4]{(x+9)^2}+4\sqrt[4]{x+9}+8)\left[\frac{1}{\sqrt{x+2}+3}-\frac{1}{\sqrt[3]{(x+20)^2}+3\sqrt[3]{x+20}+9}\right]$$

$$=(\sqrt[4]{(7+9)^3}+2\sqrt[4]{(7+9)^2}+4\sqrt[4]{7+9}+8)\left[\frac{1}{\sqrt{7+2}+3}-\frac{1}{\sqrt[3]{(7+20)^2}+3\sqrt[3]{7+20}+9}\right]$$

$$=(8+8+8+8)\left[\frac{1}{3+3}-\frac{1}{9+9+9}\right]=32\left[\frac{1}{6}-\frac{1}{27}\right]=32\left[\frac{27-6}{6\cdot 27}\right]$$

$$=16\left[\frac{21}{3\cdot 27}\right]=16\left[\frac{7}{27}\right]=\frac{112}{27}$$

90. $\lim_{x\to 4}\frac{\sqrt{2x+1}-\sqrt[3]{3x+15}}{2-\sqrt{x}}$

Solución:

$$\lim_{x\to 4}\frac{\sqrt{2x+1}-\sqrt[3]{3x+15}}{2-\sqrt{x}}=\frac{\sqrt{2\cdot 4+1}-\sqrt[3]{3\cdot 4+15}}{2-\sqrt{4}}=\frac{0}{0}\qquad\text{(Indeterminación)}$$

$$\lim_{x\to 4}\frac{\sqrt{2x+1}-\sqrt[3]{3x+15}}{2-\sqrt{x}}=\lim_{x\to 4}\frac{\sqrt{2x+1}-3-\sqrt[3]{3x+15}+3}{2-\sqrt{x}}=\lim_{x\to 4}\frac{(\sqrt{2x+1}-3)-(\sqrt[3]{3x+15}-3)}{2-\sqrt{x}}$$

$$=\lim_{x\to 4}\frac{1}{2-\sqrt{x}}\left[(\sqrt{2x+1}-3)-(\sqrt[3]{3x+15}-3)\right]$$

$$=\lim_{x\to 4}\frac{1}{2-\sqrt{x}}\times\frac{2+\sqrt{x}}{2+\sqrt{x}}\left[(\sqrt{2x+1}-3)\times\frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3}-\right.$$

$$\left.-(\sqrt[3]{3x+15}-3)\times\frac{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$=\lim_{x\to 4}\frac{2+\sqrt{x}}{\underbrace{(2-\sqrt{x})(2+\sqrt{x})}_{(a-b)(a+b)}}\left[\frac{\overbrace{(\sqrt{2x+1}-3)(\sqrt{2x+1}+3)}^{(a-b)(a+b)}}{\sqrt{2x+1}+3}-\right.$$

$$\left.-\frac{\overbrace{(\sqrt[3]{3x+15}-3)(\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= \lim_{x\to 4} \underbrace{\frac{2+\sqrt{x}}{2^2-\sqrt{x}^2}}_{a^2-b^2}\left[\frac{\overbrace{\sqrt{(2x+1)^2}-3^2}^{a^2-b^2}}{\sqrt{2x+1}+3} - \frac{\overbrace{\sqrt[3]{(3x+15)^3}-3^3}^{a^3-b^3}}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= \lim_{x\to 4} \frac{2+\sqrt{x}}{4-x}\left[\frac{2x+1-9}{\sqrt{2x+1}+3} - \frac{3x+15-27}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= \lim_{x\to 4} \frac{2+\sqrt{x}}{-x+4}\left[\frac{2x-8}{\sqrt{2x+1}+3} - \frac{3x-12}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= \lim_{x\to 4} \frac{2+\sqrt{x}}{-(x-4)}\left[\frac{2(x-4)}{\sqrt{2x+1}+3} - \frac{3(x-4)}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= \lim_{x\to 4} -\frac{2+\sqrt{x}}{x-4}\left[(x-4)\left(\frac{2}{\sqrt{2x+1}+3} - \frac{3}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right)\right]$$

$$= \lim_{x\to 4} -\frac{(2+\sqrt{x}\,)(x-4)}{x-4}\left[\frac{2}{\sqrt{2x+1}+3} - \frac{3}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= \lim_{x\to 4} -\frac{(2+\sqrt{x}\,)\cancel{(x-4)}}{\cancel{x-4}}\left[\frac{2}{\sqrt{2x+1}+3} - \frac{3}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= \lim_{x\to 4} -(2+\sqrt{x}\,)\left[\frac{2}{\sqrt{2x+1}+3} - \frac{3}{\sqrt[3]{(3x+15)^2}+3\sqrt[3]{3x+15}+9}\right]$$

$$= -(2+\sqrt{4}\,)\left[\frac{2}{\sqrt{2\cdot 4+1}+3} - \frac{3}{\sqrt[3]{(3\cdot 4+15)^2}+3\sqrt[3]{3\cdot 4+15}+9}\right]$$

$$= -(2+2)\left[\frac{2}{\sqrt{9}+3} - \frac{3}{\sqrt[3]{(27)^2}+3\sqrt[3]{27}+9}\right] = -4\left[\frac{2}{3+3} - \frac{3}{9+3\cdot 3+9}\right]$$

$$= -4\left[\frac{2}{6} - \frac{3}{9+9+9}\right] = -4\left[\frac{1}{3} - \frac{3}{27}\right] = -4\left[\frac{1}{3} - \frac{1}{9}\right] = -4\left[\frac{3-1}{9}\right] = -4\left[\frac{2}{9}\right] = -\frac{8}{9}$$

91. $\lim_{x\to 1} \dfrac{\sqrt[3]{1-x}+\sqrt{x+3}-2}{\sqrt[3]{x^2-3x+2}}$

Solución:

$$\lim_{x\to 1} \frac{\sqrt[3]{1-x}+\sqrt{x+3}-2}{\sqrt[3]{x^2-3x+2}} = \frac{\sqrt[3]{1-1}+\sqrt{1+3}-2}{\sqrt[3]{1^2-3\cdot 1+2}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1} \frac{\sqrt[3]{1-x}+\sqrt{x+3}-2}{\sqrt[3]{x^2-3x+2}} = \lim_{x\to 1}\left[\frac{\sqrt[3]{1-x}}{\sqrt[3]{x^2-3x+2}} + \frac{\sqrt{x+3}-2}{\sqrt[3]{x^2-3x+2}}\right]$$

$$= \lim_{x\to 1}\left[\frac{\sqrt[3]{-x+1}}{\sqrt[3]{(x-2)(x-1)}} + \frac{\sqrt{x+3}-2}{\sqrt[3]{(x-2)(x-1)}}\right]$$

$$= \lim_{x\to 1}\left[\frac{\sqrt[3]{-(x-1)}}{\sqrt[3]{(x-2)}\cdot\sqrt[3]{(x-1)}} + \frac{\sqrt{x+3}-2}{\sqrt[3]{(x-2)(x-1)}}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\cdot\right.$$

$$\left.\cdot\frac{\sqrt[3]{((x-2)(x-1))^2}}{\sqrt[3]{((x-2)(x-1))^2}}\right]$$

$$= \lim_{x\to 1}\left[\frac{-\sqrt[3]{x-1}}{\sqrt[3]{(x-2)}\cdot\sqrt[3]{(x-1)}} + \frac{\overbrace{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}^{(a-b)(a+b)}}{\sqrt[3]{(x-2)(x-1)}\cdot\sqrt[3]{((x-2)(x-1))^2}}\cdot\right.$$

$$\left.\cdot\frac{\sqrt[3]{((x-2)(x-1))^2}}{\sqrt{x+3}+2}\right]$$

$$= \lim_{x\to 1}\left[\frac{-\cancel{\sqrt[3]{x-1}}}{\sqrt[3]{(x-2)}\cancel{\sqrt[3]{(x-1)}}} + \frac{\overbrace{\sqrt{(x+3)^2}-2^2}^{a^2-b^2}}{\sqrt[3]{((x-2)(x-1))^3}}\cdot\frac{\sqrt[3]{((x-2)(x-1))^2}}{\sqrt{x+3}+2}\right]$$

$$= \lim_{x\to 1}\left[\frac{-1}{\sqrt[3]{(x-2)}} + \frac{x+3-4}{\sqrt[3]{((x-2)(x-1))^3}} \cdot \frac{\sqrt[3]{(x-2)^2(x-1)^2}}{\sqrt{x+3}+2}\right]$$

$$= \lim_{x\to 1}\left[\frac{-1}{\sqrt[3]{(x-2)}} + \frac{x-1}{(x-2)(x-1)} \cdot \frac{\sqrt[3]{(x-2)^2(x-1)^2}}{\sqrt{x+3}+2}\right]$$

$$= \lim_{x\to 1}\left[\frac{-1}{\sqrt[3]{(x-2)}} + \frac{\cancel{x-1}}{(x-2)\cancel{(x-1)}} \cdot \frac{\sqrt[3]{(x-2)^2(x-1)^2}}{\sqrt{x+3}+2}\right]$$

$$= \lim_{x\to 1}\left[\frac{-1}{\sqrt[3]{(x-2)}} + \frac{1}{x-2} \cdot \frac{\sqrt[3]{(x-2)^2(x-1)^2}}{\sqrt{x+3}+2}\right]$$

$$= \frac{-1}{\sqrt[3]{(1-2)}} + \frac{1}{1-2} \cdot \frac{\sqrt[3]{(1-2)^2(1-1)^2}}{\sqrt{1+3}+2}$$

$$= \frac{-1}{-1} + \frac{1}{-1} \cdot \frac{\sqrt[3]{(-1)^2(0)^2}}{2+2} = 1 - \frac{0}{4} = 1 - 0 = 1$$

93. $\lim_{x\to 1} \dfrac{\sqrt{3x+1}+\sqrt{2x-1}-3\sqrt{x}}{\sqrt{x}+2\sqrt{5x+4}-7\sqrt{3x-2}}$

Solución

$$\lim_{x\to 1} \frac{\sqrt{3x+1}+\sqrt{2x-1}-3\sqrt{x}}{\sqrt{x}+2\sqrt{5x+4}-7\sqrt{3x-2}} = \frac{\sqrt{3\cdot 1+1}+\sqrt{2\cdot 1-1}-3\sqrt{1}}{\sqrt{1}+2\sqrt{5\cdot 1+4}-7\sqrt{3\cdot 1-2}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1} \frac{\sqrt{3x+1}+\sqrt{2x-1}-3\sqrt{x}}{\sqrt{x}+2\sqrt{5x+4}-7\sqrt{3x-2}} = \lim_{x\to 1} \frac{\sqrt{3x+1}-2+\sqrt{2x-1}-1-3\sqrt{x}+3}{2\sqrt{5x+4}-6-7\sqrt{3x-2}+7+\sqrt{x}-1}$$

$$= \lim_{x\to 1} \frac{(\sqrt{3x+1}-2)+(\sqrt{2x-1}-1)-3(\sqrt{x}-1)}{2(\sqrt{5x+4}-3)-7(\sqrt{3x-2}-1)+(\sqrt{x}-1)}$$

$$= \lim_{x\to 1} \frac{(\sqrt{3x+1}-2)\cdot\dfrac{\sqrt{3x+1}+2}{\sqrt{3x+1}+2}+(\sqrt{2x-1}-1)\cdot\dfrac{\sqrt{2x-1}+1}{\sqrt{2x-1}+1}-3(\sqrt{x}-1)\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}}{2(\sqrt{5x+4}-3)\cdot\dfrac{\sqrt{5x+4}+3}{\sqrt{5x+4}+3}-7(\sqrt{3x-2}-1)\cdot\dfrac{\sqrt{3x-2}+1}{\sqrt{3x-2}+1}+(\sqrt{x}-1)\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}}$$

$$= \lim_{x\to 1} \frac{\dfrac{\overbrace{(\sqrt{3x+1}-2)(\sqrt{3x+1}+2)}^{(a-b)(a+b)}}{\sqrt{3x+1}+2}+\dfrac{\overbrace{(\sqrt{2x-1}-1)(\sqrt{2x-1}+1)}^{(a-b)(a+b)}}{\sqrt{2x-1}+1}-\dfrac{3\ \overbrace{(\sqrt{x}-1)(\sqrt{x}+1)}^{(a-b)(a+b)}}{\sqrt{x}+1}}{\dfrac{2\ \overbrace{(\sqrt{5x+4}-3)(\sqrt{5x+4}+3)}^{(a-b)(a+b)}}{\sqrt{5x+4}+3}-\dfrac{7\ \overbrace{(\sqrt{3x-2}-1)(\sqrt{3x-2}+1)}^{(a-b)(a+b)}}{\sqrt{3x-2}+1}+\dfrac{\overbrace{(\sqrt{x}-1)(\sqrt{x}+1)}^{(a-b)(a+b)}}{\sqrt{x}+1}}$$

$$= \lim_{x\to 1} \frac{\dfrac{\overbrace{\sqrt{(3x+1)^2}-2^2}^{a^2-b^2}}{\sqrt{3x+1}+2}+\dfrac{\overbrace{\sqrt{(2x-1)^2}-1^2}^{a^2-b^2}}{\sqrt{2x-1}+1}-\dfrac{3\ \overbrace{(\sqrt{x^2}-1^2)}^{a^2-b^2}}{\sqrt{x}+1}}{\dfrac{2\ \overbrace{(\sqrt{(5x+4)^2}-3^2)}^{a^2-b^2}}{\sqrt{5x+4}+3}-\dfrac{7\ \overbrace{(\sqrt{(3x-2)^2}-1^2)}^{a^2-b^2}}{\sqrt{3x-2}+1}+\dfrac{\overbrace{\sqrt{x^2}-1^2}^{a^2-b^2}}{\sqrt{x}+1}}$$

$$= \lim_{x\to 1} \frac{\dfrac{3x+1-4}{\sqrt{3x+1}+2}+\dfrac{2x-1-1}{\sqrt{2x-1}+1}-\dfrac{3(x-1)}{\sqrt{x}+1}}{\dfrac{2(5x+4-9)}{\sqrt{5x+4}+3}-\dfrac{7(3x-2-1)}{\sqrt{3x-2}+1}+\dfrac{x-1}{\sqrt{x}+1}}$$

$$= \lim_{x\to 1} \frac{\dfrac{3x-3}{\sqrt{3x+1}+2}+\dfrac{2x-2}{\sqrt{2x-1}+1}-\dfrac{3(x-1)}{\sqrt{x}+1}}{\dfrac{2(5x-5)}{\sqrt{5x+4}+3}-\dfrac{7(3x-3)}{\sqrt{3x-2}+1}+\dfrac{x-1}{\sqrt{x}+1}}$$

$$= \lim_{x\to 1} \frac{\dfrac{3(x-1)}{\sqrt{3x+1}+2} + \dfrac{2(x-1)}{\sqrt{2x-1}+1} - \dfrac{3(x-1)}{\sqrt{x}+1}}{\dfrac{2\cdot 5(x-1)}{\sqrt{5x+4}+3} - \dfrac{7\cdot 3(x-1)}{\sqrt{3x-2}+1} + \dfrac{x-1}{\sqrt{x}+1}}$$

$$= \lim_{x\to 1} \frac{\dfrac{3(x-1)}{\sqrt{3x+1}+2} + \dfrac{2(x-1)}{\sqrt{2x-1}+1} - \dfrac{3(x-1)}{\sqrt{x}+1}}{\dfrac{10(x-1)}{\sqrt{5x+4}+3} - \dfrac{21(x-1)}{\sqrt{3x-2}+1} + \dfrac{x-1}{\sqrt{x}+1}}$$

$$= \lim_{x\to 1} \frac{(x-1)\left[\dfrac{3}{\sqrt{3x+1}+2} + \dfrac{2}{\sqrt{2x-1}+1} - \dfrac{3}{\sqrt{x}+1}\right]}{(x-1)\left[\dfrac{10}{\sqrt{5x+4}+3} - \dfrac{21}{\sqrt{3x-2}+1} + \dfrac{1}{\sqrt{x}+1}\right]}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}\left[\dfrac{3}{\sqrt{3x+1}+2} + \dfrac{2}{\sqrt{2x-1}+1} - \dfrac{3}{\sqrt{x}+1}\right]}{\cancel{(x-1)}\left[\dfrac{10}{\sqrt{5x+4}+3} - \dfrac{21}{\sqrt{3x-2}+1} + \dfrac{1}{\sqrt{x}+1}\right]}$$

$$= \lim_{x\to 1} \frac{\dfrac{3}{\sqrt{3x+1}+2} + \dfrac{2}{\sqrt{2x-1}+1} - \dfrac{3}{\sqrt{x}+1}}{\dfrac{10}{\sqrt{5x+4}+3} - \dfrac{21}{\sqrt{3x-2}+1} + \dfrac{1}{\sqrt{x}+1}}$$

$$= \frac{\dfrac{3}{\sqrt{3\cdot 1+1}+2} + \dfrac{2}{\sqrt{2\cdot 1-1}+1} - \dfrac{3}{\sqrt{1}+1}}{\dfrac{10}{\sqrt{5\cdot 1+4}+3} - \dfrac{21}{\sqrt{3\cdot 1-2}+1} + \dfrac{1}{\sqrt{1}+1}}$$

$$= \frac{\dfrac{3}{2+2} + \dfrac{2}{1+1} - \dfrac{3}{1+1}}{\dfrac{10}{3+3} - \dfrac{21}{1+1} + \dfrac{1}{1+1}} = \frac{\dfrac{3}{4} + \dfrac{2}{2} - \dfrac{3}{2}}{\dfrac{10}{6} - \dfrac{21}{2} + \dfrac{1}{2}}$$

$$= \frac{\dfrac{3}{4} + 1 - \dfrac{3}{2}}{\dfrac{5}{3} - \dfrac{21}{2} + \dfrac{1}{2}} = \frac{\dfrac{3+4-6}{4}}{\dfrac{10-63+3}{6}} = \frac{\dfrac{1}{4}}{\dfrac{-50}{6}}$$

$$= \frac{6}{-200} = -\frac{3}{100}$$

95. $\displaystyle\lim_{x\to 4} \frac{\sqrt[3]{2x} - \sqrt{x} + 2x - 8}{x-4}$

Solución:

$$\lim_{x\to 4} \frac{\sqrt[3]{2x} - \sqrt{x} + 2x - 8}{x-4} = \frac{\sqrt[3]{2\cdot 4} - \sqrt{4} + 2\cdot 4 - 8}{4-4} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 4} \frac{\sqrt[3]{2x} - \sqrt{x} + 2x - 8}{x-4} = \lim_{x\to 4} \frac{\sqrt[3]{2x} - 2 - \sqrt{x} + 2 + 2(x-4)}{x-4} = \lim_{x\to 4} \frac{(\sqrt[3]{2x}-2) - (\sqrt{x}-2) + 2(x-4)}{x-4}$$

$$= \lim_{x\to 4} \frac{1}{x-4}\left[(\sqrt[3]{2x}-2) - (\sqrt{x}-2) + 2(x-4)\right]$$

$$= \lim_{x\to 4} \frac{1}{x-4}\left[(\sqrt[3]{2x}-2)\cdot\frac{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - (\sqrt{x}-2)\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2} + 2(x-4)\right]$$

$$= \lim_{x\to 4} \frac{1}{x-4}\left[\frac{\overbrace{(\sqrt[3]{2x}-2)(\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{\overbrace{(\sqrt{x}-2)(\sqrt{x}+2)}^{(a-b)(a+b)}}{\sqrt{x}+2} + 2(x-4)\right]$$

$$= \lim_{x\to 4} \frac{1}{x-4}\left[\frac{\overbrace{\sqrt[3]{(2x)^3}-2^3}^{a^3-b^3}}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{\overbrace{\sqrt{x^2}-2^2}^{a^2-b^2}}{\sqrt{x}+2} + 2(x-4)\right]$$

$$= \lim_{x\to 4} \frac{1}{x-4}\left[\frac{2x-8}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{x-4}{\sqrt{x}+2} + 2(x-4)\right]$$
$$= \lim_{x\to 4} \frac{1}{x-4}\left[\frac{2(x-4)}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{x-4}{\sqrt{x}+2} + 2(x-4)\right]$$
$$= \lim_{x\to 4} \frac{1}{x-4}\left[(x-4)\left(\frac{2}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{1}{\sqrt{x}+2} + 2\right)\right]$$
$$= \lim_{x\to 4} \frac{x-4}{x-4}\left[\frac{2}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{1}{\sqrt{x}+2} + 2\right]$$
$$= \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{x-4}}\left[\frac{2}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{1}{\sqrt{x}+2} + 2\right]$$
$$= \lim_{x\to 4} \left[\frac{2}{\sqrt[3]{(2x)^2}+2\sqrt[3]{2x}+4} - \frac{1}{\sqrt{x}+2} + 2\right]$$
$$= \frac{2}{\sqrt[3]{(2\cdot 4)^2}+2\sqrt[3]{2\cdot 4}+4} - \frac{1}{\sqrt{4}+2} + 2$$
$$= \frac{2}{4+4+4} - \frac{1}{2+2} + 2 = \frac{2}{12} - \frac{1}{4} + 2$$
$$= \frac{2-3+24}{12} = \frac{23}{12}$$

96. $\lim\limits_{x\to 0} \dfrac{x^3+x}{\sqrt[3]{1+x^2}-\sqrt{1-2x}}$

Solución:

$$\lim_{x\to 0} \frac{x^3+x}{\sqrt[3]{1+x^2}-\sqrt{1-2x}} = \frac{0^3+0}{\sqrt[3]{1+0^2}-\sqrt{1-2\cdot 0}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 0} \frac{x^3+x}{\sqrt[3]{1+x^2}-\sqrt{1-2x}} = \lim_{x\to 0} \frac{x(x^2+1)}{\sqrt[3]{1+x^2}-1-\sqrt{1-2x}+1} = \lim_{x\to 0} \frac{x(x^2+1)}{(\sqrt[3]{1+x^2}-1)-(\sqrt{1-2x}-1)}$$
$$= \lim_{x\to 0} \frac{x(x^2+1)}{(\sqrt[3]{1+x^2}-1)\cdot\dfrac{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1} - (\sqrt{1-2x}-1)\cdot\dfrac{\sqrt{1-2x}+1}{\sqrt{1-2x}+1}}$$
$$= \lim_{x\to 0} \frac{x(x^2+1)}{\dfrac{\overbrace{(\sqrt[3]{1+x^2}-1)(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1} - \dfrac{\overbrace{(\sqrt{1-2x}-1)(\sqrt{1-2x}+1)}^{(a-b)(a+b)}}{\sqrt{1-2x}+1}}$$
$$= \lim_{x\to 0} \frac{x(x^2+1)}{\dfrac{\overbrace{\sqrt[3]{(1+x^2)^3}-1^3}^{a^3-b^3}}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1} - \dfrac{\overbrace{\sqrt{(1-2x)^2}-1^2}^{a^2-b^2}}{\sqrt{1-2x}+1}}$$
$$= \lim_{x\to 0} \frac{x(x^2+1)}{\dfrac{1+x^2-1}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1} - \dfrac{1-2x-1}{\sqrt{1-2x}+1}}$$
$$= \lim_{x\to 0} \frac{x(x^2+1)}{\dfrac{x^2}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1} - \dfrac{-2x}{\sqrt{1-2x}+1}}$$
$$= \lim_{x\to 0} \frac{x(x^2+1)}{\dfrac{x^2}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1} + \dfrac{2x}{\sqrt{1-2x}+1}}$$

$$= \lim_{x\to 0} \frac{x(x^2+1)}{x\left(\frac{x}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1}+\frac{2}{\sqrt{1-2x}+1}\right)}$$

$$= \lim_{x\to 0} \frac{\cancel{x}(x^2+1)}{\cancel{x}\left(\frac{x}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1}+\frac{2}{\sqrt{1-2x}+1}\right)}$$

$$= \lim_{x\to 0} \frac{x^2+1}{\frac{x}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1}+\frac{2}{\sqrt{1-2x}+1}}$$

$$= \frac{0^2+1}{\frac{0}{\sqrt[3]{(1+0^2)^2}+\sqrt[3]{1+0^2}+1}+\frac{2}{\sqrt{1-2\cdot 0}+1}}$$

$$= \frac{0+1}{\frac{0}{1+1+1}+\frac{2}{1+1}} = \frac{1}{\frac{0}{3}+\frac{2}{2}} = \frac{1}{0+1} = \frac{1}{1} = 1$$

97. $\lim_{x\to 2} \frac{5\sqrt[3]{3x-5}-3\sqrt{3x-5}+2x-6}{x^2-2x}$

Solución:

$$\lim_{x\to 2} \frac{5\sqrt[3]{3x-5}-3\sqrt{3x-5}+2x-6}{x^2-2x} = \frac{5\sqrt[3]{3\cdot 2-5}-3\sqrt{3\cdot 2-5}+2\cdot 2-6}{2^2-2\cdot 2} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 2} \frac{5\sqrt[3]{3x-5}-3\sqrt{3x-5}+2x-6}{x^2-2x} = \lim_{x\to 2} \frac{5\sqrt[3]{3x-5}-5-3\sqrt{3x-5}+3+2x-6+5-3}{x(x-2)}$$

$$= \lim_{x\to 2} \frac{5(\sqrt[3]{3x-5}-1)-3(\sqrt{3x-5}-1)+2x-4}{x(x-2)}$$

$$= \lim_{x\to 2} \frac{5(\sqrt[3]{3x-5}-1)-3(\sqrt{3x-5}-1)+2(x-2)}{x(x-2)}$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[5(\sqrt[3]{3x-5}-1)-3(\sqrt{3x-5}-1)+2(x-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[5(\sqrt[3]{3x-5}-1)\cdot\frac{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \right.$$
$$\left. -3(\sqrt{3x-5}-1)\cdot\frac{\sqrt{3x-5}+1}{\sqrt{3x-5}+1}+2(x-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[\frac{5\overbrace{(\sqrt[3]{3x-5}-1)(\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \right.$$
$$\left. -\frac{3\overbrace{(\sqrt{3x-5}-1)(\sqrt{3x-5}+1)}^{(a-b)(a+b)}}{\sqrt{3x-5}+1}+2(x-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[\frac{5\overbrace{(\sqrt[3]{(3x-5)^3}-1^3)}^{a^3-b^3}}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{3\overbrace{(\sqrt{(3x-5)^2}-1^2)}^{a^2-b^2}}{\sqrt{3x-5}+1}+2(x-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[\frac{5(3x-5-1)}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{3(3x-5-1)}{\sqrt{3x-5}+1}+2(x-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[\frac{5(3x-6)}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{3(3x-6)}{\sqrt{3x-5}+1}+2(x-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[\frac{5\cdot 3(x-2)}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{3\cdot 3(x-2)}{\sqrt{3x-5}+1}+2(x-2)\right]$$

$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[\frac{15(x-2)}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{9(x-2)}{\sqrt{3x-5}+1} + 2(x-2)\right]$$
$$= \lim_{x\to 2} \frac{1}{x(x-2)}\left[(x-2)\left(\frac{15}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{9}{\sqrt{3x-5}+1} + 2\right)\right]$$
$$= \lim_{x\to 2} \frac{x-2}{x(x-2)}\left[\frac{15}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{9}{\sqrt{3x-5}+1} + 2\right]$$
$$= \lim_{x\to 2} \frac{\cancel{x-2}}{x\cancel{(x-2)}}\left[\frac{15}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{9}{\sqrt{3x-5}+1} + 2\right]$$
$$= \lim_{x\to 2} \frac{1}{x}\left[\frac{15}{\sqrt[3]{(3x-5)^2}+\sqrt[3]{3x-5}+1} - \frac{9}{\sqrt{3x-5}+1} + 2\right]$$
$$= \frac{1}{2}\left[\frac{15}{\sqrt[3]{(3\cdot 2-5)^2}+\sqrt[3]{3\cdot 2-5}+1} - \frac{9}{\sqrt{3\cdot 2-5}+1} + 2\right]$$
$$= \frac{1}{2}\left[\frac{15}{1+1+1} - \frac{9}{1+1} + 2\right] = \frac{1}{2}\left[\frac{15}{3} - \frac{9}{2} + 2\right] = \frac{1}{2}\left[5 - \frac{9}{2} + 2\right]$$
$$= \frac{1}{2}\left[7 - \frac{9}{2}\right] = \frac{1}{2}\left[\frac{14-9}{2}\right] = \frac{1}{2}\left[\frac{5}{2}\right] = \frac{5}{4}$$

98. $\lim\limits_{x\to 0} \dfrac{x^2}{\sqrt[3]{8+x^3}-\sqrt{4+x^2}}$

Solución:

$$\lim_{x\to 0} \frac{x^2}{\sqrt[3]{8+x^3}-\sqrt{4+x^2}} = \lim_{x\to 0} \frac{0^2}{\sqrt[3]{8+0^3}-\sqrt{4+0^2}} = \frac{0}{0} \qquad \text{(Indeterminación)}$$
$$\lim_{x\to 0} \frac{x^2}{\sqrt[3]{8+x^3}-\sqrt{4+x^2}} = \lim_{x\to 0} \frac{x^2}{\sqrt[3]{8+x^3}-2-\sqrt{4+x^2}+2} = \lim_{x\to 0} \frac{x^2}{(\sqrt[3]{8+x^3}-2)-(\sqrt{4+x^2}-2)}$$
$$= \lim_{x\to 0} \frac{x^2}{(\sqrt[3]{8+x^3}-2)\cdot\frac{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - (\sqrt{4+x^2}-2)\cdot\frac{\sqrt{4+x^2}+2}{\sqrt{4+x^2}+2}}$$
$$= \lim_{x\to 0} \frac{x^2}{\frac{\overbrace{(\sqrt[3]{8+x^3}-2)(\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - \frac{\overbrace{(\sqrt{4+x^2}-2)(\sqrt{4+x^2}+2)}^{(a-b)(a+b)}}{\sqrt{4+x^2}+2}}$$
$$= \lim_{x\to 0} \frac{x^2}{\frac{\overbrace{\sqrt[3]{(8+x^3)^3}-2^3}^{a^3-b^3}}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - \frac{\overbrace{\sqrt{(4+x^2)^2}-2^2}^{a^2-b^2}}{\sqrt{4+x^2}+2}}$$
$$= \lim_{x\to 0} \frac{x^2}{\frac{8+x^3-8}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - \frac{4+x^2-4}{\sqrt{4+x^2}+2}}$$
$$= \lim_{x\to 0} \frac{x^2}{\frac{x^3}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - \frac{x^2}{\sqrt{4+x^2}+2}}$$
$$= \lim_{x\to 0} \frac{x^2}{x^2\left(\frac{x}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - \frac{1}{\sqrt{4+x^2}+2}\right)}$$
$$= \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}\left(\frac{x}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - \frac{1}{\sqrt{4+x^2}+2}\right)}$$

$$= \lim_{x\to 0} \frac{1}{\dfrac{x}{\sqrt[3]{(8+x^3)^2}+2\sqrt[3]{8+x^3}+4} - \dfrac{1}{\sqrt{4+x^2}+2}}$$

$$= \frac{1}{\dfrac{0}{\sqrt[3]{(8+0^3)^2}+2\sqrt[3]{8+0^3}+4} - \dfrac{1}{\sqrt{4+0^2}+2}}$$

$$= \frac{1}{\dfrac{0}{4+4+4} - \dfrac{1}{2+2}} = \frac{1}{\dfrac{0}{12}-\dfrac{1}{4}} = \frac{1}{0-\dfrac{1}{4}} = \frac{1}{-\dfrac{1}{4}} = -\frac{4}{1} = -4$$

99. $\displaystyle\lim_{x\to 1} \frac{3\sqrt[4]{x^4+15}-5\sqrt[3]{x^3+7}+4x}{x^4-1}$

Solución:

$$\lim_{x\to 1} \frac{3\sqrt[4]{x^4+15}-5\sqrt[3]{x^3+7}+4x}{x^4-1} = \frac{3\sqrt[4]{1^4+15}-5\sqrt[3]{1^3+7}+4\cdot 1}{1^4-1} = \frac{0}{0} \qquad \text{(Indeterminación)}$$

$$\lim_{x\to 1} \frac{3\sqrt[4]{x^4+15}-5\sqrt[3]{x^3+7}+4x}{x^4-1} = \lim_{x\to 1} \frac{3\sqrt[4]{x^4+15}-6-5\sqrt[3]{x^3+7}+10+4x+6-10}{x^4-1}$$

$$= \lim_{x\to 1} \frac{3(\sqrt[4]{x^4+15}-2)-5(\sqrt[3]{x^3+7}-2)+4x-4}{x^4-1}$$

$$= \lim_{x\to 1} \frac{3(\sqrt[4]{x^4+15}-2)-5(\sqrt[3]{x^3+7}-2)+4(x-1)}{x^4-1}$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\Big[3(\sqrt[4]{x^4+15}-2)-5(\sqrt[3]{x^3+7}-2)+4(x-1)\Big]$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\Bigg[3(\sqrt[4]{x^4+15}-2)\cdot \frac{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - 5(\sqrt[3]{x^3+7}-2)\cdot\frac{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4(x-1)\Bigg]$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\Bigg[\frac{3\,\overbrace{(\sqrt[4]{x^4+15}-2)(\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8)}^{(a-b)(a^3+a^2b+ab^2+b^3)}}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \frac{5\,\overbrace{(\sqrt[3]{x^3+7}-2)(\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4)}^{(a-b)(a^2+ab+b^2)}}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4(x-1)\Bigg]$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\Bigg[\frac{3\,\overbrace{(\sqrt[4]{(x^4+15)^4}-2^4)}^{a^4-b^4}}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \frac{5\,\overbrace{(\sqrt[3]{(x^3+7)^3}-2^3)}^{a^3-b^3}}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4(x-1)\Bigg]$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\Bigg[\frac{3(x^4+15-16)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \frac{5(x^3+7-8)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4(x-1)\Bigg]$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\Bigg[\frac{3(x^4-1)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \frac{5(x^3-1)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4(x-1)\Bigg]$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\left[\frac{3(x-1)(x^3+x^2+x+1)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \right.$$

$$\left. -\frac{5(x-1)(x^2+x+1)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4(x-1)\right]$$

$$= \lim_{x\to 1} \frac{1}{x^4-1}\left[(x-1)\left(\frac{3(x^3+x^2+x+1)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \right.\right.$$

$$\left.\left. -\frac{5(x^2+x+1)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4\right)\right]$$

$$= \lim_{x\to 1} \frac{(x-1)}{x^4-1}\left[\frac{3(x^3+x^2+x+1)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \right.$$

$$\left. -\frac{5(x^2+x+1)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4\right]$$

$$= \lim_{x\to 1} \frac{(x-1)}{(x-1)(x^3+x^2+x+1)}\left[\frac{3(x^3+x^2+x+1)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \right.$$

$$\left. -\frac{5(x^2+x+1)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4\right]$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x^3+x^2+x+1)}\left[\frac{3(x^3+x^2+x+1)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \right.$$

$$\left. -\frac{5(x^2+x+1)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4\right]$$

$$= \lim_{x\to 1} \frac{1}{x^3+x^2+x+1}\left[\frac{3(x^3+x^2+x+1)}{\sqrt[4]{(x^4+15)^3}+2\sqrt[4]{(x^4+15)^2}+4\sqrt[4]{x^4+15}+8} - \right.$$

$$\left. -\frac{5(x^2+x+1)}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}+4\right]$$

$$= \frac{1}{1^3+1^2+1+1}\left[\frac{3(1^3+1^2+1+1)}{\sqrt[4]{(1^4+15)^3}+2\sqrt[4]{(1^4+15)^2}+4\sqrt[4]{x^1+15}+8} - \right.$$

$$\left. -\frac{5(1^2+1+1)}{\sqrt[3]{(1^3+7)^2}+2\sqrt[3]{1^3+7}+4}+4\right]$$

$$= \frac{1}{4}\left[\frac{3(4)}{8+8+8+8}-\frac{5(3)}{4+4+4}+4\right] = \frac{1}{4}\left[\frac{12}{32}-\frac{15}{12}+4\right] = \frac{1}{4}\left[\frac{3}{8}-\frac{5}{4}+4\right]$$

$$= \frac{1}{4}\left[\frac{3-10+32}{8}\right] = \frac{1}{4}\left[\frac{25}{8}\right] = \frac{25}{32}$$