Capítulo 2
Transformaciones algebraicas
Propiedades de la potenciacion
Formulas basicas

Para cualquier número natural x y y, y números naturales positivos a y b se verifica la igualdad:

$$\begin{aligned}
a^0 &= 1 && (1)\\
a^x \cdot a^y &= a^{x+y} && (2)\\
a^x : a^y &= a^{x-y} && (3)\\
(a^x)^y &= a^{xy} && (4)\\
(ab)^x &= a^x b^x && (5)\\
\left(\frac{a}{b}\right)^x &= \frac{a^x}{b^x} && (6)\\
a^{-x} &= \frac{1}{a^x} && (7)
\end{aligned}$$

Polinomios

Para cualquier número natural $a,\ b$ y c se verifica la igualdad:

$$\begin{aligned}
a^2 - b^2 &= (a-b)(a+b) && (8)\\
(a+b)^2 &= a^2 + 2ab + b^2 && (9)\\
(a-b)^2 &= a^2 - 2ab + b^2 && (10)\\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 && (11)\\
(a+b)^3 &= a^3 + b^3 + 3ab(a+b) &&\\
(a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 && (12)\\
(a-b)^3 &= a^3 - b^3 - 3ab(a-b) &&\\
a^3 + b^3 &= (a+b)(a^2 - ab + b^2) && (13)\\
a^3 - b^3 &= (a-b)(a^2 + ab + b^2) && (14)\\
ax^2 + bx + c &= a(x-x_1)(x-x_2) && (15)
\end{aligned}$$

donde x_1 y x_2 - son las raices de la ecuación $ax^2 + bx + c = 0$

Propiedad de la reducción aritmética

Para cualquier número natural n y k mayores a 1, y cualquier número natural no negativo a y b se verifican las siguientes igualdades:

$$\begin{aligned}
\sqrt[n]{ab} &= \sqrt[n]{a}\ \sqrt[n]{b} && (16)\\
\sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (b \neq 0) && (17)\\
(\sqrt[n]{a})^k &= \sqrt[n]{a^k} && (18)\\
\sqrt[n]{\sqrt[k]{a}} &= \sqrt[kn]{a} && (19)\\
\sqrt[n]{a} &= \sqrt[kn]{a^k} && (20)\\
(\sqrt[n]{a})^n &= a\ (a \geq 0) && (21)\\
\sqrt[n]{a} &< \sqrt[n]{b} \quad 0 \leq a < b && (22)\\
\sqrt{a^2} = \mid a \mid &= \begin{cases} a & \text{si} \quad a > 0 \\ -a & \text{si} \quad a < 0 \end{cases} && (23)\\
\sqrt[2n]{a^{2n}} &= \mid a \mid && (24)\\
\sqrt[2n+1]{-a} &= -\sqrt[2n+1]{a} \quad (a \geq 0) && (25)
\end{aligned}$$

Simplificar las expresiones siguientes

46. $(a^2-b^2-c^2+2bc):\dfrac{a+b-c}{a+b+c}$, evaluar el resultado para $a=8,6$; $b=\sqrt{3}$; $c=3\frac{1}{3}$

Solución.

$$\begin{aligned}
(a^2-b^2-c^2+2bc):\frac{a+b-c}{a+b+c} &= (a^2-b^2+2bc-c^2)\cdot\frac{a+b+c}{a+b-c}\\
&= (a^2-(b^2-2bc+c^2))\cdot\frac{a+b+c}{a+b-c}\\
&= (a^2-(b-c)^2)\cdot\frac{a+b+c}{a+b-c}\\
&= (a-(b-c))(a+(b-c))\cdot\frac{a+b+c}{a+b-c}\\
&= (a-b+c)(a+b-c)\cdot\frac{a+b+c}{a+b-c}\\
&= \frac{(a-b+c)(a+b-c)(a+b+c)}{a+b-c}\\
&= \frac{(a-b+c)\cancel{(a+b-c)}(a+b+c)}{\cancel{a+b-c}}\\
&= (a-b+c)(a+b+c)\\
&= \big((a+c)-b\big)\big((a+c)+b\big)\\
&= \boxed{(a+c)^2-b^2}
\end{aligned}$$

reemplazando: $a=8,6=\dfrac{43}{5}$; $b=\sqrt{3}$; $c=3\dfrac{1}{3}=\dfrac{10}{3}$

$$(a+c)^2-b^2=\left(\frac{43}{5}+\frac{10}{3}\right)^2-(\sqrt{3})^2=\left(\frac{129+50}{15}\right)^2-3=\left(\frac{179}{15}\right)^2-3=\frac{32041}{225}-3$$

$$=\frac{32041-675}{225}=\frac{31366}{225}=\boxed{139\frac{91}{225}}$$

47. $\dfrac{a^2-1}{n^2+an}\left(\dfrac{1}{1-\frac{1}{n}}-1\right)\dfrac{a-an^3-n^4+n}{1-a^2}$

Solución.

$$\begin{aligned}
\frac{a^2-1}{n^2+an}\left(\frac{1}{1-\frac{1}{n}}-1\right)\frac{a-an^3-n^4+n}{1-a^2} &= \frac{a^2-1}{n(n+a)}\left(\frac{1}{\frac{n-1}{n}}-1\right)\frac{a+n-an^3-n^4}{-a^2+1}\\
&= \frac{a^2-1}{n(n+a)}\left(\frac{n}{n-1}-1\right)\frac{(a+n)-n^3(a+n)}{-(a^2-1)}\\
&= \frac{a^2-1}{n(n+a)}\left(\frac{n-(n-1)}{n-1}\right)\frac{(a+n)(1-n^3)}{-(a^2-1)}\\
&= \frac{a^2-1}{n(n+a)}\left(\frac{n-n+1}{n-1}\right)\frac{(a+n)(-n^3+1)}{-(a^2-1)}\\
&= \frac{a^2-1}{n(n+a)}\left(\frac{1}{n-1}\right)\frac{-(a+n)(n^3-1)}{-(a^2-1)}
\end{aligned}$$

$$= \frac{a^2-1}{n(a+n)}\left(\frac{1}{n-1}\right)\frac{(a+n)(n-1)(n^2+n+1)}{a^2-1}$$

$$= \frac{(a^2-1)(a+n)(n-1)(n^2+n+1)}{n(a+n)(n-1)(a^2-1)}$$

$$= \frac{\cancel{(a^2-1)}\,\cancel{(a+n)}\,\cancel{(n-1)}(n^2+n+1)}{n\cancel{(a+n)}\,\cancel{(n-1)}\,\cancel{(a^2-1)}}$$

$$= \boxed{\frac{n^2+n+1}{n}}$$

48. $\dfrac{x}{ax-2a^2} - \dfrac{2}{x^2+x-2ax-2a}\left(1+\dfrac{3x+x^2}{3+x}\right)$

Solución.

$$\frac{x}{ax-2a^2} - \frac{2}{x^2+x-2ax-2a}\left(1+\frac{3x+x^2}{3+x}\right) = \frac{x}{a(x-2a)} - \frac{2}{x(x+1)-2a(x+1)}\left(1+\frac{x(3+x)}{3+x}\right)$$

$$= \frac{x}{a(x-2a)} - \frac{2}{(x+1)(x-2a)}\left(1+\frac{x\cancel{(3+x)}}{\cancel{3+x}}\right)$$

$$= \frac{x}{a(x-2a)} - \frac{2}{(x+1)(x-2a)}(x+1)$$

$$= \frac{x}{a(x-2a)} - \frac{2(x+1)}{(x+1)(x-2a)}$$

$$= \frac{x}{a(x-2a)} - \frac{2\cancel{(x+1)}}{\cancel{(x+1)}(x-2a)}$$

$$= \frac{x}{a(x-2a)} - \frac{2}{x-2a}$$

$$= \frac{x-2a}{a(x-2a)}$$

$$= \frac{\cancel{x-2a}}{a\cancel{(x-2a)}}$$

$$= \boxed{\frac{1}{a}}$$

49. $\dfrac{2a}{a^2-4x^2} + \dfrac{1}{2x^2+6x-ax-3x}\left(x+\dfrac{3x-6}{x-2}\right)$

Solución.

$$\frac{2a}{a^2-4x^2} + \frac{1}{2x^2+6x-ax-3a}\left(x+\frac{3x-6}{x-2}\right) = \frac{2a}{-4x^2+a^2} + \frac{1}{2x(x+3)-a(x+3)}\left(x+\frac{3(x-2)}{x-2}\right)$$

$$= \frac{2a}{-4x^2+a^2} + \frac{1}{(x+3)(2x-a)}\left(x+\frac{3\cancel{(x-2)}}{\cancel{x-2}}\right)$$

$$= \frac{2a}{-(4x^2-a^2)} + \frac{1}{(x+3)(2x-a)}(x+3)$$

$$= \frac{2a}{-(4x^2-a^2)} + \frac{x+3}{(x+3)(2x-a)}$$

$$= -\frac{2a}{(2x-a)(2x+a)} + \frac{\cancel{x+3}}{\cancel{(x+3)}(2x-a)}$$

$$= -\frac{2a}{(2x-a)(2x+a)} + \frac{1}{2x-a}$$

$$= \frac{-2a+2x+a}{(2x-a)(2x+a)}$$

$$= \frac{2x-a}{(2x-a)(2x+a)}$$

$$= \frac{\cancel{2x-a}}{\cancel{(2x-a)}(2x+a)}$$

$$= \boxed{\frac{1}{2x+a}}$$

50. $\left(\frac{2a+10}{3a-1}+\frac{130-a}{1-3a}+\frac{30}{a}-3\right)\frac{3a^3+8a^2-3a}{1-\frac{1}{4}a^2}$

Solución.

$$\left(\frac{2a+10}{3a-1}+\frac{130-a}{1-3a}+\frac{30}{a}-3\right)\frac{3a^3+8a^2-3a}{1-\frac{1}{4}a^2}=$$

$$= \left(\frac{2a+10}{3a-1}+\frac{-a+130}{-3a+1}+\frac{30}{a}-3\right)\frac{a(3a^2+8a-3)}{1-\frac{a^2}{4}}$$

$$= \left(\frac{2a+10}{3a-1}+\frac{-(a-130)}{-(3a-1)}+\frac{30}{a}-3\right)\frac{a(a+3)(3a-1)}{\frac{4-a^2}{4}}$$

$$= \left(\frac{2a+10}{3a-1}+\frac{a-130}{3a-1}+\frac{30}{a}-3\right)\frac{4a(a+3)(3a-1)}{4-a^2}$$

$$= \frac{a(2a+10)+a(a-130)+30(3a-1)-3a(2a-1)}{(3a-1)a}\cdot\frac{4a(a+3)(3a-1)}{-a^2+4}$$

$$= \frac{2a^2+10a+a^2-130a+90a-30-9a^2+3a}{(3a-1)a}\cdot\frac{4a(a+3)(3a-1)}{-(a^2-4)}$$

$$= \frac{-6a^2-27a-30}{(3a-1)a}\cdot\frac{4a(a+3)(3a-1)}{-(a-2)(a+2)}$$

$$= \frac{-3(2a^2+9a+10)}{(3a-1)a}\cdot\frac{4a(a+3)(3a-1)}{-(a-2)(a+2)}$$

$$= \frac{-3(2a+5)(a+2)}{(3a-1)a}\cdot\frac{4a(a+3)(3a-1)}{-(a-2)(a+2)}$$

$$= \frac{-12a(2a+5)(a+2)(a+3)(3a-1)}{-a(3a-1)(a-2)(a+2)}$$

$$= \frac{-12\cancel{a}(2a+5)\cancel{(a+2)}(a+3)\cancel{(3a-1)}}{-\cancel{a}\cancel{(3a-1)}(a-2)\cancel{(a+2)}}$$

$$= \boxed{\frac{12(2a+5)(a+3)}{a-2}}$$

51. $\frac{a^2-b^2}{a-b}-\frac{a^3-b^3}{a^2-b^2}$

Solución.

$$\frac{a^2-b^2}{a-b}-\frac{a^3-b^3}{a^2-b^2}=\frac{(a-b)(a+b)}{a-b}-\frac{(a-b)(a^2+ab+b^2)}{(a-b)(a+b)}$$

$$= \frac{\cancel{(a-b)}(a+b)}{\cancel{a-b}} - \frac{\cancel{(a-b)}(a^2+ab+b^2)}{\cancel{(a-b)}(a+b)}$$

$$= (a+b) - \frac{a^2+ab+b^2}{a+b}$$

$$= \frac{(a+b)(a+b)-(a^2+ab+b^2)}{a+b}$$

$$= \frac{a^2+ab+ab+b^2-a^2-ab-b^2}{a+b}$$

$$= \boxed{\frac{ab}{a+b}}$$

52. $\dfrac{x}{x^2+y^2} - \dfrac{y(x-y)^2}{x^4-y^4}$

Solución.

$$\frac{x}{x^2+y^2} - \frac{y(x-y)^2}{x^4-y^4} = \frac{x}{x^2+y^2} - \frac{y(x-y)^2}{(x^2)^2-(y^2)^2}$$

$$= \frac{x}{x^2+y^2} - \frac{y(x-y)^2}{(x^2-y^2)(x^2+y^2)}$$

$$= \frac{x(x^2-y^2)-y(x-y)^2}{(x^2-y^2)(x^2+y^2)}$$

$$= \frac{x(x-y)(x+y)-y(x-y)^2}{(x-y)(x+y)(x^2+y^2)}$$

$$= \frac{(x-y)\big(x(x+y)-y(x-y)\big)}{(x-y)(x+y)(x^2+y^2)}$$

$$= \frac{\cancel{(x-y)}\big(x(x+y)-y(x-y)\big)}{\cancel{(x-y)}(x+y)(x^2+y^2)}$$

$$= \frac{x(x+y)-y(x-y)}{(x+y)(x^2+y^2)}$$

$$= \frac{x^2+xy-xy+y^2}{(x+y)(x^2+y^2)}$$

$$= \frac{x^2+y^2}{(x+y)(x^2+y^2)}$$

$$= \frac{\cancel{x^2+y^2}}{(x+y)\cancel{(x^2+y^2)}}$$

$$= \boxed{\frac{1}{x+y}}$$

53. $\dfrac{2}{3}\left[\dfrac{1}{1+\left(\dfrac{2x+1}{\sqrt{3}}\right)^2} + \dfrac{1}{1+\left(\dfrac{2x-1}{\sqrt{3}}\right)^2}\right]$

Solución.

$$\frac{2}{3}\left[\frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}+\frac{1}{1+\left(\frac{2x-1}{\sqrt{3}}\right)^2}\right]=\frac{2}{3}\left[\frac{1}{1+\frac{(2x+1)^2}{(\sqrt{3})^2}}+\frac{1}{1+\frac{(2x-1)^2}{(\sqrt{3})^2}}\right]$$

$$=\frac{2}{3}\left[\frac{1}{1+\frac{4x^2+4x+1}{3}}+\frac{1}{1+\frac{4x^2-4x+1}{3}}\right]$$

$$=\frac{2}{3}\left[\frac{1}{\frac{3+4x^2+4x+1}{3}}+\frac{1}{\frac{3+4x^2-4x+1}{3}}\right]$$

$$=\frac{2}{3}\left[\frac{1}{\frac{4x^2+4x+4}{3}}+\frac{1}{\frac{4x^2-4x+4}{3}}\right]$$

$$=\frac{2}{3}\left[\frac{3}{4x^2+4x+4}+\frac{3}{4x^2-4x+4}\right]$$

$$=\frac{2}{3}\left[\frac{3}{4(x^2+x+1)}+\frac{3}{4(x^2-x+1)}\right]$$

$$=\frac{2}{3}\cdot\frac{3}{4}\left[\frac{1}{x^2+x+1}+\frac{1}{x^2-x+1}\right]$$

$$=\frac{6}{12}\left[\frac{x^2-x+1+x^2+x+1}{(x^2+x+1)(x^2-x+1)}\right]$$

$$=\frac{1}{2}\left[\frac{2x^2+2}{\big((x^2+1)+x\big)\big((x^2+1)-x\big)}\right]$$

$$=\frac{1}{2}\left[\frac{2(x^2+1)}{(x^2+1)^2-x^2}\right]$$

$$=\frac{1}{2}\left[\frac{2(x^2+1)}{2x^2+2x+1-x^2}\right]$$

$$=\frac{1}{2}\left[\frac{2(x^2+1)}{x^2+2x+1}\right]$$

$$=\frac{2(x^2+1)}{2(x^2+2x+1)}$$

$$=\frac{\not{2}(x^2+1)}{\not{2}(x^2+2x+1)}$$

$$=\boxed{\frac{x^2+1}{x^2+2x+1}}$$

54. $\left(\frac{a-1}{a^2-2a+1}+\frac{2(a-1)}{a^2-4}-\frac{4(a+1)}{a^2+a-2}+\frac{a}{a^2-3a+2}\right)\frac{36a^3-144a-36a^2+144}{a^3+27}$

Solución.

$$\left(\frac{a-1}{a^2-2a+1}+\frac{2(a-1)}{a^2-4}-\frac{4(a+1)}{a^2+a-2}+\frac{a}{a^2-3a+2}\right)\frac{36a^3-144a-36a^2+144}{a^3+27}=$$

$$=\left(\frac{a-1}{(a-1)^2}+\frac{2(a-1)}{(a-2)(a+2)}-\frac{4(a+1)}{(a+2)(a-1)}+\frac{a}{(a-2)(a-1)}\right)\frac{36a^3-36a^2-144a+144}{a^3+3^3}$$

$$= \left(\frac{1}{a-1} + \frac{2(a-1)}{(a-2)(a+2)} - \frac{4(a+1)}{(a+2)(a-1)} + \frac{a}{(a-2)(a-1)}\right)\frac{36a^2(a-1)-144(a-1)}{(a+3)(a^2-3a+9)}$$

$$= \left(\frac{(a-2)(a+2)+2(a-1)(a-1)-4(a+1)(a-2)+a(a+2)}{(a-1)(a-2)(a+2)}\right)\frac{(a-1)(36a^2-144)}{(a+3)(a^2-3a+9)}$$

$$= \left(\frac{a^2-4+2(a-1)^2-4(a^2-2a+a-2)+a^2+2a}{(a-1)(a-2)(a+2)}\right)\frac{(a-1)(6a-12)(6a+12)}{(a+3)(a^2-3a+9)}$$

$$= \left(\frac{2a^2+2a-4+2(a^2-2a+1)-4(a^2-a-2)}{(a-1)(a-2)(a+2)}\right)\frac{(a-1)6(a-2)6(a+2)}{(a+3)(a^2-3a+9)}$$

$$= \left(\frac{2a^2+2a-4+2a^2-4a+2-4a^2+4a+8}{(a-1)(a-2)(a+2)}\right)\frac{36(a-1)(a-2)(a+2)}{(a+3)(a^2-3a+9)}$$

$$= \left(\frac{2a+6}{(a-1)(a-2)(a+2)}\right)\frac{36(a-1)(a-2)(a+2)}{(a+3)(a^2-3a+9)}$$

$$= \left(\frac{2(a+3)}{(a-1)(a-2)(a+2)}\right)\frac{36(a-1)(a-2)(a+2)}{(a+3)(a^2-3a+9)}$$

$$= \frac{72(a+3)(a-1)(a-2)(a+2)}{(a-1)(a-2)(a+2)(a+3)(a^2-3a+9)}$$

$$= \frac{72\cancel{(a+3)}\cancel{(a-1)}\cancel{(a-2)}\cancel{(a+2)}}{\cancel{(a-1)}\cancel{(a-2)}\cancel{(a+2)}\cancel{(a+3)}(a^2-3a+9)}$$

$$= \boxed{\frac{72}{a^2-3a+9}}$$

55. $\left[\frac{3(x+2)}{2(x^3+x^2+x+1)} + \frac{2x^2-x-10}{2(x^3-x^2+x-1)}\right] : \left[\frac{5}{x^2+1} + \frac{3}{2(x+1)} - \frac{3}{2x-1}\right]$

Solución.

$$\left[\frac{3(x+2)}{2(x^3+x^2+x+1)} + \frac{2x^2-x-10}{2(x^3-x^2+x-1)}\right] : \left[\frac{5}{x^2+1} + \frac{3}{2(x+1)} - \frac{3}{2x-1}\right] =$$

$$= \overbrace{\left[\frac{3(x+2)}{2(x^3+x^2+x+1)} + \frac{2x^2-x-10}{2(x^3-x^2+x-1)}\right]}^{A} : \overbrace{\left[\frac{5}{x^2+1} + \frac{3}{2(x+1)} - \frac{3}{2x-1}\right]}^{B}$$

Realizando operaciones en forma separada:

$$A = \left[\frac{3(x+2)}{2(x^3+x^2+x+1)} + \frac{2x^2-x-10}{2(x^3-x^2+x-1)}\right]$$

$$= \left[\frac{3(x+2)}{2(x^2(x+1)+(x+1))} + \frac{(2x-5)(x+2)}{2(x^2(x-1)+(x-1))}\right]$$

$$= \left[\frac{3(x+2)}{2(x+1)(x^2+1)} + \frac{(2x-5)(x+2)}{2(x-1)(x^2+1)}\right]$$

$$= \frac{x+2}{2(x^2+1)}\left[\frac{3}{x+1} + \frac{2x-5}{x-1}\right]$$

$$= \frac{x+2}{2(x^2+1)}\left[\frac{3(x-1)+(x+1)(2x-5)}{(x+1)(x-1)}\right]$$

$$= \frac{x+2}{2(x^2+1)}\left[\frac{3x-3+2x^2-5x+2x-5}{x^2-1}\right]$$

$$= \frac{x+2}{2(x^2+1)}\left[\frac{2x^2-8}{x^2-1}\right]$$

$$= \frac{x+2}{2(x^2+1)}\left[\frac{2(x^2-4)}{x^2-1}\right]$$

$$= \left[\frac{2(x+2)(x^2-4)}{2(x^2+1)(x^2-1)}\right]$$

$$= \left[\frac{\not{2}(x+2)(x^2-4)}{\not{2}(x^2+1)(x^2-1)}\right]$$

$$\boxed{A = \left[\frac{(x+2)(x^2-4)}{(x^2+1)(x^2-1)}\right]}$$

$$B = \left[\frac{5}{x^2+1} + \frac{3}{2(x+1)} - \frac{3}{2x-1}\right]$$

$$= \left[\frac{10(x+1)(x-1) + 3(x^2+1)(x-1) - 3(x^2+1)(x+1)}{2(x^2+1)(x+1)(x-1)}\right]$$

$$= \left[\frac{10(x^2-1) + 3(x^3-x^2+x-1) - 3(x^3+x^2+x-1)}{2(x^2+1)(x^2-1)}\right]$$

$$= \left[\frac{10x^2-10+3x^3-3x^2+3x-3-3x^3-3x^2-3x-3}{2(x^2+1)(x^2-1)}\right]$$

$$= \left[\frac{4x^2-16}{2(x^2+1)(x^2-1)}\right]$$

$$= \left[\frac{4(x^2-4)}{2(x^2+1)(x^2-1)}\right]$$

$$= \left[\frac{\not{4}(x^2-4)}{\not{2}(x^2+1)(x^2-1)}\right]$$

$$\boxed{B = \left[\frac{2(x^2-4)}{(x^2+1)(x^2-1)}\right]}$$

reemplazando:

$$\left[\frac{3(x+2)}{2(x^3+x^2+x+1)} + \frac{2x^2-x-10}{2(x^3-x^2+x-1)}\right] : \left[\frac{5}{x^2+1} + \frac{3}{2(x+1)} - \frac{3}{2x-1}\right] =$$

$$= \left[\frac{(x+2)(x^2-4)}{(x^2+1)(x^2-1)}\right] : \left[\frac{2(x^2-4)}{(x^2+1)(x^2-1)}\right]$$

$$= \frac{(x+2)(x^2-4)}{(x^2+1)(x^2-1)} \cdot \frac{(x^2+1)(x^2-1)}{2(x^2-4)}$$

$$= \frac{(x+2)(x^2-4)(x^2+1)(x^2-1)}{2(x^2+1)(x^2-1)(x^2-4)}$$

$$= \frac{(x+2)\cancel{(x^2-4)(x^2+1)(x^2-1)}}{2\cancel{(x^2+1)(x^2-1)(x^2-4)}}$$

$$= \boxed{\frac{x+2}{2}}$$

56. $\left(\dfrac{x-y}{2y-x} - \dfrac{x^2+y^2+y-2}{x^2-xy-2y^2}\right) : \dfrac{4x^4+4x^2y+y^2-4}{x^2+y+xy+x}$

Solución.

$$\left(\frac{x-y}{2y-x} - \frac{x^2+y^2+y-2}{x^2-xy-2y^2}\right) : \frac{4x^4+4x^2y+y^2-4}{x^2+y+xy+x} = \overbrace{\left(\frac{x-y}{2y-x} - \frac{x^2+y^2+y-2}{x^2-xy-2y^2}\right)}^{A} : \overbrace{\frac{4x^4+4x^2y+y^2-4}{x^2+y+xy+x}}^{B}$$

Realizando operaciones en forma separada:

$$\begin{aligned}
A &= \frac{x-y}{2y-x} - \frac{x^2+y^2+y-2}{x^2-xy-2y^2}\\
&= \frac{x-y}{-x+2y} - \frac{x^2+y^2+y-2}{(x-2y)(x+y)}\\
&= \frac{x-y}{-(x-2y)} - \frac{x^2+y^2+y-2}{(x-2y)(x+y)}\\
&= -\frac{x-y}{x-2y} - \frac{x^2+y^2+y-2}{(x-2y)(x+y)}\\
&= \frac{-(x+y)(x-y)-(x^2+y^2+y-2)}{(x-2y)(x+y)}\\
&= \frac{-x^2+xy-xy+y^2-x^2-y^2-y+2}{(x-2y)(x+y)}\\
&= \frac{-2x^2-y+2}{(x-2y)(x+y)}
\end{aligned}$$

$$\boxed{A = \frac{-(2x^2+y-2)}{(x-2y)(x+y)}}$$

$$\begin{aligned}
B &= \frac{4x^4+4x^2y+y^2-4}{x^2+y+xy+x}\\
&= \frac{(2x^2)^2+2\cdot 2x^2\cdot y+y^2-4}{x^2+xy+x+y}\\
&= \frac{(2x^2+y)^2-4}{x(x+y)+(x+y)}
\end{aligned}$$

$$\boxed{B = \frac{(2x^2+y-2)(2x^2+y+2)}{(x+y)(x+1)}}$$

reemplazando:

$$\begin{aligned}
\left(\frac{x-y}{2y-x} - \frac{x^2+y^2+y-2}{x^2-xy-2y^2}\right) : \frac{4x^4+4x^2y+y^2-4}{x^2+y+xy+x} &= \frac{-(2x^2+y-2)}{(x-2y)(x+y)} : \frac{(2x^2+y-2)(2x^2+y+2)}{(x+y)(x+1)}\\
&= \frac{-(2x^2+y-2)}{(x-2y)(x+y)} \cdot \frac{(x+y)(x+1)}{(2x^2+y-2)(2x^2+y+2)}\\
&= \frac{-(2x^2+y-2)(x+y)(x+1)}{(x-2y)(x+y)(2x^2+y-2)(2x^2+y+2)}\\
&= \frac{-\cancel{(2x^2+y-2)}\cancel{(x+y)}(x+1)}{(x-2y)\cancel{(x+y)}\cancel{(2x^2+y-2)}(2x^2+y+2)}\\
&= \boxed{-\frac{x+1}{(x-2y)(2x^2+y+2)}}
\end{aligned}$$

57. $\dfrac{a^2+a-2}{a^{n+1}-3a^n}\left(\dfrac{(a+2)^2-a^2}{4a^2-4} - \dfrac{3}{a^2-a}\right)$

Solución.

$$\begin{aligned}
\frac{a^2+a-2}{a^{n+1}-3a^n}\left(\frac{(a+2)^2-a^2}{4a^2-4} - \frac{3}{a^2-a}\right) &= \frac{(a+2)(a-1)}{a^n\cdot a-3a^n}\left(\frac{a^2+4a+4-a^2}{4(a^2-1)} - \frac{3}{a(a-1)}\right)\\
&= \frac{(a+2)(a-1)}{a^n(a-3)}\left(\frac{4a+4}{4(a-1)(a+1)} - \frac{3}{a(a-1)}\right)
\end{aligned}$$

$$= \frac{(a+2)(a-1)}{a^n(a-3)}\left(\frac{4(a+1)}{4(a-1)(a+1)} - \frac{3}{a(a-1)}\right)$$

$$= \frac{(a+2)(a-1)}{a^n(a-3)}\left(\frac{\cancel{4}\cancel{(a+1)}}{\cancel{4}(a-1)\cancel{(a+1)}} - \frac{3}{a(a-1)}\right)$$

$$= \frac{(a+2)(a-1)}{a^n(a-3)}\left(\frac{1}{a-1} - \frac{3}{a(a-1)}\right)$$

$$= \frac{(a+2)(a-1)}{a^n(a-3)} \cdot \frac{1}{a-1}\left(1 - \frac{3}{a}\right)$$

$$= \frac{(a+2)(a-1)}{a^n(a-3)(a-1)}\left(\frac{a-3}{a}\right)$$

$$= \frac{(a+2)\cancel{(a-1)}}{a^n(a-3)\cancel{(a-1)}}\left(\frac{a-3}{a}\right)$$

$$= \frac{a+2}{a^n(a-3)}\left(\frac{a-3}{a}\right)$$

$$= \frac{(a+2)(a-3)}{a^n \cdot a(a-3)}$$

$$= \frac{(a+2)\cancel{(a-3)}}{a^n \cdot a\cancel{(a-3)}}$$

$$= \boxed{\frac{a+2}{a^{n+1}}}$$

58. $\dfrac{2a^2(b+c)^{2n} - \dfrac{1}{2}}{an^2 - a^3 - 2a^2 - a} : \dfrac{2a(b+c)^n - 1}{a^2c - a(nc-c)}$

Solución.

$$\frac{2a^2(b+c)^{2n} - \frac{1}{2}}{an^2 - a^3 - 2a^2 - a} : \frac{2a(b+c)^n - 1}{a^2c - a(nc-c)} = \overbrace{\frac{2a^2(b+c)^{2n} - \frac{1}{2}}{an^2 - a^3 - 2a^2 - a}}^{A} : \overbrace{\frac{2a(b+c)^n - 1}{a^2c - a(nc-c)}}^{B}$$

$$A = \frac{2a^2(b+c)^{2n} - \frac{1}{2}}{an^2 - a^3 - 2a^2 - a}$$

$$= \frac{\frac{4a^2(b+c)^{2n} - 1}{2}}{a(n^2 - a^2 - 2a - 1)}$$

$$= \frac{\frac{\big(2a(b+c)^n\big)^2 - 1}{2}}{a\big(n^2 - (a^2 + 2a + 1)\big)}$$

$$= \frac{\frac{\big(2a(b+c)^n - 1\big)\big(2a(b+c)^n + 1\big)}{2}}{a\big(n^2 - (a+1)^2\big)}$$

$$= \frac{\frac{\big(2a(b+c)^n - 1\big)\big(2a(b+c)^n + 1\big)}{2}}{a\big(n - (a+1)\big)\big(n + (a+1)\big)}$$

$$= \frac{\frac{\big(2a(b+c)^n - 1\big)\big(2a(b+c)^n + 1\big)}{2}}{a(n - a - 1)(n + a + 1)}$$

$$\boxed{A=\frac{\big(2a(b+c)^n-1\big)\big(2a(b+c)^n+1\big)}{2a(n-a-1)(n+a+1)}}$$

$$\begin{aligned}B&=\frac{2a(b+c)^n-1}{a^2c-a(nc-c)}\\&=\frac{2a(b+c)^n-1}{a^2c-ac(n-1)}\\&=\frac{2a(b+c)^n-1}{ac\big(a-(n-1)\big)}\\&=\frac{2a(b+c)^n-1}{ac(a-n+1)}\\&=\frac{2a(b+c)^n-1}{ac(-n+a+1)}\end{aligned}$$

$$\boxed{B=\frac{2a(b+c)^n-1}{-ac(n-a-1)}}$$

reemplazando:

$$\begin{aligned}\frac{2a^2(b+c)^{2n}-\frac{1}{2}}{an^2-a^3-2a^2-a}:\frac{2a(b+c)^n-1}{a^2c-a(nc-c)}&=\frac{\big(2a(b+c)^n-1\big)\big(2a(b+c)^n+1\big)}{2a(n-a-1)(n+a+1)}:\frac{2a(b+c)^n-1}{-ac(n-a-1)}\\&=\frac{\big(2a(b+c)^n-1\big)\big(2a(b+c)^n+1\big)}{2a(n-a-1)(n+a+1)}\cdot\frac{-ac(n-a-1)}{2a(b+c)^n-1}\\&=\frac{-ac(n-a-1)\big(2a(b+c)^n-1\big)\big(2a(b+c)^n+1\big)}{2a(n-a-1)(n+a+1)\big(2a(b+c)^n-1\big)}\\&=-\frac{\cancel{a}c\cancel{(n-a-1)}\cancel{\big(2a(b+c)^n-1\big)}\big(2a(b+c)^n+1\big)}{2\cancel{a}\cancel{(n-a-1)}(n+a+1)\cancel{\big(2a(b+c)^n-1\big)}}\\&=\boxed{-\frac{c\big(2a(b+c)^n+1\big)}{2(n+a+1)}}\end{aligned}$$

59. $\dfrac{1}{a(a-b)(a-c)}+\dfrac{1}{b(b-a)(b-c)}+\dfrac{1}{c(c-a)(c-b)}$

Solución.

$$\begin{aligned}\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}&=\\&=\frac{1}{a(-c+a)(a-b)}+\frac{1}{b(b-c)(-a+b)}+\frac{1}{c(-b+c)(c-a)}\\&=\frac{1}{-a(c-a)(a-b)}+\frac{1}{-b(b-c)(a-b)}+\frac{1}{-c(b-c)(c-a)}\\&=-\frac{1}{a(c-a)(a-b)}-\frac{1}{b(b-c)(a-b)}-\frac{1}{c(b-c)(c-a)}\\&=\frac{-bc(b-c)-ac(c-a)-ab(a-b)}{abc(b-c)(c-a)(a-b)}\\&=\frac{-bc(b-c)-ac^2+a^2c-a^2b+ab^2}{abc(b-c)(c-a)(a-b)}\\&=\frac{-bc(b-c)-a^2b+a^2c+ab^2-ac^2}{abc(b-c)(c-a)(a-b)}\end{aligned}$$

$$= \frac{-bc(b-c)-a^2(b-c)+a(b^2-c^2)}{abc(b-c)(c-a)(a-b)}$$

$$= \frac{-bc(b-c)-a^2(b-c)+a(b-c)(b+c)}{abc(b-c)(c-a)(a-b)}$$

$$= \frac{(b-c)\big(-bc-a^2+a(b+c)\big)}{abc(b-c)(c-a)(a-b)}$$

$$= \frac{(b-c)(-bc-a^2+ab+ac)}{abc(b-c)(c-a)(a-b)}$$

$$= \frac{(b-c)(ac-bc-a^2+ab)}{abc(b-c)(c-a)(a-b)}$$

$$= \frac{(b-c)\big(c(a-b)-a(a-b)\big)}{abc(b-c)(c-a)(a-b)}$$

$$= \frac{(b-c)(a-b)(c-a)}{abc(b-c)(c-a)(a-b)}$$

$$= \frac{\cancel{(b-c)}\cancel{(a-b)}\cancel{(c-a)}}{abc\cancel{(b-c)}\cancel{(c-a)}\cancel{(a-b)}}$$

$$= \boxed{\frac{1}{abc}}$$

60. $\dfrac{1+(a+x)^{-1}}{1-(a+x)^{-1}}\left(1-\dfrac{1-(a^2+x^2)}{2ax}\right)$, evaluar para $x=\dfrac{1}{a-1}$

Solución.

$$\frac{1+(a+x)^{-1}}{1-(a+x)^{-1}}\left(1-\frac{1-(a^2+x^2)}{2ax}\right) = \frac{1+\dfrac{1}{a+x}}{1-\dfrac{1}{a+x}}\left(\frac{2ax-(1-(a^2+x^2))}{2ax}\right)$$

$$= \frac{\dfrac{a+x+1}{a+x}}{\dfrac{a+x-1}{a+x}}\left(\frac{2ax-1+a^2+x^2}{2ax}\right)$$

$$= \frac{(a+x+1)(a+x)}{(a+x)(a+x-1)}\left(\frac{x^2+2ax+a^2-1}{2ax}\right)$$

$$= \frac{(a+x+1)\cancel{(a+x)}}{\cancel{(a+x)}(a+x-1)}\left(\frac{(x+a)^2-1}{2ax}\right)$$

$$= \frac{a+x+1}{a+x-1}\left(\frac{(x+a)^2-1}{2ax}\right)$$

$$= \frac{x+a+1}{x+a-1}\cdot\frac{(x+a-1)(x+a+1)}{2ax}$$

$$= \frac{(x+a-1)(x+a+1)^2}{2ax(a+x-1)}$$

$$= \frac{\cancel{(x+a-1)}(x+a+1)^2}{2ax\cancel{(a+x-1)}}$$

$$= \boxed{\frac{(x+a+1)^2}{2ax}}$$

reemplazando $x=\dfrac{1}{a-1}$

$$\begin{aligned}
\frac{(x+a+1)^2}{2ax} &= \frac{\left(\dfrac{1}{a-1}+a+1\right)^2}{2a\left(\dfrac{1}{a-1}\right)} \\
&= \frac{\left(\dfrac{1+(a-1)(a+1)}{a-1}\right)^2}{\dfrac{2a}{a-1}} \\
&= \frac{\left(\dfrac{1+a^2-1}{a-1}\right)^2}{\dfrac{2a}{a-1}} \\
&= \frac{\left(\dfrac{a^2}{a-1}\right)^2}{\dfrac{2a}{a-1}} \\
&= \frac{\dfrac{a^4}{(a-1)^2}}{\dfrac{2a}{a-1}} \\
&= \frac{a^4(a-1)}{(a-1)^2\cdot 2a} \\
&= \frac{a^{\not{4}}\cancel{(a-1)}}{(a-1)^{\not{2}}\cdot 2\not{a}} \\
&= \boxed{\frac{a^3}{2(a-1)}}
\end{aligned}$$

61. $\left[\dfrac{2+ba^{-1}}{a+2b}-6b(4b^2-a^2)^{-1}\right]\left(2a^nb+3a^{n+1}-\dfrac{6a^{n+2}}{2a-b}\right)$

Solución.

$\left[\dfrac{2+ba^{-1}}{a+2b}-6b(4b^2-a^2)^{-1}\right]\left(2a^nb+3a^{n+1}-\dfrac{6a^{n+2}}{2a-b}\right)=$

$$= \overbrace{\left[\frac{2+ba^{-1}}{a+2b}-6b(4b^2-a^2)^{-1}\right]}^{A}\overbrace{\left(2a^nb+3a^{n+1}-\frac{6a^{n+2}}{2a-b}\right)}^{B}$$

$$\begin{aligned}
A &= \frac{2+ba^{-1}}{a+2b}-6b(4b^2-a^2)^{-1} \\
&= \frac{2+\dfrac{b}{a}}{a+2b}-\frac{6b}{4b^2-a^2} \\
&= \frac{\dfrac{2a+b}{a}}{a+2b}-\frac{6b}{-a^2+4b^2} \\
&= \frac{2a+b}{a(a+2b)}-\frac{6b}{-(a^2-4b^2)} \\
&= \frac{2a+b}{a(a+2b)}+\frac{6b}{a^2-4b^2}
\end{aligned}$$

$$= \frac{2a+b}{a(a+2b)} + \frac{6b}{(a-2b)(a+2b)}$$

$$= \frac{(a-2b)(2a+b)+6ab}{a(a-2b)(a+2b)}$$

$$= \frac{2a^2+ab-4ab-2b^2+6ab}{a(a-2b)(a+2b)}$$

$$= \frac{2a^2+3ab-2b^2}{a(a-2b)(a+2b)}$$

$$= \frac{(a+2b)(2a-b)}{a(a-2b)(a+2b)}$$

$$= \frac{\cancel{(a+2b)}(2a-b)}{a(a-2b)\cancel{(a+2b)}}$$

$$\boxed{A = \frac{2a-b}{a(a-2b)}}$$

$$B = 2a^nb + 3a^{n+1} - \frac{6a^{n+2}}{2a-b}$$

$$= 2a^nb + 3a^n \cdot a - \frac{6a^n \cdot a^2}{2a-b}$$

$$= a^n\left(2b+3a-\frac{6a^2}{2a-b}\right)$$

$$= a^n\left(\frac{(2a-b)(2b+3a)-6a^2}{2a-b}\right)$$

$$= a^n\left(\frac{4ab+6a^2-2b^2-3ab-6a^2}{2a-b}\right)$$

$$= a^n\left(\frac{ab-2b^2}{2a-b}\right)$$

$$= a^n\left(\frac{b(a-2b)}{2a-b}\right)$$

$$\boxed{B = \frac{a^nb(a-2b)}{2a-b}}$$

reemplazando:

$$\left[\frac{2+ba^{-1}}{a+2b} - 6b(4b^2-a^2)^{-1}\right]\left(2a^nb+3a^{n+1}-\frac{6a^{n+2}}{2a-b}\right) = \frac{2a-b}{a(a-2b)} \cdot \frac{a^nb(a-2b)}{2a-b}$$

$$= \frac{a^nb(a-2b)(2a-b)}{a(a-2b)(2a-b)}$$

$$= \frac{a^nb\cancel{(a-2b)}\cancel{(2a-b)}}{a\cancel{(a-2b)}\cancel{(2a-b)}}$$

$$= \frac{a^nb}{a}$$

$$= a^n \cdot a^{-1}b$$

$$= \boxed{a^{n-1}b}$$

62. $$\frac{\left[1-\left(\frac{a}{b}\right)^{-2}\right]a^2}{(\sqrt{a}-\sqrt{b})^2+2\sqrt{ab}}$$

Solución.

$$\frac{\left[1-\left(\frac{a}{b}\right)^{-2}\right]a^2}{(\sqrt{a}-\sqrt{b})^2+2\sqrt{ab}} = \frac{\left[1-\frac{1}{\left(\frac{a}{b}\right)^2}\right]a^2}{(\sqrt{a})^2-2\cdot\sqrt{a}\cdot\sqrt{b}+(\sqrt{b})^2+2\sqrt{ab}}$$

$$= \frac{\left[1-\frac{1}{\frac{a^2}{b^2}}\right]a^2}{a-2\sqrt{ab}+b+2\sqrt{ab}}$$

$$= \frac{\left[1-\frac{b^2}{a^2}\right]a^2}{a+b} = \frac{\left[\frac{a^2-b^2}{a^2}\right]a^2}{a+b}$$

$$= \frac{\frac{a^2(a^2-b^2)}{a^2}}{a+b} = \frac{\frac{\cancel{a^2}(a^2-b^2)}{\cancel{a^2}}}{a+b}$$

$$= \frac{a^2-b^2}{a+b} = \frac{(a-b)(a+b)}{a+b}$$

$$= \frac{(a-b)\cancel{(a+b)}}{\cancel{a+b}} = \boxed{a-b}$$

63. $\frac{b}{a-b}\cdot\sqrt[3]{(a^2-2ab+b^2)(a^2-b^2)(a+b)}\cdot\frac{a^3-b^3}{\sqrt[3]{(a+b)^2}}$

Solución.

$$\frac{b}{a-b}\cdot\sqrt[3]{(a^2-2ab+b^2)(a^2-b^2)(a+b)}\cdot\frac{a^3-b^3}{\sqrt[3]{(a+b)^2}} = \frac{b(a^3-b^3)\sqrt[3]{(a^2-2ab+b^2)(a^2-b^2)(a+b)}}{(a-b)\sqrt[3]{(a+b)^2}}$$

$$= \frac{b(a^3-b^3)\sqrt[3]{(a-b)^2(a-b)(a+b)(a+b)}}{(a-b)\sqrt[3]{(a+b)^2}}$$

$$= \frac{b(a^3-b^3)\sqrt[3]{(a-b)^3(a+b)^2}}{(a-b)\sqrt[3]{(a+b)^2}}$$

$$= \frac{b(a^3-b^3)(a-b)\sqrt[3]{(a+b)^2}}{(a-b)\sqrt[3]{(a+b)^2}}$$

$$= \frac{b(a^3-b^3)\cancel{(a-b)}\cancel{\sqrt[3]{(a+b)^2}}}{\cancel{(a-b)}\cancel{\sqrt[3]{(a+b)^2}}}$$

$$= \boxed{b(a^3-b^3)}$$

64. $\sqrt[6]{8x(7+4\sqrt{3})}\,\sqrt[3]{2\sqrt{6x}-4\sqrt{2x}}$

Solución.

$$\sqrt[6]{8x(7+4\sqrt{3})}\,\sqrt[3]{2\sqrt{6x}-4\sqrt{2x}} = \sqrt[6]{8x(7+4\sqrt{3})}\,\sqrt[3\cdot 2]{(2\sqrt{6x}-4\sqrt{2x})^2}$$

$$= \sqrt[6]{8x(7+4\sqrt{3})}\,\sqrt[6]{4\cdot 6x-16\cdot\sqrt{6x}\cdot\sqrt{2x}+16\cdot 2x}$$

$$= \sqrt[6]{8x(7+4\sqrt{3})}\,\sqrt[6]{24x-16\sqrt{12x^2}+32x}$$

$$= \sqrt[6]{8x(7+4\sqrt{3})}\,\sqrt[6]{56x-32x\sqrt{3}}$$

$$= \sqrt[6]{8x(7+4\sqrt{3})}\sqrt[6]{8x(7-4\sqrt{3})}$$
$$= \sqrt[6]{8x(7+4\sqrt{3})\cdot 8x(7-4\sqrt{3})}$$
$$= \sqrt[6]{64x(49+16\cdot 3)}$$
$$= 2\sqrt[6]{x(49+48)}$$
$$= 2\sqrt[6]{x(1)}$$
$$= \boxed{2\sqrt[6]{x}}$$

65. $\dfrac{a}{2}\sqrt[4]{(a+1)(a^2-1)(1+2a+a^2)}\left(\dfrac{a^2-3a+2}{\sqrt{a-1}}\right)^{-1}$

Solución.

$$\frac{a}{2}\sqrt[4]{(a+1)(a^2-1)(1+2a+a^2)}\left(\frac{a^2-3a+2}{\sqrt{a-1}}\right)^{-1} = \overbrace{\frac{a}{2}\sqrt[4]{(a+1)(a^2-1)(1+2a+a^2)}}^{A}\overbrace{\left(\frac{a^2-3a+2}{\sqrt{a-1}}\right)^{-1}}^{B}$$

$$A = \frac{a}{2}\sqrt[4]{(a+1)(a^2-1)(1+2a+a^2)}$$
$$= \frac{a}{2}\sqrt[4]{(a+1)(a-1)(a+1)(a^2+2a+1)}$$
$$= \frac{a}{2}\sqrt[4]{(a+1)^2(a-1)(a+1)^2}$$
$$= \frac{a}{2}\sqrt[4]{(a+1)^4(a-1)}$$
$$\boxed{A = \frac{a}{2}(a+1)\sqrt[4]{a-1}}$$

$$B = \left(\frac{a^2-3a+2}{\sqrt{a-1}}\right)^{-1}$$
$$= \frac{1}{\dfrac{a^2-3a+2}{\sqrt{a-1}}}$$
$$= \frac{\sqrt{a-1}}{a^2-3a+2}$$
$$= \frac{\sqrt[2\cdot 2]{(a-1)^2}}{(a-2)(a-1)}$$
$$\boxed{B = \frac{\sqrt[4]{(a-1)^2}}{(a-2)(a-1)}}$$

reemplazando:

$$\frac{a}{2}\sqrt[4]{(a+1)(a^2-1)(1+2a+a^2)}\left(\frac{a^2-3a+2}{\sqrt{a-1}}\right)^{-1} = \frac{a}{2}(a+1)\sqrt[4]{a-1}\cdot\frac{\sqrt[4]{(a-1)^2}}{(a+2)(a+1)}$$
$$= \frac{a(a+1)\sqrt[4]{a-1}\sqrt[4]{(a-1)^2}}{2a(a+2)(a+1)}$$
$$= \frac{\cancel{a}\cancel{(a+1)}\sqrt[4]{(a-1)^3}}{2\cancel{a}(a+2)\cancel{(a+1)}}$$

$$= \boxed{\frac{a\sqrt[4]{(a-1)^3}}{2(a+2)}}$$

66. $\sqrt{\frac{(1+a)\sqrt[3]{1+a}}{3a}} \cdot \sqrt[3]{\frac{\sqrt{3}}{9+18a^{-1}+9a^{-2}}}$

Solución.

$$\sqrt{\frac{(1+a)\sqrt[3]{1+a}}{3a}} \cdot \sqrt[3]{\frac{\sqrt{3}}{9+18a^{-1}+9a^{-2}}} = \overbrace{\sqrt{\frac{(1+a)\sqrt[3]{1+a}}{3a}}}^{A} \cdot \overbrace{\sqrt[3]{\frac{\sqrt{3}}{9+18a^{-1}+9a^{-2}}}}^{B}$$

$$A = \sqrt{\frac{(1+a)\sqrt[3]{1+a}}{3a}}$$

$$= \sqrt[2\cdot 3]{\left(\frac{(1+a)\sqrt[3]{1+a}}{3a}\right)^3}$$

$$= \sqrt[6]{\frac{(1+a)^3(1+a)}{27a^3}}$$

$$\boxed{A = \sqrt[6]{\frac{(1+a)^4}{27a^3}}}$$

$$B = \sqrt[3]{\frac{\sqrt{3}}{9+18a^{-1}+9a^{-2}}}$$

$$= \sqrt[3]{\frac{\sqrt{3}}{2}9 + \frac{18}{a} + \frac{9}{a^2}}$$

$$= \sqrt[3]{\frac{\sqrt{3}}{\frac{9a^2+18a+9}{a^2}}}$$

$$= \sqrt[3]{\frac{\sqrt{3}}{\frac{9(a^2+2a+1)}{a^2}}}$$

$$= \sqrt[3]{\frac{\sqrt{3}}{\frac{9(a+1)^2}{a^2}}}$$

$$= \sqrt[3]{\frac{\sqrt{3}a^2}{9(a+1)^2}}$$

$$= \sqrt[3\cdot 2]{\left(\frac{\sqrt{3}a^2}{9(a+1)^2}\right)^2}$$

$$\boxed{B = \sqrt[6]{\frac{3a^4}{81(a+1)^4}}}$$

reemplazando:

$$\sqrt{\frac{(1+a)\sqrt[3]{1+a}}{3a}}\cdot\sqrt[3]{\frac{\sqrt{3}}{9+18a^{-1}+9a^{-2}}}=\sqrt[6]{\frac{(1+a)^4}{3^3\cdot a^3}}\cdot\sqrt[6]{\frac{3a^4}{3^4(a+1)^4}}$$

$$=\sqrt[6]{\frac{(1+a)^4}{3^3\cdot a^3}\cdot\frac{3a^4}{3^4(a+1)^4}}$$

$$=\sqrt[6]{\frac{3a^4(1+a)^4}{3^7\cdot a^3(a+1)^4}}$$

$$=\sqrt[6]{\frac{\cancel{3}\cancel{a^4}\cancel{(1+a)^4}}{3^{\cancel{7}}\cdot\cancel{a^3}\cancel{(a+1)^4}}}$$

$$=\sqrt[6]{\frac{a}{3^6}}$$

$$=\frac{\sqrt[6]{a}}{\sqrt[6]{3^6}}$$

$$=\boxed{\frac{\sqrt[6]{a}}{3}}$$

67. $ab\sqrt[n]{a^{1-n}b^{-n}-a^{-n}b^{1-n}}\cdot\sqrt[n]{(a-b)^{-1}}$

Solución.

$$ab\sqrt[n]{a^{1-n}b^{-n}-a^{-n}b^{1-n}}\cdot\sqrt[n]{(a-b)^{-1}}=ab\sqrt[n]{a\cdot a^{-n}b^{-n}-a^{-n}b\cdot b^{-n}}\cdot\sqrt[n]{\frac{1}{a+b}}$$

$$=ab\sqrt[n]{a\cdot\frac{1}{a^n}\cdot\frac{1}{b^n}-\frac{1}{a^n}\cdot b\cdot\frac{1}{b^n}}\cdot\frac{1}{\sqrt[n]{a-b}}$$

$$=ab\sqrt[n]{\frac{a}{a^nb^n}-\frac{b}{a^nb^n}}\cdot\frac{1}{\sqrt[n]{a-b}}$$

$$=ab\sqrt[n]{\frac{a-b}{a^nb^n}}\cdot\frac{1}{\sqrt[n]{a-b}}$$

$$=ab\cdot\frac{\sqrt[n]{a-b}}{ab}\cdot\frac{1}{\sqrt[n]{a-b}}$$

$$=\frac{ab\sqrt[n]{a-b}}{ab\sqrt[n]{a-b}}$$

$$=\boxed{1}$$

68. $\left(\frac{15}{\sqrt{6}+1}+\frac{4}{\sqrt{6}-2}-\frac{12}{3-\sqrt{6}}\right)(\sqrt{6}+11)$

Solución.

Racionalizando denominadores:

$$\frac{15}{\sqrt{6}+1}=\frac{15}{\sqrt{6}+1}\cdot\frac{\sqrt{6}-1}{\sqrt{6}-1}$$

$$=\frac{15(\sqrt{6}-1)}{(\sqrt{6}+1)(\sqrt{6}-1)}$$

$$=\frac{15(\sqrt{6}-1)}{(\sqrt{6})^2-1^2}$$

$$= \frac{15(\sqrt{6}-1)}{6-1}$$

$$= \frac{15(\sqrt{6}-1)}{5}$$

$$= 3(\sqrt{6}-1)$$

$$= \boxed{3\sqrt{6}-3}$$

$$\frac{4}{\sqrt{6}-2} = \frac{4}{\sqrt{6}-2} \cdot \frac{\sqrt{6}+2}{\sqrt{6}+2}$$

$$= \frac{4(\sqrt{6}+2)}{(\sqrt{6}-2)(\sqrt{6}+2)}$$

$$= \frac{4(\sqrt{6}+2)}{(\sqrt{6})^2-2^2}$$

$$= \frac{4(\sqrt{6}+2)}{6-4}$$

$$= \frac{4(\sqrt{6}+2)}{2}$$

$$= 2(\sqrt{6}+2)$$

$$= \boxed{2\sqrt{6}+4}$$

$$-\frac{12}{3-\sqrt{6}} = -\frac{12}{-\sqrt{6}+3}$$

$$= -\frac{12}{-(\sqrt{6}+3)}$$

$$= \frac{12}{\sqrt{6}+3}$$

$$= \frac{12}{\sqrt{6}+3} \cdot \frac{\sqrt{6}-3}{\sqrt{6}-3}$$

$$= \frac{12(\sqrt{6}-3)}{(\sqrt{6}+3)(\sqrt{6}-3)}$$

$$= \frac{12(\sqrt{6}-3)}{(\sqrt{6})^2-3^2}$$

$$= \frac{12(\sqrt{6}+2)}{6-9}$$

$$= \frac{12(\sqrt{6}+2)}{-3}$$

$$= -4(\sqrt{6}+2)$$

$$= \boxed{-4\sqrt{6}-12}$$

reemplazando:

$$\left(\frac{15}{\sqrt{6}+1} + \frac{4}{\sqrt{6}-2} - \frac{12}{3-\sqrt{6}}\right)(\sqrt{6}+11) = (3\sqrt{6}-3+2\sqrt{6}+4-4\sqrt{6}-12)(\sqrt{6}+11)$$

$$= (\sqrt{6}-11)(\sqrt{6}+11)$$

$$= (\sqrt{6})^2 - 11^2 = 6 - 121$$

$$= \boxed{-115}$$

69. $\left(\dfrac{1}{\sqrt{a}+\sqrt{a+b}}+\dfrac{1}{\sqrt{a}-\sqrt{a-b}}\right):\left(1+\sqrt{\dfrac{a+b}{a-b}}\right)$

Solución.

$$\begin{aligned}
\left(\frac{1}{\sqrt{a}+\sqrt{a+b}}+\frac{1}{\sqrt{a}-\sqrt{a-b}}\right):\left(1+\sqrt{\frac{a+b}{a-b}}\right) &= \\
&= \left(\frac{\sqrt{a}-\sqrt{a-b}+\sqrt{a}+\sqrt{a+b}}{(\sqrt{a}+\sqrt{a+b})(\sqrt{a}-\sqrt{a-b})}\right):\left(1+\frac{\sqrt{a+b}}{\sqrt{a-b}}\right)\\
&= \left(\frac{2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b}}{(\sqrt{a})^2-\sqrt{a}\sqrt{a-b}+\sqrt{a}\sqrt{a+b}-\sqrt{a+b}\sqrt{a-b}}\right):\left(\frac{\sqrt{a-b}+\sqrt{a+b}}{\sqrt{a-b}}\right)\\
&= \frac{2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b}}{a-\sqrt{a(a-b)}+\sqrt{a(a+b)}-\sqrt{(a+b)(a-b)}}\cdot\frac{\sqrt{a-b}}{\sqrt{a-b}+\sqrt{a+b}}\\
&= \frac{(2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b})\sqrt{a-b}}{(a-\sqrt{a(a-b)}+\sqrt{a(a+b)}-\sqrt{(a+b)(a-b)})(\sqrt{a-b}+\sqrt{a+b})}
\end{aligned}$$

realizando operaciones en el denominador:

$$\begin{aligned}
&(a-\sqrt{a(a-b)}+\sqrt{a(a+b)}-\sqrt{(a+b)(a-b)}\,)(\sqrt{a-b}+\sqrt{a+b}) = a\sqrt{a-b}+a\sqrt{a+b}-\\
&-\sqrt{a(a-b)^2}-\sqrt{a(a^2-b^2)}+\sqrt{a(a^2-b^2)}+\sqrt{a(a+b)^2}-\sqrt{(a+b)(a-b)^2}-\sqrt{(a+b)^2(a-b)}=\\
&= a\sqrt{a-b}+a\sqrt{a+b}-(a-b)\sqrt{a}+(a+b)\sqrt{a}-(a-b)\sqrt{a+b}-(a+b)\sqrt{a-b}\\
&= a\sqrt{a-b}+a\sqrt{a+b}-a\sqrt{a}+b\sqrt{a}+a\sqrt{a}+b\sqrt{a}-a\sqrt{a+b}+b\sqrt{a+b}-a\sqrt{a-b}-b\sqrt{a-b}\\
&= 2b\sqrt{a}-b\sqrt{a-b}+b\sqrt{a+b}\\
&= b(2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b})
\end{aligned}$$

reemplazando:

$$\begin{aligned}
\left(\frac{1}{\sqrt{a}+\sqrt{a+b}}+\frac{1}{\sqrt{a}-\sqrt{a-b}}\right):\left(1+\sqrt{\frac{a+b}{a-b}}\right) &= \frac{(2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b})\sqrt{a-b}}{b(2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b})}\\
&= \frac{\cancel{(2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b})}\sqrt{a-b}}{b\cancel{(2\sqrt{a}-\sqrt{a-b}+\sqrt{a+b})}}\\
&= \boxed{\frac{\sqrt{a-b}}{b}}
\end{aligned}$$

70. $\left(\dfrac{1}{b-\sqrt{a}}+\dfrac{1}{b+\sqrt{a}}\right):\dfrac{\sqrt[-2]{\dfrac{1}{9}a^{-2}b^{-1}}}{a^{-2}-a^{-1}b^{-2}}$

Solución.

$$\left(\frac{1}{b-\sqrt{a}}+\frac{1}{b+\sqrt{a}}\right):\frac{\sqrt[-2]{\frac{1}{9}a^{-2}b^{-1}}}{a^{-2}-a^{-1}b^{-2}} = \overbrace{\left(\frac{1}{b-\sqrt{a}}+\frac{1}{b+\sqrt{a}}\right)}^{A}:\overbrace{\frac{\sqrt[-2]{\frac{1}{9}a^{-2}b^{-1}}}{a^{-2}-a^{-1}b^{-2}}}^{B}$$

$$\begin{aligned}
A &= \left(\frac{1}{b-\sqrt{a}}+\frac{1}{b+\sqrt{a}}\right)\\
&= \frac{b+\sqrt{a}+b-\sqrt{a}}{(b+\sqrt{a})(b-\sqrt{a})}\\
&= \frac{2b}{b^2-(\sqrt{a})^2}
\end{aligned}$$

$$\boxed{A = \frac{2b}{b^2 - a}}$$

$$\begin{aligned}
B &= \frac{\sqrt[-2]{\frac{1}{9}a^{-2}b^{-1}}}{a^{-2} - a^{-1}b^{-2}} \\
&= \frac{\sqrt[-2]{\frac{1}{3^2}} \cdot \frac{1}{a^2} \cdot \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{a} \cdot \frac{1}{b^2}} \\
&= \frac{\sqrt[-2]{3^{-2}} \cdot \frac{1}{a^2 b}}{\frac{1}{a^2} - \frac{1}{ab^2}} \\
&= \frac{3 \cdot \frac{1}{a^2 b}}{\frac{b^2 - a}{a^2 b^2}} \\
&= \frac{\frac{3}{a^2 b}}{\frac{b^2 - a}{a^2 b^2}} \\
&= \frac{3a^2b^2}{a^2b(b^2 - a)} \\
&= \frac{3\cancel{a^2}b^{\cancel{2}}}{\cancel{a^2}\cancel{b}(b^2 - a)}
\end{aligned}$$

$$\boxed{B = \frac{3b}{b^2 - a}}$$

reemplazando:

$$\begin{aligned}
\left(\frac{1}{b - \sqrt{a}} + \frac{1}{b + \sqrt{a}}\right) : \frac{\sqrt[-2]{\frac{1}{9}a^{-2}b^{-1}}}{a^{-2} - a^{-1}b^{-2}} &= \frac{2b}{b^2 - a} : \frac{3b}{b^2 - a} \\
&= \frac{2b}{b^2 - a} \cdot \frac{b^2 - a}{3b} \\
&= \frac{2b(b^2 - a)}{3b(b^2 - a)} \\
&= \frac{2\cancel{b}\cancel{(b^2 - a)}}{3\cancel{b}\cancel{(b^2 - a)}} \\
&= \boxed{\frac{2}{3}}
\end{aligned}$$

71. $\dfrac{\sqrt{\dfrac{1+a}{1-a}} + \sqrt{\dfrac{1-a}{1+a}}}{\sqrt{\dfrac{1+a}{1-a}} - \sqrt{\dfrac{1-a}{1+a}}} - \dfrac{1}{a}$

Solución.

$$\frac{\sqrt{\frac{1+a}{1-a}}+\sqrt{\frac{1-a}{1+a}}}{\sqrt{\frac{1+a}{1-a}}-\sqrt{\frac{1-a}{1+a}}}-\frac{1}{a}=\frac{\frac{\sqrt{1+a}}{\sqrt{1-a}}+\frac{\sqrt{1-a}}{\sqrt{1+a}}}{\frac{\sqrt{1+a}}{\sqrt{1-a}}-\frac{\sqrt{1-a}}{\sqrt{1+a}}}-\frac{1}{a}$$

$$=\frac{\frac{(\sqrt{1+a})^2+(\sqrt{1-a})^2}{\sqrt{1-a}\sqrt{1+a}}}{\frac{(\sqrt{1+a})^2-(\sqrt{1-a})^2}{\sqrt{1-a}\sqrt{1+a}}}-\frac{1}{a}$$

$$=\frac{\frac{(1+a)+(1-a)}{\sqrt{(1-a)(1+a)}}}{\frac{(1+a)-(1-a)}{\sqrt{(1-a)(1+a)}}}-\frac{1}{a}$$

$$=\frac{\frac{1+a+1-a}{\sqrt{1-a^2}}}{\frac{1+a-1+a}{\sqrt{1-a^2}}}-\frac{1}{a}$$

$$=\frac{\frac{2}{\sqrt{1-a^2}}}{\frac{2a}{\sqrt{1-a^2}}}-\frac{1}{a}$$

$$=\frac{2\sqrt{1-a^2}}{2a\sqrt{1-a^2}}-\frac{1}{a}$$

$$=\frac{\cancel{2}\cancel{\sqrt{1-a^2}}}{\cancel{2}a\cancel{\sqrt{1-a^2}}}-\frac{1}{a}$$

$$=\frac{1}{a}-\frac{1}{a}$$

$$=\boxed{0}$$

72. Evaluar la expresión: $\dfrac{xy-\sqrt{x^2-1}\sqrt{y^2-1}}{xy+\sqrt{x^2-1}\sqrt{y^2-1}}$ si $x=\dfrac{1}{2}\left(a+\dfrac{1}{a}\right)$, $y=\dfrac{1}{2}\left(b+\dfrac{1}{b}\right)$

Solución.

$$xy=\frac{1}{2}\left(a+\frac{1}{a}\right)\cdot\frac{1}{2}\left(b+\frac{1}{b}\right)$$

$$=\frac{1}{4}\left(\frac{a^2+1}{a}\right)\left(\frac{b^2+1}{b}\right)$$

$$=\boxed{\frac{(a^2+1)(b^2+1)}{4ab}}$$

$$\sqrt{x^2-1}=\sqrt{\left[\frac{1}{2}\left(a+\frac{1}{a}\right)\right]^2-1}$$

$$=\sqrt{\frac{1}{4}\left(a+\frac{1}{a}\right)^2-1}$$

$$=\sqrt{\frac{1}{4}\left(\frac{a^2+1}{a}\right)^2-1}$$

$$=\sqrt{\frac{1}{4}\cdot\frac{(a^2+1)^2}{a^2}-1}$$

$$= \sqrt{\frac{(a^2+1)^2}{4a^2} - 1}$$

$$= \sqrt{\frac{(a^2+1)^2 - 4a^2}{4a^2}}$$

$$= \sqrt{\frac{a^4+2a^2+1-4a^2}{4a^2}}$$

$$= \sqrt{\frac{a^4-2a^2+1}{4a^2}}$$

$$= \sqrt{\frac{(a^2-1)^2}{4a^2}}$$

$$= \sqrt{\left(\frac{a^2-1}{2a}\right)^2}$$

$$= \boxed{\frac{a^2-1}{2a}}$$

$$\sqrt{y^2-1} = \sqrt{\left[\frac{1}{2}\left(b+\frac{1}{b}\right)\right]^2 - 1}$$

$$= \sqrt{\frac{1}{4}\left(b+\frac{1}{b}\right)^2 - 1}$$

$$= \sqrt{\frac{1}{4}\left(\frac{b^2+1}{a}\right)^2 - 1}$$

$$= \sqrt{\frac{1}{4}\cdot\frac{(b^2+1)^2}{b^2} - 1}$$

$$= \sqrt{\frac{(b^2+1)^2}{4b^2} - 1}$$

$$= \sqrt{\frac{(b^2+1)^2 - 4b^2}{4b^2}}$$

$$= \sqrt{\frac{b^4+2b^2+1-4b^2}{4b^2}}$$

$$= \sqrt{\frac{b^4-2b^2+1}{4b^2}}$$

$$= \sqrt{\frac{(b^2-1)^2}{4b^2}}$$

$$= \sqrt{\left(\frac{b^2-1}{2b}\right)^2}$$

$$= \boxed{\frac{b^2-1}{2b}}$$

reemplazando:

$$\frac{xy-\sqrt{x^2-1}\sqrt{y^2-1}}{xy+\sqrt{x^2-1}\sqrt{y^2-1}} = \frac{\frac{(a^2+1)(b^2+1)}{4ab} - \frac{a^2-1}{2a}\cdot\frac{b^2-1}{2b}}{\frac{(a^2+1)(b^2+1)}{4ab} + \frac{a^2-1}{2a}\cdot\frac{b^2-1}{2b}}$$

$$= \frac{\dfrac{(a^2+1)(b^2+1)}{4ab} - \dfrac{(a^2-1)(b^2-1)}{4ab}}{\dfrac{(a^2+1)(b^2+1)}{4ab} + \dfrac{(a^2-1)(b^2-1)}{4ab}}$$

$$= \frac{\dfrac{(a^2+1)(b^2+1) - (a^2-1)(b^2-1)}{4ab}}{\dfrac{(a^2+1)(b^2+1) + (a^2-1)(b^2-1)}{4ab}}$$

$$= \frac{\dfrac{a^3b^2 + a^2 + b^2 + 1 - a^2b^2 + a^2 + b^2 - 1}{4ab}}{\dfrac{a^2b^2 + a^2 + b^2 + 1 + a^2b^2 - a^2 - b^2 + 1}{4ab}}$$

$$= \frac{\dfrac{2a^2 + 2b^2}{4ab}}{\dfrac{2a^2b^2 + 2}{4ab}}$$

$$= \frac{\dfrac{2(a^2 + b^2)}{4ab}}{\dfrac{2(a^2b^2 + 1)}{4ab}}$$

$$= \frac{\dfrac{a^2 + b^2}{2ab}}{\dfrac{a^2b^2 + 1}{2ab}}$$

$$= \frac{(a^2+b^2)2ab}{2ab(a^2b^2+1)}$$

$$= \frac{(a^2+b^2)\cancel{2ab}}{\cancel{2ab}(a^2b^2+1)}$$

$$= \boxed{\frac{a^2+b^2}{a^2b^2+1}}$$

73. Evaluar la expresión: $\dfrac{\sqrt{a+bx}+\sqrt{a-bx}}{\sqrt{a+bx}+\sqrt{a-bx}}$ para $x = \dfrac{2am}{b(1+m^2)}$

Solución.

Realizando operaciones en forma separada:

$$\sqrt{a+bx} = \sqrt{a + b \cdot \frac{2am}{b(1+m^2)}}$$

$$= \sqrt{a + \frac{2abm}{b(1+m^2)}}$$

$$= \sqrt{a + \frac{2am}{1+m^2}}$$

$$= \sqrt{\frac{a(1+m^2) + 2am}{1+m^2}}$$

$$= \sqrt{\frac{a + am^2 + 2am}{1+m^2}}$$

$$= \sqrt{\frac{am^2 + 2am + a}{1+m^2}}$$

$$= \sqrt{\frac{a(m^2+2m+1)}{1+m^2}}$$

$$= \sqrt{\frac{a(m+1)^2}{1+m^2}}$$

$$= \boxed{(m+1)\sqrt{\frac{a}{1+m^2}}}$$

$$\sqrt{a-bx} = \sqrt{a - b \cdot \frac{2am}{b(1+m^2)}}$$

$$= \sqrt{a - \frac{2abm}{b(1+m^2)}}$$

$$= \sqrt{a - \frac{2am}{1+m^2}}$$

$$= \sqrt{\frac{a(1+m^2)-2am}{1+m^2}}$$

$$= \sqrt{\frac{a+am^2-2am}{1+m^2}}$$

$$= \sqrt{\frac{am^2-2am+a}{1+m^2}}$$

$$= \sqrt{\frac{a(m^2-2m+1)}{1+m^2}}$$

$$= \sqrt{\frac{a(m-1)^2}{1+m^2}}$$

$$= \boxed{(m-1)\sqrt{\frac{a}{1+m^2}}}$$

reemplazando:

$$\frac{\sqrt{a+bx}+\sqrt{a-bx}}{\sqrt{a+bx}+\sqrt{a-bx}} = \frac{(m+1)\sqrt{\frac{a}{1+m^2}} + (m-1)\sqrt{\frac{a}{1+m^2}}}{(m+1)\sqrt{\frac{a}{1+m^2}} - (m-1)\sqrt{\frac{a}{1+m^2}}}$$

$$= \frac{\sqrt{\frac{a}{1+m^2}}\big((m+1)+(m-1)\big)}{\sqrt{\frac{a}{1+m^2}}\big((m+1)-(m-1)\big)}$$

$$= \frac{\cancel{\sqrt{\frac{a}{1+m^2}}}\big((m+1)+(m-1)\big)}{\cancel{\sqrt{\frac{a}{1+m^2}}}\big((m+1)-(m-1)\big)}$$

$$= \frac{(m+1)+(m-1)}{(m+1)-(m-1)}$$

$$= \frac{m+1+m-1}{m+1-m+1}$$

$$= \frac{2m}{2}$$

$$= \frac{\cancel{2}m}{\cancel{2}}$$

$$= \boxed{m}$$

Simplicar las expresiones siguientes:

74. $\dfrac{(m+1)^{\frac{1}{2}}+(m-1)^{\frac{1}{2}}}{(m+1)^{\frac{1}{2}}-(m-1)^{\frac{1}{2}}}$ si $x=\dfrac{2mn}{n^2+1}$ y $m>0$, $0<n<1$

Solución.

$$\begin{aligned}
(m+x)^{\frac{1}{2}} &= \sqrt{m+x}\\
&= \sqrt{m+\frac{2mn}{n^2+1}}\\
&= \sqrt{\frac{m(n^2+1)+2mn}{n^2+1}}\\
&= \sqrt{\frac{(n^2+1+2n)m}{n^2+1}}\\
&= \sqrt{\frac{(n^2+2n+1)m}{n^2+1}}\\
&= \sqrt{\frac{(n+1)^2m}{n^2+1}}\\
&= \boxed{(n+1)\sqrt{\frac{m}{n^2+1}}}
\end{aligned}$$

$$\begin{aligned}
(m-x)^{\frac{1}{2}} &= \sqrt{m-x}\\
&= \sqrt{m-\frac{2mn}{n^2+1}}\\
&= \sqrt{\frac{m(n^2+1)-2mn}{n^2+1}}\\
&= \sqrt{\frac{(n^2+1-2n)m}{n^2+1}}\\
&= \sqrt{\frac{(n^2-2n+1)m}{n^2+1}}\\
&= \sqrt{\frac{(n-1)^2m}{n^2+1}}\\
&= \boxed{(n-1)\sqrt{\frac{m}{n^2+1}}}
\end{aligned}$$

reemplazando:

$$\begin{aligned}
\frac{(m+1)^{\frac{1}{2}}+(m-1)^{\frac{1}{2}}}{(m+1)^{\frac{1}{2}}-(m-1)^{\frac{1}{2}}} &= \frac{(n+1)\sqrt{\frac{m}{n^2+1}}+(n-1)\sqrt{\frac{m}{n^2+1}}}{(n+1)\sqrt{\frac{m}{n^2+1}}-(n-1)\sqrt{\frac{m}{n^2+1}}}\\
&= \frac{\sqrt{\frac{m}{n^2+1}}\big((n+1)+(n-1)\big)}{\sqrt{\frac{m}{n^2+1}}\big((n+1)-(n-1)\big)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\cancel{\sqrt{\frac{m}{n^2+1}}}\big((n+1)+(n-1)\big)}{\cancel{\sqrt{\frac{m}{n^2+1}}}\big((n+1)-(n-1)\big)} \\
&= \frac{(n+1)+(n-1)}{(n+1)-(n-1)} \\
&= \frac{n+1+n-1}{n+1-n+1} \\
&= \frac{2n}{2} \\
&= \frac{\cancel{2}n}{\cancel{2}} \\
&= \boxed{n}
\end{aligned}
$$

75. $\left[\dfrac{(1-x^2)^{-\frac{1}{2}}+1}{2}\right]^{-\frac{1}{2}} + \left[\dfrac{(1-x^2)^{-\frac{1}{2}}-1}{2}\right]^{-\frac{1}{2}}$ si $x = 2k^{\frac{1}{2}}(1+k)^{-1}$ y $k > 0$

Solución.

$$
\begin{aligned}
(1-x^2)^{-\frac{1}{2}} &= (1-(2k^{\frac{1}{2}}(1+k)^{-1})^2)^{-\frac{1}{2}} \\
&= (1-2^2k^{\frac{2}{2}}(1+k)^{-2})^{-\frac{1}{2}} \\
&= \left(1-4k\cdot\frac{1}{(1+k)^2}\right)^{-\frac{1}{2}} \\
&= \left(1-\frac{4k}{(1+k)^2}\right)^{-\frac{1}{2}} \\
&= \left(\frac{(1+k)^2-4k}{(1+k)^2}\right)^{-\frac{1}{2}} \\
&= \left(\frac{1+2k+k^2-4k}{(k+1)^2}\right)^{-\frac{1}{2}} \\
&= \left(\frac{k^2-2k+1}{(k+1)^2}\right)^{-\frac{1}{2}} \\
&= \left(\frac{(k-1)^2}{(k+1)^2}\right)^{-\frac{1}{2}} \\
&= \left(\left(\frac{k-1}{k+1}\right)^2\right)^{-\frac{1}{2}} \\
&= \left(\frac{k-1}{k+1}\right)^{-1} \\
&= \frac{1}{\frac{k-1}{k+1}} \\
&= \boxed{\frac{k+1}{k-1}}
\end{aligned}
$$

reemplazando:

$$\left[\frac{(1-x^2)^{-\frac{1}{2}}+1}{2}\right]^{-\frac{1}{2}}+\left[\frac{(1-x^2)^{-\frac{1}{2}}-1}{2}\right]^{-\frac{1}{2}}=\left[\frac{\frac{k+1}{k-1}+1}{2}\right]^{-\frac{1}{2}}+\left[\frac{\frac{k+1}{k-1}-1}{2}\right]^{-\frac{1}{2}}$$

$$=\left[\frac{\frac{k+1+k-1}{k-1}}{2}\right]^{-\frac{1}{2}}+\left[\frac{\frac{k+1-k+1}{k-1}}{2}\right]^{-\frac{1}{2}}$$

$$=\left[\frac{\frac{2k}{k-1}}{2}\right]^{-\frac{1}{2}}+\left[\frac{\frac{2}{k-1}}{2}\right]^{-\frac{1}{2}}$$

$$=\left[\frac{2k}{2(k-1)}\right]^{-\frac{1}{2}}+\left[\frac{2}{2(k-1)}\right]^{-\frac{1}{2}}$$

$$=\left[\frac{\cancel{2}k}{\cancel{2}(k-1)}\right]^{-\frac{1}{2}}+\left[\frac{\cancel{2}}{\cancel{2}(k-1)}\right]^{-\frac{1}{2}}$$

$$=\left[\frac{k}{k-1}\right]^{-\frac{1}{2}}+\left[\frac{1}{k-1}\right]^{-\frac{1}{2}}$$

$$=\frac{1}{\left[\frac{k}{k-1}\right]^{\frac{1}{2}}}+\frac{1}{\left[\frac{1}{k-1}\right]^{\frac{1}{2}}}$$

$$=\frac{1}{\sqrt{\frac{k}{k-1}}}+\frac{1}{\sqrt{\frac{1}{k-1}}}$$

$$=\frac{1}{\frac{\sqrt{k}}{\sqrt{k-1}}}+\frac{1}{\frac{1}{\sqrt{k-1}}}$$

$$=\frac{\sqrt{k-1}}{\sqrt{k}}+\sqrt{k-1}$$

$$=\boxed{\sqrt{k-1}\left(\frac{1}{\sqrt{k}}+1\right)}$$

76. $\left(\frac{1}{2}-\frac{1}{4a^{-1}}-\frac{2^{-2}}{a}\right)\left[(a-1)\sqrt[3]{(a+1)^{-3}}-\frac{(a+1)^{\frac{3}{2}}}{\sqrt{(a^2-1)(a-1)}}\right]$

Solución.

$$\left(\frac{1}{2}-\frac{1}{4a^{-1}}-\frac{2^{-2}}{a}\right)\left[(a-1)\sqrt[3]{(a+1)^{-3}}-\frac{(a+1)^{\frac{3}{2}}}{\sqrt{(a^2-1)(a-1)}}\right]=$$

$$=\overbrace{\left(\frac{1}{2}-\frac{1}{4a^{-1}}-\frac{2^{-2}}{a}\right)}^{A}\overbrace{\left[(a-1)\sqrt[3]{(a+1)^{-3}}-\frac{(a+1)^{\frac{3}{2}}}{\sqrt{(a^2-1)(a-1)}}\right]}^{B}$$

$$A=\frac{1}{2}-\frac{1}{4a^{-1}}-\frac{2^{-2}}{a}$$

$$=\frac{1}{2}-\frac{a}{4}-\frac{1}{2^2\cdot a}$$

$$
\begin{aligned}
&= \frac{1}{2} - \frac{a}{4} - \frac{1}{4a} \\
&= \frac{2a - a^2 - 1}{4a} \\
&= \frac{-a^2 + 2a - 1}{4a} \\
&= \frac{-(a^2 - 2a + 1)}{4a}
\end{aligned}
$$

$$\boxed{A = \frac{-(a-1)^2}{4a}}$$

$$
\begin{aligned}
B &= (a-1)\sqrt[3]{(a+1)^{-3}} - \frac{(a+1)^{\frac{3}{2}}}{\sqrt{(a^2-1)(a-1)}} \\
&= (a-1)(a+1)^{\frac{-3}{3}} - \frac{\sqrt{(a+1)^3}}{\sqrt{(a-1)(a+1)(a-1)}} \\
&= (a-1)(a+1)^{-1} - \frac{(a+1)\sqrt{a+1}}{\sqrt{(a-1)^2(a+1)}} \\
&= (a-1)\cdot\frac{1}{a+1} - \frac{(a+1)\sqrt{a+1}}{(a-1)\sqrt{a+1}} \\
&= \frac{a-1}{a+1} - \frac{(a+1)\cancel{\sqrt{a+1}}}{(a-1)\cancel{\sqrt{a+1}}} \\
&= \frac{a-1}{a+1} - \frac{a+1}{a-1} \\
&= \frac{(a-1)^2 - (a+1)^2}{(a+1)(a-1)} \\
&= \frac{(a^2 - 2a + 1) - (a^2 + 2a + 1)}{(a+1)(a-1)} \\
&= \frac{a^2 - 2a + 1 - a^2 - 2a - 1}{(a+1)(a-1)}
\end{aligned}
$$

$$\boxed{B = \frac{-4a}{(a+1)(a-1)}}$$

reemplazando:

$$
\begin{aligned}
\left(\frac{1}{2} - \frac{1}{4a^{-1}} - \frac{2^{-2}}{a}\right)\left[(a-1)\sqrt[3]{(a+1)^{-3}} - \frac{(a+1)^{\frac{3}{2}}}{\sqrt{(a^2-1)(a-1)}}\right] &= \frac{-(a-1)^2}{4a}\cdot\frac{-4a}{(a+1)(a-1)} \\
&= \frac{4a(a-1)^2}{4a(a+1)(a-1)} \\
&= \frac{\cancel{4a}(a-1)^{\cancel{2}}}{\cancel{4a}(a+1)\cancel{(a-1)}} \\
&= \boxed{\frac{a-1}{a+1}}
\end{aligned}
$$

77. $\left(2\sqrt{x^4 - a^2x^2} - \dfrac{2a^2}{\sqrt{1 - a^2x^{-2}}}\right)\dfrac{(x^2a^{-2} - 4 + a^2x^{-2})^{-\frac{1}{2}}}{2ax(x^2 - a^2)^{-\frac{1}{2}}}$

Solución.

$$\left(2\sqrt{x^4-a^2x^2}-\frac{2a^2}{\sqrt{1-a^2x^{-2}}}\right)\frac{(x^2a^{-2}-4+a^2x^{-2})^{-\frac{1}{2}}}{2ax(x^2-a^2)^{-\frac{1}{2}}}=$$

$$=\overbrace{\left(2\sqrt{x^4-a^2x^2}-\frac{2a^2}{\sqrt{1-a^2x^{-2}}}\right)}^{A}\overbrace{\frac{(x^2a^{-2}-4+a^2x^{-2})^{-\frac{1}{2}}}{2ax(x^2-a^2)^{-\frac{1}{2}}}}^{B}$$

$$A=\left(2\sqrt{x^4-a^2x^2}-\frac{2a^2}{\sqrt{1-a^2x^{-2}}}\right)$$

$$=2\left(\sqrt{x^4-a^2x^2}-\frac{a^2}{\sqrt{1-a^2x^{-2}}}\right)$$

$$=2\left(\sqrt{x^2(x^2-a^2)}-\frac{a^2}{\sqrt{1-\frac{a^2}{x^2}}}\right)$$

$$=2\left(x\sqrt{x^2-a^2}-\frac{a^2}{\sqrt{\frac{x^2-a^2}{x^2}}}\right)$$

$$=2\left(x\sqrt{x^2-a^2}-\frac{a^2}{\frac{\sqrt{x^2-a^2}}{x}}\right)$$

$$=2\left(x\sqrt{x^2-a^2}-\frac{a^2x}{\sqrt{x^2-a^2}}\right)$$

$$=2x\left(\sqrt{x^2-a^2}-\frac{a^2}{\sqrt{x^2-a^2}}\right)$$

$$=2x\left(\frac{(\sqrt{x^2-a^2})^2-a^2}{\sqrt{x^2-a^2}}\right)$$

$$=2x\left(\frac{x^2-a^2-a^2}{\sqrt{x^2-a^2}}\right)$$

$$=2x\left(\frac{x^2-2a^2}{\sqrt{x^2-a^2}}\right)$$

$$\boxed{A=\frac{2x(x^2-2a^2)}{\sqrt{x^2-a^2}}}$$

$$B=\frac{(x^2a^{-2}-4+a^2x^{-2})^{-\frac{1}{2}}}{2ax(x^2-a^2)^{-\frac{1}{2}}}$$

$$=\frac{(x^2-a^2)^{\frac{1}{2}}}{2ax(x^2a^{-2}-4+a^2x^{-2})^{\frac{1}{2}}}$$

$$=\frac{(x^2-a^2)^{\frac{1}{2}}}{2ax\left(\frac{x^2}{a^2}-4+\frac{a^2}{x^2}\right)^{\frac{1}{2}}}$$

$$=\frac{(x^2-a^2)^{\frac{1}{2}}}{2ax\left(\frac{x^4-4a^2x^2+4a^4}{a^2x^2}\right)^{\frac{1}{2}}}$$

$$= \frac{(x^2-a^2)^{\frac{1}{2}}}{2ax\left(\dfrac{(x^2-2a^2)^2}{a^2x^2}\right)^{\frac{1}{2}}}$$

$$= \frac{\sqrt{x^2-a^2}}{2ax\sqrt{\dfrac{(x^2-2a^2)^2}{a^2x^2}}}$$

$$= \frac{\sqrt{x^2-a^2}}{2ax\sqrt{\left(\dfrac{x^2-2a^2}{ax}\right)^2}}$$

$$= \frac{\sqrt{x^2-a^2}}{2ax\left(\dfrac{x^2-2a^2}{ax}\right)}$$

$$= \frac{\sqrt{x^2-a^2}}{\dfrac{2ax(x^2-2a^2)}{ax}}$$

$$= \frac{\sqrt{x^2-a^2}}{\dfrac{2\cancel{ax}(x^2-2a^2)}{\cancel{ax}}}$$

$$\boxed{B = \frac{\sqrt{x^2-a^2}}{2(x^2-2a^2)}}$$

reemplazando

$$\left(2\sqrt{x^4-a^2x^2} - \frac{2a^2}{\sqrt{1-a^2x^{-2}}}\right)\frac{(x^2a^{-2}-4+a^2x^{-2})^{-\frac{1}{2}}}{2ax(x^2-a^2)^{-\frac{1}{2}}} = \frac{2x(x^2-2a^2)}{\sqrt{x^2-a^2}}\cdot\frac{\sqrt{x^2-a^2}}{2(x^2-2a^2)}$$

$$= \frac{2x(x^2-2a^2)\sqrt{x^2-a^2}}{2\sqrt{x^2-a^2}(x^2-2a^2)}$$

$$= \frac{\cancel{2}x\cancel{(x^2-2a^2)}\cancel{\sqrt{x^2-a^2}}}{\cancel{2}\cancel{\sqrt{x^2-a^2}}\cancel{(x^2-2a^2)}}$$

$$= \boxed{x}$$

78. $$\frac{a\left(\dfrac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b\left(\dfrac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}}{\left(\dfrac{a+\sqrt{ab}}{2ab}\right)^{-1} - \left(\dfrac{b+\sqrt{ab}}{2ab}\right)^{-1}}$$

Solución.

$$\frac{a\left(\dfrac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b\left(\dfrac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}}{\left(\dfrac{a+\sqrt{ab}}{2ab}\right)^{-1} - \left(\dfrac{b+\sqrt{ab}}{2ab}\right)^{-1}} = \frac{\overbrace{a\left(\dfrac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b\left(\dfrac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}}^{A}}{\underbrace{\left(\dfrac{a+\sqrt{ab}}{2ab}\right)^{-1} - \left(\dfrac{b+\sqrt{ab}}{2ab}\right)^{-1}}_{B}}$$

$$A = a\left(\frac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b\left(\frac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}$$

$$= a \cdot \frac{1}{\frac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}} + b \cdot \frac{1}{\frac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}}$$

$$= a \cdot \frac{2b\sqrt{a}}{\sqrt{a}+\sqrt{b}} + b \cdot \frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{2ab\sqrt{a}}{\sqrt{a}+\sqrt{b}} + \frac{2ab\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{2ab\sqrt{a}+2ab\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{2ab(\sqrt{a}+\sqrt{b})}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{2ab\cancel{(\sqrt{a}+\sqrt{b})}}{\cancel{\sqrt{a}+\sqrt{b}}}$$

$$\boxed{A = 2ab}$$

$$B = \left(\frac{a+\sqrt{ab}}{2ab}\right)^{-1} - \left(\frac{b+\sqrt{ab}}{2ab}\right)^{-1}$$

$$= \frac{1}{\frac{a+\sqrt{ab}}{2ab}} - \frac{1}{\frac{b+\sqrt{ab}}{2ab}}$$

$$= \frac{2ab}{a+\sqrt{ab}} - \frac{2ab}{b+\sqrt{ab}}$$

$$= 2ab\left(\frac{1}{a+\sqrt{ab}} - \frac{1}{b+\sqrt{ab}}\right)$$

$$= 2ab\left(\frac{1}{\sqrt{a^2}+\sqrt{ab}} - \frac{1}{\sqrt{b^2}+\sqrt{ab}}\right)$$

$$= 2ab\left(\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{b})} - \frac{1}{\sqrt{b}(\sqrt{b}+\sqrt{a})}\right)$$

$$= 2ab \cdot \frac{1}{\sqrt{a}+\sqrt{b}}\left(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}}\right)$$

$$= \frac{2ab}{\sqrt{a}+\sqrt{b}}\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}\sqrt{b}}\right)$$

$$= \frac{2ab(\sqrt{a}+\sqrt{b})}{(\sqrt{a}+\sqrt{b})\sqrt{a}\sqrt{b}}$$

$$= \frac{2ab\cancel{(\sqrt{a}+\sqrt{b})}}{\cancel{(\sqrt{a}+\sqrt{b})}\sqrt{a}\sqrt{b}}$$

$$= \frac{2ab}{\sqrt{a}\sqrt{b}}$$

$$\boxed{B = \frac{2ab}{\sqrt{ab}}}$$

reemplazando:

$$\frac{a\left(\dfrac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1}+b\left(\dfrac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}}{\left(\dfrac{a+\sqrt{ab}}{2ab}\right)^{-1}-\left(\dfrac{b+\sqrt{ab}}{2ab}\right)^{-1}}=\frac{2ab}{\dfrac{2ab}{\sqrt{ab}}}$$

$$=\frac{2ab\sqrt{ab}}{2ab}$$

$$=\frac{\cancel{2ab}\sqrt{ab}}{\cancel{2ab}}$$

$$=\boxed{\sqrt{ab}}$$

79. $\left(\dfrac{\sqrt{a}+\sqrt{x}}{\sqrt{a+x}}-\dfrac{\sqrt{a+x}}{\sqrt{a}+\sqrt{x}}\right)^{-2}-\left(\dfrac{\sqrt{a}-\sqrt{x}}{\sqrt{a+x}}-\dfrac{\sqrt{a+x}}{\sqrt{a}-\sqrt{x}}\right)^{-2}$

Solución.

$$\left(\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}+\sqrt{x}}\right)^{-2}-\left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}-\sqrt{x}}\right)^{-2}=$$

$$=\overbrace{\left(\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}+\sqrt{x}}\right)^{-2}}^{A}-\overbrace{\left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}-\sqrt{x}}\right)^{-2}}^{B}$$

$$A=\left(\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}+\sqrt{x}}\right)^{-2}$$

$$=\left(\frac{(\sqrt{a}+\sqrt{x})^2-(\sqrt{a+x})^2}{\sqrt{a+x}(\sqrt{a}+\sqrt{x})}\right)^{-2}$$

$$=\left(\frac{(\sqrt{a})^2+2\sqrt{a}\sqrt{x}+(\sqrt{x})^2-(a+x)}{\sqrt{a+x}(\sqrt{a}+\sqrt{x})}\right)^{-2}$$

$$=\left(\frac{a+2\sqrt{ax}+x-a-x}{\sqrt{a+x}(\sqrt{a}+\sqrt{x})}\right)^{-2}$$

$$=\left(\frac{2\sqrt{ax}}{\sqrt{a+x}(\sqrt{a}+\sqrt{x})}\right)^{-2}$$

$$=\frac{1}{\left(\dfrac{2\sqrt{ax}}{\sqrt{a+x}(\sqrt{a}+\sqrt{x})}\right)^{2}}$$

$$=\frac{1}{\dfrac{4ax}{(a+x)(\sqrt{a}+\sqrt{x})^2}}$$

$$\boxed{A=\frac{(a+x)(\sqrt{a}+\sqrt{x})^2}{4ax}}$$

$$B=\left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a+x}}-\frac{\sqrt{a+x}}{\sqrt{a}-\sqrt{x}}\right)^{-2}$$

$$=\left(\frac{(\sqrt{a}-\sqrt{x})^2-(\sqrt{a+x})^2}{\sqrt{a+x}(\sqrt{a}-\sqrt{x})}\right)^{-2}$$

$$=\left(\frac{(\sqrt{a})^2-2\sqrt{a}\sqrt{x}+(\sqrt{x})^2-(a+x)}{\sqrt{a+x}(\sqrt{a}-\sqrt{x})}\right)^{-2}$$

$$= \left(\frac{a - 2\sqrt{ax} + x - a - x}{\sqrt{a+x}(\sqrt{a} - \sqrt{x})} \right)^{-2}$$

$$= \left(\frac{-2\sqrt{ax}}{\sqrt{a+x}(\sqrt{a} - \sqrt{x})} \right)^{-2}$$

$$= \frac{1}{\left(\frac{2\sqrt{ax}}{\sqrt{a+x}(\sqrt{a} - \sqrt{x})} \right)^{2}}$$

$$= \frac{1}{\frac{4ax}{(a+x)(\sqrt{a} - \sqrt{x})^2}}$$

$$\boxed{B = \frac{(a+x)(\sqrt{a} - \sqrt{x})^2}{4ax}}$$

reemplazando:

$$\left(\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a+x}} - \frac{\sqrt{a+x}}{\sqrt{a} + \sqrt{x}} \right)^{-2} - \left(\frac{\sqrt{a} - \sqrt{x}}{\sqrt{a+x}} - \frac{\sqrt{a+x}}{\sqrt{a} - \sqrt{x}} \right)^{-2} =$$

$$= \frac{(a+x)(\sqrt{a} + \sqrt{x})^2}{4ax} - \frac{(a+x)(\sqrt{a} - \sqrt{x})^2}{4ax}$$

$$= \frac{(a+x)}{4ax} \left[(\sqrt{a} + \sqrt{x})^2 - (\sqrt{a} - \sqrt{x})^2 \right]$$

$$= \frac{(a+x)}{4ax} \left[\left((\sqrt{a})^2 + 2\sqrt{a}\sqrt{x} + (\sqrt{x})^2 \right) - \left((\sqrt{a})^2 - 2\sqrt{a}\sqrt{x} + (\sqrt{x})^2 \right) \right]$$

$$= \frac{(a+x)}{4ax} \left[(a + 2\sqrt{ax} + x) - (a - 2\sqrt{ax} + x) \right]$$

$$= \frac{(a+x)}{4ax} [a + 2\sqrt{ax} + x - a + 2\sqrt{ax} - x]$$

$$= \frac{(a+x)}{4ax} \left[4\sqrt{ax} \right]$$

$$= \frac{4\sqrt{ax}(a+x)}{4ax}$$

$$= \frac{(ax)^{\frac{1}{2}}(a+x)}{ax}$$

$$= \frac{a+x}{(ax)(ax)^{-\frac{1}{2}}}$$

$$= \frac{a+x}{(ax)^{\frac{1}{2}}}$$

$$= \boxed{\frac{a+x}{\sqrt{ab}}}$$

80. $\frac{1}{2} \left(\sqrt{x^2 + a} + \frac{x^2}{\sqrt{x^2 + a}} \right) + \frac{a}{2} \cdot \frac{1 + \frac{x}{\sqrt{x^2 + a}}}{x + \sqrt{x^2 + a}}$

Solución.

$$\frac{1}{2} \left(\sqrt{x^2 + a} + \frac{x^2}{\sqrt{x^2 + a}} \right) + \frac{a}{2} \cdot \frac{1 + \frac{x}{\sqrt{x^2 + a}}}{x + \sqrt{x^2 + a}} = \overbrace{\frac{1}{2} \left(\sqrt{x^2 + a} + \frac{x^2}{\sqrt{x^2 + a}} \right)}^{A} + \overbrace{\frac{a}{2} \cdot \frac{1 + \frac{x}{\sqrt{x^2 + a}}}{x + \sqrt{x^2 + a}}}^{B}$$

$$
\begin{aligned}
A &= \frac{1}{2}\left(\sqrt{x^2+a}+\frac{x^2}{\sqrt{x^2+a}}\right)\\
&= \frac{1}{2}\left(\frac{(\sqrt{x^2+a})^2+x^2}{\sqrt{x^2+a}}\right)\\
&= \frac{1}{2}\left(\frac{x^2+a+x^2}{\sqrt{x^2+a}}\right)\\
&= \frac{1}{2}\left(\frac{2x^2+a}{\sqrt{x^2+a}}\right)
\end{aligned}
$$

$$
\boxed{A = \frac{2x^2+a}{2\sqrt{x^2+a}}}
$$

$$
\begin{aligned}
B &= \frac{a}{2}\cdot\frac{1+\dfrac{x}{\sqrt{x^2+a}}}{x+\sqrt{x^2+a}}\\
&= \frac{a}{2}\cdot\frac{\dfrac{\sqrt{x^2+a}+x}{\sqrt{x^2+a}}}{x+\sqrt{x^2+a}}\\
&= \frac{a}{2}\cdot\frac{\sqrt{x^2+a}+x}{\sqrt{a^2+a}(x+\sqrt{x^2+a})}\\
&= \frac{a(x+\sqrt{x^2+a})}{2\sqrt{a^2+a}(x+\sqrt{x^2+a})}\\
&= \frac{a\cancel{(x+\sqrt{x^2+a})}}{2\sqrt{a^2+a}\cancel{(x+\sqrt{x^2+a})}}
\end{aligned}
$$

$$
\boxed{B = \frac{a}{2\sqrt{a^2+a}}}
$$

reemplazando:

$$
\begin{aligned}
\frac{1}{2}\left(\sqrt{x^2+a}+\frac{x^2}{\sqrt{x^2+a}}\right)+\frac{a}{2}\cdot\frac{1+\dfrac{x}{\sqrt{x^2+a}}}{x+\sqrt{x^2+a}} &= \frac{2x^2+a}{2\sqrt{x^2+a}}+\frac{a}{2\sqrt{a^2+a}}\\
&= \frac{2x^2+a+a}{2\sqrt{x^2+a}}\\
&= \frac{2x^2+2a}{2\sqrt{x^2+a}}\\
&= \frac{2(x^2+a)}{2\sqrt{x^2+a}}\\
&= \frac{\cancel{2}(x^2+a)}{\cancel{2}\sqrt{x^2+a}}\\
&= \frac{(x^2+a)}{\sqrt{x^2+a}}\\
&= \frac{(x^2+a)}{(x^2+a)^{\frac{1}{2}}}\\
&= (x^2+a)(x^2+a)^{-\frac{1}{2}}\\
&= (x^2+a)^{\frac{1}{2}}
\end{aligned}
$$

$$= \boxed{\sqrt{x^2+a}}$$

81. $2x+\sqrt{x^2-1}\left(1+\dfrac{x^2}{x^2-1}\right)-\dfrac{1+\dfrac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}$

Solución.

$$2x+\sqrt{x^2-1}\left(1+\frac{x^2}{x^2-1}\right)-\frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}} = 2x+\overbrace{\sqrt{x^2-1}\left(1+\frac{x^2}{x^2-1}\right)}^{A}-\frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}$$

$$\begin{aligned}
A &= \sqrt{x^2-1}\left(1+\frac{x^2}{x^2-1}\right)\\
&= \sqrt{x^2-1}\left(\frac{x^2-1+x^2}{x^2-1}\right)\\
&= (x^2-1)^{\frac{1}{2}}\left(\frac{2x^2-1}{x^2-1}\right)\\
&= \frac{(x^2-1)^{\frac{1}{2}}(2x^2-1)}{x^2-1}\\
&= \frac{2x^2-1}{(x^2-1)^{-\frac{1}{2}}(x^2-1)}\\
&= \frac{2x^2-1}{(x^2-1)^{\frac{1}{2}}}
\end{aligned}$$

$$\boxed{A=\frac{2x^2-1}{\sqrt{x^2-1}}}$$

$$\begin{aligned}
B &= \frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}\\
&= \frac{\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}\\
&= \frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}(x+\sqrt{x^2-1})}\\
&= \frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}(x+\sqrt{x^2-1})}\\
&= \frac{\cancel{x+\sqrt{x^2-1}}}{\sqrt{x^2-1}\cancel{(x+\sqrt{x^2-1})}}
\end{aligned}$$

$$\boxed{B=\frac{1}{\sqrt{x^2-1}}}$$

reemplazando:

$$\begin{aligned}
2x+\sqrt{x^2-1}\left(1+\frac{x^2}{x^2-1}\right)-\frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}} &= 2x+\frac{2x^2-1}{\sqrt{x^2-1}}-\frac{1}{\sqrt{x^2-1}}\\
&= \frac{2x\sqrt{x^2-1}+2x^2-1-1}{\sqrt{x^2-1}}
\end{aligned}$$

$$= \frac{2x\sqrt{x^2-1}+2x^2-2}{\sqrt{x^2-1}}$$
$$= \frac{2x\sqrt{x^2-1}+2(x^2-1)}{\sqrt{x^2-1}}$$
$$= \frac{2x\sqrt{x^2-1}+2\sqrt{(x^2-1)^2}}{\sqrt{x^2-1}}$$
$$= \frac{2\sqrt{x^2-1}(x+\sqrt{x^2-1})}{\sqrt{x^2-1}}$$
$$= \frac{2\cancel{\sqrt{x^2-1}}(x+\sqrt{x^2-1})}{\cancel{\sqrt{x^2-1}}}$$
$$= \boxed{2(x+\sqrt{x^2-1})}$$

82. Calcular $[a^{-\frac{3}{2}}b(ab^{-2})^{-\frac{1}{2}}(a^{-1})^{-\frac{2}{3}}]^3$ para $a=\dfrac{\sqrt{2}}{2}, b=\dfrac{1}{\sqrt[3]{2}}$

Solución.

$$\begin{aligned}
[a^{-\frac{3}{2}}b(ab^{-2})^{-\frac{1}{2}}(a^{-1})^{-\frac{2}{3}}]^3 &= [a^{-\frac{3}{2}}\cdot b\cdot a^{-\frac{1}{2}}\cdot b^{\frac{2}{2}}\cdot a^{\frac{2}{3}}]^3\\
&= [a^{-\frac{3}{2}}a^{-\frac{1}{2}}\cdot a^{\frac{2}{3}}\cdot b\cdot b]^3\\
&= [a^{-\frac{4}{3}}b^2]^3\\
&= \boxed{a^{-4}b^6}
\end{aligned}$$

reemplazando: $a=\dfrac{\sqrt{2}}{2}$, $b=\dfrac{1}{\sqrt[3]{2}}$

$$\begin{aligned}
a^{-4}b^6 &= \left(\frac{\sqrt{2}}{2}\right)^{-4}\left(\frac{1}{\sqrt[3]{2}}\right)^6\\
&= \left(\frac{2^{\frac{1}{2}}}{2}\right)^{-4}\left(\frac{1}{2^{\frac{1}{3}}}\right)^6\\
&= (2^{\frac{1}{2}}\cdot 2^{-1})^{-4}(2^{-\frac{1}{3}})^6\\
&= (2^{-\frac{1}{2}})^{-4}(2^{-\frac{6}{3}})\\
&= (2^{\frac{4}{2}})(2^{-2})\\
&= 2^2\cdot 2^{-2}\\
&= 2^{2-2}\\
&= 2^0\\
&= \boxed{1}
\end{aligned}$$

83. Evaluar la expresión $(a+1)^{-1}(b+1)^{-1}$ para $a=(2+\sqrt{3})^{-1}$ y $a=(2-\sqrt{3})^{-1}$

Solución.

$$\begin{aligned}
a+1 &= (2+\sqrt{3})^{-1}+1\\
&= \frac{1}{2+\sqrt{3}}+1\\
&= \frac{2+\sqrt{3}+1}{2+\sqrt{3}}
\end{aligned}$$

$$= \boxed{\frac{3+\sqrt{3}}{2+\sqrt{3}}}$$

$$b+1 = (2-\sqrt{3})^{-1}+1$$
$$= \frac{1}{2-\sqrt{3}}+1$$
$$= \frac{2-\sqrt{3}+1}{2-\sqrt{3}}$$
$$= \boxed{\frac{3-\sqrt{3}}{2-\sqrt{3}}}$$

reemplazando:

$$(a+1)^{-1}(b+1)^{-1} = \left(\frac{3+\sqrt{3}}{2+\sqrt{3}}\right)^{-1} + \left(\frac{3-\sqrt{3}}{2-\sqrt{3}}\right)^{-1}$$
$$= \frac{1}{\frac{3+\sqrt{3}}{2+\sqrt{3}}} + \frac{1}{\frac{3-\sqrt{3}}{2-\sqrt{3}}}$$
$$= \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{3-\sqrt{3}}$$
$$= \frac{(2+\sqrt{3})(3-\sqrt{3})+(2-\sqrt{3})(3+\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$
$$= \frac{6-2\sqrt{3}+3\sqrt{3}-(\sqrt{3})^2+6+2\sqrt{3}-3\sqrt{3}-(\sqrt{3})^2}{3^2-(\sqrt{3})^2}$$
$$= \frac{12-3-3}{9-3}$$
$$= \frac{6}{6}$$
$$= \boxed{1}$$

84. $\frac{x+\sqrt{x^2-4x}}{x-\sqrt{x^2-4x}} - \frac{x-\sqrt{x^2-4x}}{x+\sqrt{x^2-4x}}$

Solución.

$$\frac{x+\sqrt{x^2-4x}}{x-\sqrt{x^2-4x}} - \frac{x-\sqrt{x^2-4x}}{x+\sqrt{x^2-4x}} = \frac{(x+\sqrt{x^2-4x})^2-(x-\sqrt{x^2-4x})^2}{(x-\sqrt{x^2-4x})(x+\sqrt{x^2-4x})}$$
$$= \frac{(x^2+2x\sqrt{x^2-4x}+x^2-4x)-(x^2-2x\sqrt{x^2-4x}+x^2-4x)}{x^2-(x^2-4x)}$$
$$= \frac{2x^2+2x\sqrt{x^2-4x}-4x-2x^2+2x\sqrt{x^2-4x}+4x}{x^2-x^2+4x}$$
$$= \frac{4x\sqrt{x^2-4x}}{4x}$$
$$= \frac{\cancel{4x}\sqrt{x^2-4x}}{\cancel{4x}}$$
$$= \boxed{\sqrt{x^2-4x}}$$

85. $\dfrac{n+2+\sqrt{n^2-4}}{n+2-\sqrt{n^2-4}}+\dfrac{n+2-\sqrt{n^2-4}}{n+2+\sqrt{n^2-4}}$

Solución.

$$\frac{n+2+\sqrt{n^2-4}}{n+2-\sqrt{n^2-4}}+\frac{n+2-\sqrt{n^2-4}}{n+2+\sqrt{n^2-4}}=$$

$$=\frac{\sqrt{(n+2)^2}+\sqrt{(n-2)(n+2)}}{\sqrt{(n+2)^2}-\sqrt{(n-2)(n+2)}}+\frac{\sqrt{(n+2)^2}-\sqrt{(n-2)(n+4)}}{\sqrt{(n+2)^2}+\sqrt{(n-2)(n+4)}}$$

$$=\frac{\sqrt{n+2}(\sqrt{n+2}+\sqrt{n-2})}{\sqrt{n+2}(\sqrt{n+2}-\sqrt{n-2})}+\frac{\sqrt{n+2}(\sqrt{n+2}-\sqrt{n-2})}{\sqrt{n+2}(\sqrt{n+2}+\sqrt{n-2})}$$

$$=\frac{\cancel{\sqrt{n+2}}(\sqrt{n+2}+\sqrt{n-2})}{\cancel{\sqrt{n+2}}(\sqrt{n+2}-\sqrt{n-2})}+\frac{\cancel{\sqrt{n+2}}(\sqrt{n+2}-\sqrt{n-2})}{\cancel{\sqrt{n+2}}(\sqrt{n+2}+\sqrt{n-2})}$$

$$=\frac{\sqrt{n+2}+\sqrt{n-2}}{\sqrt{n+2}-\sqrt{n-2}}+\frac{\sqrt{n+2}-\sqrt{n-2}}{\sqrt{n+2}+\sqrt{n-2}}$$

$$=\frac{(\sqrt{n+2}+\sqrt{n-2})^2+(\sqrt{n+2}-\sqrt{n-2})^2}{(\sqrt{n+2}-\sqrt{n-2})(\sqrt{n+2}+\sqrt{n-2})}$$

$$=\frac{(\sqrt{n+2})^2+2\sqrt{n+2}\sqrt{n-2}+(\sqrt{n-2})^2+(\sqrt{n+2})^2-2\sqrt{n+2}\sqrt{n-2}+(\sqrt{n-2})^2}{(\sqrt{n+2})^2-(\sqrt{n-2})^2}$$

$$=\frac{2(\sqrt{n+2})^2+2(\sqrt{n-2})^2}{(n+2)-(n-2)}$$

$$=\frac{2(n+2)+2(n-2)}{n+2-n+2}$$

$$=\frac{2n+4+2n-4}{4}$$

$$=\frac{\cancel{4}n}{\cancel{4}}$$

$$=\boxed{n}$$

86. $\sqrt{\dfrac{x}{x-a^2}}:\left(\dfrac{\sqrt{x}-\sqrt{x-a^2}}{\sqrt{x}+\sqrt{x-a^2}}-\dfrac{\sqrt{x}+\sqrt{x-a^2}}{\sqrt{x}-\sqrt{x-a^2}}\right)$

Solución.

$$\sqrt{\frac{x}{x-a^2}}:\left(\frac{\sqrt{x}-\sqrt{x-a^2}}{\sqrt{x}+\sqrt{x-a^2}}-\frac{\sqrt{x}+\sqrt{x-a^2}}{\sqrt{x}-\sqrt{x-a^2}}\right)=\frac{\sqrt{x}}{\sqrt{x-a^2}}:\frac{(\sqrt{x}-\sqrt{x-a^2})^2-(\sqrt{x}+\sqrt{x-a^2})^2}{(\sqrt{x}+\sqrt{x-a^2})(\sqrt{x}-\sqrt{x-a^2})}$$

$$=\frac{\sqrt{x}}{\sqrt{x-a^2}}:\frac{\left((\sqrt{x})^2-2\sqrt{x}\sqrt{x-a^2}+(\sqrt{x-a^2})^2\right)-\left((\sqrt{x})^2+2\sqrt{x}\sqrt{x-a^2}+(\sqrt{x-a^2})^2\right)}{(\sqrt{x})^2-(\sqrt{x-a^2})^2}$$

$$=\frac{\sqrt{x}}{\sqrt{x-a^2}}:\frac{(x-2\sqrt{x}\sqrt{x-a^2}+x-a^2)-(x+2\sqrt{x}\sqrt{x-a^2}+x-a^2)}{x-(x-a^2)}$$

$$=\frac{\sqrt{x}}{\sqrt{x-a^2}}:\frac{x-2\sqrt{x}\sqrt{x-a^2}+x-a^2-x-2\sqrt{x}\sqrt{x-a^2}-x+a^2}{x-x+a^2}$$

$$=\frac{\sqrt{x}}{\sqrt{x-a^2}}:\frac{-4\sqrt{x}\sqrt{x-a^2}}{a^2}$$

$$=\frac{\sqrt{x}}{\sqrt{x-a^2}}\cdot\frac{a^2}{-4\sqrt{x}\sqrt{x-a^2}}$$

$$= \frac{a^2\sqrt{x}}{-4\sqrt{x}\sqrt{x-a^2}}$$

$$= -\frac{a^2\cancel{\sqrt{x}}}{4\cancel{\sqrt{x}}\sqrt{x-a^2}}$$

$$= \boxed{-\frac{a^2}{4\sqrt{x-a^2}}}$$

87. $\dfrac{x^{\frac{1}{2}}+1}{x+x^{\frac{1}{2}}+1} : \dfrac{1}{x^{1,5}-1}$

Solución.

$$\frac{x^{\frac{1}{2}}+1}{x+x^{\frac{1}{2}}+1} : \frac{1}{x^{1,5}-1} = \frac{x^{\frac{1}{2}}+1}{x+x^{\frac{1}{2}}+1} : \overbrace{\frac{1}{x^{1,5}-1}}^{A}$$

$$A = \frac{1}{x^{1,5}-1}$$

$$= \frac{1}{x^{\frac{3}{2}}-1}$$

$$= \frac{1}{(x^{\frac{1}{2}})^3-1}$$

$$= \frac{1}{(x^{\frac{1}{2}}-1)\big((x^{\frac{1}{2}})^2+x^{\frac{1}{2}}\cdot 1+1^2\big)}$$

$$\boxed{A = \frac{1}{(x^{\frac{1}{2}}-1)(x+x^{\frac{1}{2}}+1)}}$$

reemplazando:

$$\frac{x^{\frac{1}{2}}+1}{x+x^{\frac{1}{2}}+1} : \frac{1}{x^{1,5}-1} = \frac{x^{\frac{1}{2}}+1}{x+x^{\frac{1}{2}}+1} : \frac{1}{(x^{\frac{1}{2}}-1)(x+x^{\frac{1}{2}}+1)}$$

$$= \frac{x^{\frac{1}{2}}+1}{x+x^{\frac{1}{2}}+1} \cdot \frac{(x^{\frac{1}{2}}-1)(x+x^{\frac{1}{2}}+1)}{1}$$

$$= \frac{(x^{\frac{1}{2}}+1)(x^{\frac{1}{2}}-1)(x+x^{\frac{1}{2}}+1)}{x+x^{\frac{1}{2}}+1}$$

$$= \frac{(x^{\frac{1}{2}}+1)(x^{\frac{1}{2}}-1)\cancel{(x+x^{\frac{1}{2}}+1)}}{\cancel{x+x^{\frac{1}{2}}+1}}$$

$$= (x^{\frac{1}{2}}+1)(x^{\frac{1}{2}}-1)$$

$$= (x^{\frac{1}{2}})^2-1^2$$

$$= \boxed{x-1}$$

88. $(2^{\frac{3}{2}}+27y^{\frac{3}{5}}) : \left[\left(\dfrac{1}{2}\right)^{-\frac{1}{2}}+3y^{\frac{1}{5}}\right]$

Solución.

$$(2^{\frac{3}{2}}+27y^{\frac{3}{5}}) : \left[\left(\frac{1}{2}\right)^{-\frac{1}{2}}+3y^{\frac{1}{5}}\right] = \overbrace{(2^{\frac{3}{2}}+27y^{\frac{3}{5}})}^{A} : \overbrace{\left[\left(\frac{1}{2}\right)^{-\frac{1}{2}}+3y^{\frac{1}{5}}\right]}^{B}$$

$$\begin{aligned}
A &= (2^{\frac{3}{2}} + 27y^{\frac{3}{5}}) = (2^{\frac{1}{2}})^3 + 3^3 \cdot (y^{\frac{1}{5}})^3 \\
&= (2^{\frac{1}{2}})^3 + (3y^{\frac{1}{5}})^3 \\
&= (2^{\frac{1}{2}} + 3y^{\frac{1}{5}})\big((2^{\frac{1}{2}})^2 - 2^{\frac{1}{2}} \cdot 3y^{\frac{1}{5}} + (3y^{\frac{1}{5}})^2\big) \\
&= (2^{\frac{1}{2}} + 3y^{\frac{1}{5}})(2 - 3 \cdot 2^{\frac{1}{2}} y^{\frac{1}{5}} + 9y^{\frac{2}{5}}) \\
&= (\sqrt{2} + 3\sqrt[5]{y})(2 - 3\sqrt{2}\sqrt[5]{y} + 9\sqrt[5]{y^2}) \\
&= (\sqrt{2} + 3\sqrt[5]{y})(2 - 3\sqrt[2\cdot 5]{2^5}\sqrt[5\cdot 2]{y^2} + 9\sqrt[5]{y^2}) \\
&= (\sqrt{2} + 3\sqrt[5]{y})(2 - 3\sqrt[10]{32}\sqrt[10]{y^2} + 9\sqrt[5]{y^2})
\end{aligned}$$

$$\boxed{A = (\sqrt{2} + 3\sqrt[5]{y})(2 - 3\sqrt[10]{32y^2} + 9\sqrt[5]{y^2})}$$

$$\begin{aligned}
B &= \left[\left(\frac{1}{2}\right)^{-\frac{1}{2}} + 3y^{\frac{1}{5}}\right] \\
&= \left[(2^{-1})^{-\frac{1}{2}} + 3y^{\frac{1}{5}}\right] \\
&= [2^{\frac{1}{2}} + 3y^{\frac{1}{5}}]
\end{aligned}$$

$$\boxed{B = [\sqrt{2} + 3\sqrt[5]{y}]}$$

reemplazando:

$$\begin{aligned}
(2^{\frac{3}{2}} + 27y^{\frac{3}{5}}) : \left[\left(\frac{1}{2}\right)^{-\frac{1}{2}} + 3y^{\frac{1}{5}}\right] &= (\sqrt{2} + 3\sqrt[5]{y})(2 - 3\sqrt[10]{32y^2} + 9\sqrt[5]{y^2}) : [\sqrt{2} + 3\sqrt[5]{y}] \\
&= (\sqrt{2} + 3\sqrt[5]{y})(2 - 3\sqrt[10]{32y^2} + 9\sqrt[5]{y^2}) \cdot \frac{1}{\sqrt{2} + 3\sqrt[5]{y}} \\
&= \frac{(\sqrt{2} + 3\sqrt[5]{y})(2 - 3\sqrt[10]{32y^2} + 9\sqrt[5]{y^2})}{\sqrt{2} + 3\sqrt[5]{y}} \\
&= \frac{\cancel{(\sqrt{2} + 3\sqrt[5]{y})}(2 - 3\sqrt[10]{32y^2} + 9\sqrt[5]{y^2})}{\cancel{\sqrt{2} + 3\sqrt[5]{y}}} \\
&= \boxed{2 - 3\sqrt[10]{32y^2} + 9\sqrt[5]{y^2}}
\end{aligned}$$

89. Comprobar la identidad: $a^{\frac{1}{2}} - \dfrac{a - a^{-2}}{a^{\frac{1}{2}} - a^{-\frac{1}{2}}} + \dfrac{1 - a^{-2}}{a^{\frac{1}{2}} + a^{-\frac{1}{2}}} + \dfrac{2}{a^{\frac{3}{2}}} = 0$

Solución.

$$a^{\frac{1}{2}} - \overbrace{\frac{a - a^{-2}}{a^{\frac{1}{2}} - a^{-\frac{1}{2}}}}^{A} + \overbrace{\frac{1 - a^{-2}}{a^{\frac{1}{2}} + a^{-\frac{1}{2}}}}^{B} + \frac{2}{a^{\frac{3}{2}}} = 0$$

$$\begin{aligned}
A &= \frac{a - a^{-2}}{a^{\frac{1}{2}} - a^{-\frac{1}{2}}} \\
&= \frac{a - \frac{1}{a^2}}{a^{\frac{1}{2}} - \frac{1}{a^{\frac{1}{2}}}} \\
&= \frac{\frac{a^3 - 1}{a^2}}{\frac{(a^{\frac{1}{2}})^2 - 1}{a^{\frac{1}{2}}}}
\end{aligned}$$

$$= \frac{\dfrac{(a-1)(a^2+a+1)}{a^2}}{\dfrac{a-1}{a^{\frac{1}{2}}}}$$

$$= \frac{\dfrac{(a-1)(a^2+a+1)}{a^2}}{\dfrac{a-1}{a^{\frac{1}{2}}}}$$

$$= \frac{(a-1)(a^2+a+1)a^{\frac{1}{2}}}{a^2(a-1)}$$

$$= \frac{\cancel{(a-1)}(a^2+a+1)a^{\frac{1}{2}}}{a^2\cancel{(a-1)}}$$

$$= \frac{(a^2+a+1)a^{\frac{1}{2}}}{a^2}$$

$$= \frac{a^2+a+1}{a^2 \cdot a^{-\frac{1}{2}}}$$

$$\boxed{A = \frac{a^2+a+1}{a^{\frac{3}{2}}}}$$

$$B = \frac{1-a^{-2}}{a^{\frac{1}{2}}+a^{-\frac{1}{2}}}$$

$$= \frac{1-\dfrac{1}{a^2}}{a^{\frac{1}{2}}+\dfrac{1}{a^{\frac{1}{2}}}}$$

$$= \frac{\dfrac{a^2-1}{a^2}}{\dfrac{(a^{\frac{1}{2}})^2+1}{a^{\frac{1}{2}}}}$$

$$= \frac{\dfrac{(a-1)(a+1)}{a^2}}{\dfrac{a+1}{a^{\frac{1}{2}}}}$$

$$= \frac{(a-1)(a+1)a^{\frac{1}{2}}}{a^2(a+1)}$$

$$= \frac{(a-1)\cancel{(a+1)}a^{\frac{1}{2}}}{a^2\cancel{(a+1)}}$$

$$= \frac{(a-1)a^{\frac{1}{2}}}{a^2}$$

$$= \frac{a-1}{a^2 \cdot a^{-\frac{1}{2}}}$$

$$\boxed{B = \frac{a-1}{a^{\frac{3}{2}}}}$$

reemplazando:

$$\begin{aligned}
a^{\frac{1}{2}} - \frac{a-a^{-2}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}} + \frac{1-a^{-2}}{a^{\frac{1}{2}}+a^{-\frac{1}{2}}} + \frac{2}{a^{\frac{3}{2}}} &= 0\\
a^{\frac{1}{2}} - \frac{a^2+a+1}{a^{\frac{3}{2}}} + \frac{a-1}{a^{\frac{3}{2}}} + \frac{2}{a^{\frac{3}{2}}} &= 0\\
\frac{a^{\frac{3}{2}}\cdot a^{\frac{1}{2}} - (a^2+a+1)+(a-1)+2}{a^{\frac{3}{2}}} &= 0\\
a^{\frac{3}{2}}\cdot a^{\frac{1}{2}} - (a^2+a+1)+(a-1)+2 &= 0\\
a^2-a^2-a-1+a-1+2 &= 0\\
0 &= 0
\end{aligned}$$

90. Calcular $\dfrac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{(a^2-ab)^{\frac{2}{3}}} : \dfrac{a^{-\frac{2}{3}}\sqrt[3]{a-b}}{a\sqrt{a}-b\sqrt{b}}$ para $a=1,2$ y $b=\dfrac{3}{5}$

Solución.

$$\frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{(a^2-ab)^{\frac{2}{3}}} : \frac{a^{-\frac{2}{3}}\sqrt[3]{a-b}}{a\sqrt{a}-b\sqrt{b}} = \overbrace{\frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{(a^2-ab)^{\frac{2}{3}}}}^{A} : \overbrace{\frac{a^{-\frac{2}{3}}\sqrt[3]{a-b}}{a\sqrt{a}-b\sqrt{b}}}^{B}$$

$$\begin{aligned}
A &= \frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{(a^2-ab)^{\frac{2}{3}}}\\
&= \frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{\big[a(a-b)\big]^{\frac{2}{3}}}
\end{aligned}$$

$$\boxed{A = \frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{a^{\frac{2}{3}}(a-b)^{\frac{2}{3}}}}$$

$$\begin{aligned}
B &= \frac{a^{-\frac{2}{3}}\sqrt[3]{a-b}}{a\sqrt{a}-b\sqrt{b}}\\
&= \frac{(a-b)^{\frac{1}{3}}}{a^{\frac{2}{3}}(a\cdot a^{\frac{1}{2}}-b\cdot b^{\frac{1}{2}})}
\end{aligned}$$

$$\boxed{B = \frac{(a-b)^{\frac{1}{3}}}{a^{\frac{2}{3}}(a^{\frac{3}{2}}-b^{\frac{3}{2}})}}$$

reemplazando:

$$\begin{aligned}
\frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{(a^2-ab)^{\frac{2}{3}}} : \frac{a^{-\frac{2}{3}}\sqrt[3]{a-b}}{a\sqrt{a}-b\sqrt{b}} &= \frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{a^{\frac{2}{3}}(a-b)^{\frac{2}{3}}} : \frac{(a-b)^{\frac{1}{3}}}{a^{\frac{2}{3}}(a^{\frac{3}{2}}-b^{\frac{3}{2}})}\\
&= \frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{a^{\frac{2}{3}}(a-b)^{\frac{2}{3}}} \cdot \frac{a^{\frac{2}{3}}(a^{\frac{3}{2}}-b^{\frac{3}{2}})}{(a-b)^{\frac{1}{3}}}\\
&= \frac{a^{\frac{2}{3}}(a^{\frac{3}{2}}+b^{\frac{3}{2}})(a^{\frac{3}{2}}-b^{\frac{3}{2}})}{a^{\frac{2}{3}}(a-b)^{\frac{2}{3}}(a-b)^{\frac{1}{3}}}\\
&= \frac{\cancel{a^{\frac{2}{3}}}\big((a^{\frac{3}{2}})^2-(b^{\frac{3}{2}})^2\big)}{\cancel{a^{\frac{2}{3}}}(a-b)}\\
&= \frac{a^3-b^3}{a-b}\\
&= \frac{(a-b)(a^2+ab+b^2)}{a-b}
\end{aligned}$$

$$= \frac{\cancel{(a-b)}(a^2+ab+b^2)}{\cancel{a-b}}$$

$$= \boxed{a^2+ab+b^2}$$

Simplificar las siguientes espresiones:

91. $\left[(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}+5b^{\frac{1}{2}})-(a^{\frac{1}{2}}+2b^{\frac{1}{2}})(a^{\frac{1}{2}}-2b^{\frac{1}{2}})\right]:(2a+3a^{\frac{1}{2}}b^{\frac{1}{2}})$. Evaluar el resultado para $a=54$ y $b=6$

Solución.

$\left[(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}+5b^{\frac{1}{2}})-(a^{\frac{1}{2}}+2b^{\frac{1}{2}})(a^{\frac{1}{2}}-2b^{\frac{1}{2}})\right]:(2a+3a^{\frac{1}{2}}b^{\frac{1}{2}})=$

$$=\overbrace{\left[(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}+5b^{\frac{1}{2}})-(a^{\frac{1}{2}}+2b^{\frac{1}{2}})(a^{\frac{1}{2}}-2b^{\frac{1}{2}})\right]}^{A}:\overbrace{(2a+3a^{\frac{1}{2}}b^{\frac{1}{2}})}^{B}$$

$$\begin{aligned}
A &= \left[(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}+5b^{\frac{1}{2}})-(a^{\frac{1}{2}}+2b^{\frac{1}{2}})(a^{\frac{1}{2}}-2b^{\frac{1}{2}})\right]\\
&= \left[\left((a^{\frac{1}{2}})^2+5a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{2}}b^{\frac{1}{2}}+5(b^{\frac{1}{2}})^2\right)-\left((a^{\frac{1}{2}})^2-(2b^{\frac{1}{2}})^2\right)\right]\\
&= \left[(a+6a^{\frac{1}{2}}b^{\frac{1}{2}}+5b)-(a-4b)\right]\\
&= [a+6a^{\frac{1}{2}}b^{\frac{1}{2}}+5b-a+4b]\\
&= [6a^{\frac{1}{2}}b^{\frac{1}{2}}+9b]\\
&= [6a^{\frac{1}{2}}b^{\frac{1}{2}}+9b^{\frac{2}{2}}]\\
&= [6a^{\frac{1}{2}}b^{\frac{1}{2}}+9(b^{\frac{1}{2}})^2]\\
&= [6a^{\frac{1}{2}}b^{\frac{1}{2}}+9(b^{\frac{1}{2}})^2]
\end{aligned}$$

$$\boxed{A=3b^{\frac{1}{2}}[2a^{\frac{1}{2}}+3b^{\frac{1}{2}}]}$$

$$\begin{aligned}
B &= (2a+3a^{\frac{1}{2}}b^{\frac{1}{2}})\\
&= (2a^{\frac{2}{2}}+3a^{\frac{1}{2}}b^{\frac{1}{2}})\\
&= \left(2(a^{\frac{1}{2}})^2+3a^{\frac{1}{2}}b^{\frac{1}{2}}\right)
\end{aligned}$$

$$\boxed{B=a^{\frac{1}{2}}(2a^{\frac{1}{2}}+3b^{\frac{1}{2}})}$$

reemplazando:

$$\begin{aligned}
[(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}+5b^{\frac{1}{2}})-(a^{\frac{1}{2}}+2b^{\frac{1}{2}})(a^{\frac{1}{2}}-2b^{\frac{1}{2}})]:(2a+3a^{\frac{1}{2}}b^{\frac{1}{2}}) &= 3b^{\frac{1}{2}}[2a^{\frac{1}{2}}+3b^{\frac{1}{2}}]:a^{\frac{1}{2}}(a^{\frac{1}{2}}+3b^{\frac{1}{2}})\\
&= 3b^{\frac{1}{2}}[2a^{\frac{1}{2}}+3b^{\frac{1}{2}}]\cdot\frac{1}{a^{\frac{1}{2}}(2a^{\frac{1}{2}}+3b^{\frac{1}{2}})}\\
&= \frac{3b^{\frac{1}{2}}[2a^{\frac{1}{2}}+3b^{\frac{1}{2}}]}{a^{\frac{1}{2}}(2a^{\frac{1}{2}}+3b^{\frac{1}{2}})}\\
&= \frac{3b^{\frac{1}{2}}\cancel{[2a^{\frac{1}{2}}+3b^{\frac{1}{2}}]}}{a^{\frac{1}{2}}\cancel{(2a^{\frac{1}{2}}+3b^{\frac{1}{2}})}}\\
&= \frac{3b^{\frac{1}{2}}}{a^{\frac{1}{2}}}\\
&= 3\frac{b^{\frac{1}{2}}}{a^{\frac{1}{2}}}\\
&= 3\frac{\sqrt{b}}{\sqrt{a}}
\end{aligned}$$

$$= \boxed{3\sqrt{\frac{a}{b}}}$$

sustituyendo: $a = 54$ y $b = 6$

$$3\sqrt{\frac{a}{b}} = 3\sqrt{\frac{6}{54}} = 3\sqrt{\frac{1}{9}} = 3\sqrt{\left(\frac{1}{3}\right)^2} = 3 \cdot \frac{1}{3} = \frac{3}{3} = \boxed{1}$$

92. $\dfrac{[(a+b)^{-\frac{1}{2}} + (a-b)^{-\frac{1}{2}}]^{-1} + [(a+b)^{-\frac{1}{2}} - (a-b)^{-\frac{1}{2}}]^{-1}}{[(a+b)^{-\frac{1}{2}} + (a-b)^{-\frac{1}{2}}]^{-1} - [(a+b)^{-\frac{1}{2}} - (a-b)^{-\frac{1}{2}}]^{-1}}$

Solución.

$$\begin{aligned}
[(a+b)^{-\frac{1}{2}} + (a-b)^{-\frac{1}{2}}]^{-1} &= \left[\frac{1}{(a+b)^{\frac{1}{2}}} + \frac{1}{(a-b)^{\frac{1}{2}}}\right]^{-1} \\
&= \left[\frac{1}{\sqrt{a+b}} + \frac{1}{\sqrt{a-b}}\right]^{-1} \\
&= \left[\frac{\sqrt{a-b} + \sqrt{a+b}}{\sqrt{a+b}\sqrt{a-b}}\right]^{-1} \\
&= \frac{1}{\dfrac{\sqrt{a-b} + \sqrt{a+b}}{\sqrt{a+b}\sqrt{a-b}}} \\
&= \boxed{\frac{\sqrt{a+b}\sqrt{a-b}}{\sqrt{a-b} + \sqrt{a+b}}}
\end{aligned}$$

$$\begin{aligned}
[(a+b)^{-\frac{1}{2}} - (a-b)^{-\frac{1}{2}}]^{-1} &= \left[\frac{1}{(a+b)^{\frac{1}{2}}} - \frac{1}{(a-b)^{\frac{1}{2}}}\right]^{-1} \\
&= \left[\frac{1}{\sqrt{a+b}} - \frac{1}{\sqrt{a-b}}\right]^{-1} \\
&= \left[\frac{\sqrt{a-b} - \sqrt{a+b}}{\sqrt{a+b}\sqrt{a-b}}\right]^{-1} \\
&= \frac{1}{\dfrac{\sqrt{a+b} - \sqrt{a-b}}{\sqrt{a+b}\sqrt{a-b}}} \\
&= \boxed{\frac{\sqrt{a+b}\sqrt{a-b}}{\sqrt{a+b} - \sqrt{a-b}}}
\end{aligned}$$

reemplazando:

$$\begin{aligned}
\frac{[(a+b)^{-\frac{1}{2}} + (a-b)^{-\frac{1}{2}}]^{-1} + [(a+b)^{-\frac{1}{2}} - (a-b)^{-\frac{1}{2}}]^{-1}}{[(a+b)^{-\frac{1}{2}} + (a-b)^{-\frac{1}{2}}]^{-1} - [(a+b)^{-\frac{1}{2}} - (a-b)^{-\frac{1}{2}}]^{-1}} &= \frac{\dfrac{\sqrt{a+b}\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}} + \dfrac{\sqrt{a+b}\sqrt{a-b}}{\sqrt{a+b}-\sqrt{a-b}}}{\dfrac{\sqrt{a+b}\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}} - \dfrac{\sqrt{a+b}\sqrt{a-b}}{\sqrt{a+b}-\sqrt{a-b}}} \\
&= \frac{\sqrt{a+b}\sqrt{a-b}\left(\dfrac{1}{\sqrt{a+b}+\sqrt{a-b}} + \dfrac{1}{\sqrt{a+b}-\sqrt{a-b}}\right)}{\sqrt{a+b}\sqrt{a-b}\left(\dfrac{1}{\sqrt{a+b}+\sqrt{a-b}} - \dfrac{1}{\sqrt{a+b}-\sqrt{a-b}}\right)} \\
&= \frac{\cancel{\sqrt{a+b}\sqrt{a-b}}\left(\dfrac{1}{\sqrt{a+b}+\sqrt{a-b}} + \dfrac{1}{\sqrt{a+b}-\sqrt{a-b}}\right)}{\cancel{\sqrt{a+b}\sqrt{a-b}}\left(\dfrac{1}{\sqrt{a+b}+\sqrt{a-b}} - \dfrac{1}{\sqrt{a+b}-\sqrt{a-b}}\right)}
\end{aligned}$$

$$= \frac{\dfrac{1}{\sqrt{a+b}+\sqrt{a-b}} + \dfrac{1}{\sqrt{a+b}-\sqrt{a-b}}}{\dfrac{1}{\sqrt{a+b}+\sqrt{a-b}} - \dfrac{1}{\sqrt{a+b}-\sqrt{a-b}}}$$

$$= \frac{\dfrac{\sqrt{a+b}-\sqrt{a-b}+\sqrt{a+b}+\sqrt{a-b}}{(\sqrt{a+b}+\sqrt{a-b})(\sqrt{a+b}-\sqrt{a-b})}}{\dfrac{\sqrt{a+b}-\sqrt{a-b}-\sqrt{a+b}-\sqrt{a-b}}{(\sqrt{a+b}+\sqrt{a-b})(\sqrt{a+b}-\sqrt{a-b})}}$$

$$= \frac{\dfrac{2\sqrt{a+b}}{(\sqrt{a+b})^2-(\sqrt{a-b})^2}}{\dfrac{-2\sqrt{a-b}}{(\sqrt{a+b})^2-(\sqrt{a-b})^2}}$$

$$= \frac{\dfrac{2\sqrt{a+b}}{a+b-a+b}}{\dfrac{-2\sqrt{a-b}}{a+b-a+b}}$$

$$= \frac{\dfrac{2\sqrt{a+b}}{2b}}{\dfrac{-2\sqrt{a-b}}{2b}}$$

$$= \frac{\dfrac{\sqrt{a+b}}{b}}{\dfrac{-\sqrt{a-b}}{b}}$$

$$= \frac{b\sqrt{a+b}}{-b\sqrt{a-b}}$$

$$= -\frac{\not{b}\sqrt{a+b}}{\not{b}\sqrt{a-b}}$$

$$= -\frac{\sqrt{a+b}}{\sqrt{a-b}}$$

$$= \boxed{-\sqrt{\frac{a+b}{a-b}}}$$

93. $a^2(1-a^2)^{-\frac{1}{2}} - \dfrac{1}{1+[a(1-a^2)^{-\frac{1}{2}}]^2} \cdot \dfrac{(1-a^2)^{\frac{1}{2}}+a^2(1-a^2)^{-\frac{1}{2}}}{1-a^2}$

Solución.

$$a^2(1-a^2)^{-\frac{1}{2}} - \frac{1}{1+[a(1-a^2)^{-\frac{1}{2}}]^2} \cdot \frac{(1-a^2)^{\frac{1}{2}}+a^2(1-a^2)^{-\frac{1}{2}}}{1-a^2} =$$

$$= \overbrace{a^2(1-a^2)^{-\frac{1}{2}}}^{A} - \overbrace{\frac{1}{1+[a(1-a^2)^{-\frac{1}{2}}]^2} \cdot \frac{(1-a^2)^{\frac{1}{2}}+a^2(1-a^2)^{-\frac{1}{2}}}{1-a^2}}^{B}$$

$$A = a^2(1-a^2)^{-\frac{1}{2}}$$

$$= \frac{a^2}{(1-a^2)^{\frac{1}{2}}}$$

$$\boxed{A = \frac{a^2}{\sqrt{1-a^2}}}$$

$$
\begin{aligned}
B &= \frac{1}{1+[a(1-a^2)^{-\frac{1}{2}}]^2} \cdot \frac{(1-a^2)^{\frac{1}{2}} + a^2(1-a^2)^{-\frac{1}{2}}}{1-a^2} \\
&= \frac{1}{1+a^2(1-a^2)^{-\frac{2}{2}}} \cdot \frac{(1-a^2)^{\frac{1}{2}} + \dfrac{a^2}{(1-a^2)^{\frac{1}{2}}}}{1-a^2} \\
&= \frac{1}{1+a^2(1-a^2)^{-1}} \cdot \frac{\sqrt{1-a^2} + \dfrac{a^2}{\sqrt{1-a^2}}}{1-a^2} \\
&= \frac{1}{1+\dfrac{a^2}{1-a^2}} \cdot \frac{\dfrac{(\sqrt{1-a^2})^2 + a^2}{\sqrt{1-a^2}}}{1-a^2} \\
&= \frac{1}{\dfrac{1-a^2+a^2}{1-a^2}} \cdot \frac{\dfrac{1-a^2+a^2}{(1-a^2)^{\frac{1}{2}}}}{1-a^2} \\
&= \frac{1}{\dfrac{1}{1-a^2}} \cdot \frac{\dfrac{1}{(1-a^2)^{\frac{1}{2}}}}{1-a^2} \\
&= \frac{1-a^2}{1} \cdot \frac{1}{(1-a^2)^{\frac{1}{2}}(1-a^2)} \\
&= \frac{1-a^2}{(1-a^2)^{\frac{1}{2}}(1-a^2)} \\
&= \frac{\cancel{1-a^2}}{(1-a^2)^{\frac{1}{2}}\cancel{(1-a^2)}} \\
&= \frac{1}{(1-a^2)^{\frac{1}{2}}}
\end{aligned}
$$

$$\boxed{B = \frac{1}{\sqrt{1-a^2}}}$$

reemplazando:

$$
\begin{aligned}
a^2(1-a^2)^{-\frac{1}{2}} - \frac{1}{1+[a(1-a^2)^{-\frac{1}{2}}]^2} \cdot \frac{(1-a^2)^{\frac{1}{2}} + a^2(1-a^2)^{-\frac{1}{2}}}{1-a^2} &= \frac{a^2}{\sqrt{1-a^2}} - \frac{1}{\sqrt{1-a^2}} \\
&= \frac{a^2-1}{\sqrt{1-a^2}} \\
&= \frac{-1+a^2}{(1-a^2)^{\frac{1}{2}}} \\
&= -(1-a^2)(1-a^2)^{-\frac{1}{2}} \\
&= -(1-a^2)^{\frac{1}{2}} \\
&= \boxed{-\sqrt{1-a^2}}
\end{aligned}
$$

94. $\dfrac{x^{\frac{5}{2}}-x^{-\frac{1}{2}}}{(x+1)(x^2+1)}-\left(x+\dfrac{x^3}{1+x^2}\right)^{-\frac{1}{2}}\cdot\dfrac{x^2\sqrt{(1+x^2)^{-1}}-\sqrt{1+x^2}}{1+x^2}$

Solución

$$\frac{x^{\frac{5}{2}}-x^{-\frac{1}{2}}}{(x+1)(x^2+1)}-\left(x+\frac{x^3}{1+x^2}\right)^{-\frac{1}{2}}\cdot\frac{x^2\sqrt{(1+x^2)^{-1}}-\sqrt{1+x^2}}{1+x^2}=$$

$$=\overbrace{\frac{x^{\frac{5}{2}}-x^{-\frac{1}{2}}}{(x+1)(x^2+1)}}^{A}-\overbrace{\left(x+\frac{x^3}{1+x^2}\right)^{-\frac{1}{2}}\cdot\frac{x^2\sqrt{(1+x^2)^{-1}}-\sqrt{1+x^2}}{1+x^2}}^{B}$$

$$A=\frac{x^{\frac{5}{2}}-x^{-\frac{1}{2}}}{(x+1)(x^2+1)}$$

$$=\frac{x^{\frac{5}{2}}-\dfrac{1}{x^{\frac{1}{2}}}}{(x+1)(x^2+1)}$$

$$=\frac{\dfrac{x^{\frac{5}{2}}\cdot x^{\frac{1}{2}}-1}{x^{\frac{1}{2}}}}{(x+1)(x^2+1)}$$

$$=\frac{\dfrac{x^3-1}{\sqrt{x}}}{(x+1)(x^2+1)}$$

$$\boxed{A=\frac{x^3-1}{\sqrt{x}(x+1)(x^2+1)}}$$

$$B=\left(x+\frac{x^3}{1+x^2}\right)^{-\frac{1}{2}}\cdot\frac{x^2\sqrt{(1+x^2)^{-1}}-\sqrt{1+x^2}}{1+x^2}$$

$$=\left(\frac{x(1+x^2)-x^3}{1+x^2}\right)^{-\frac{1}{2}}\cdot\frac{x^2\sqrt{\dfrac{1}{1+x^2}}-\sqrt{1+x^2}}{1+x^2}$$

$$=\left(\frac{x+x^3-x^3}{1+x^2}\right)^{-\frac{1}{2}}\cdot\frac{x^2\cdot\dfrac{1}{\sqrt{1+x^2}}-\sqrt{1+x^2}}{1+x^2}$$

$$=\left(\frac{x}{1+x^2}\right)^{-\frac{1}{2}}\cdot\frac{\dfrac{x^2}{\sqrt{1+x^2}}-\sqrt{1+x^2}}{1+x^2}$$

$$=\frac{1}{\left(\dfrac{x}{1+x^2}\right)^{\frac{1}{2}}}\cdot\frac{\dfrac{x^2-(\sqrt{1+x^2})^2}{\sqrt{1+x^2}}}{1+x^2}$$

$$=\frac{1}{\sqrt{\dfrac{x}{1+x^2}}}\cdot\frac{\dfrac{x^2-(1+x^2)}{\sqrt{1+x^2}}}{1+x^2}$$

$$=\frac{1}{\dfrac{\sqrt{x}}{\sqrt{1+x^2}}}\cdot\frac{\dfrac{x^2-1-x^2}{\sqrt{1+x^2}}}{1+x^2}$$

$$= \frac{\sqrt{1+x^2}}{\sqrt{x}} \cdot \frac{\frac{-1}{\sqrt{1+x^2}}}{1+x^2}$$

$$= \frac{\sqrt{1+x^2}}{\sqrt{x}} \cdot \frac{-1}{\sqrt{1+x^2}(1+x^2)}$$

$$= \frac{-\sqrt{1+x^2}}{\sqrt{x}\sqrt{1+x^2}(1+x^2)}$$

$$= \frac{-\cancel{\sqrt{1+x^2}}}{\sqrt{x}\cancel{\sqrt{1+x^2}}(1+x^2)}$$

$$\boxed{B = \frac{-1}{\sqrt{x}(1+x^2)}}$$

reemplazando:

$$\frac{x^{\frac{5}{2}} - x^{-\frac{1}{2}}}{(x+1)(x^2+1)} - \left(x + \frac{x^3}{1+x^2}\right)^{-\frac{1}{2}} \cdot \frac{x^2\sqrt{(1+x^2)^{-1}} - \sqrt{1+x^2}}{1+x^2} = \frac{x^3-1}{\sqrt{x}(x+1)(x^2+1)} - \frac{-1}{\sqrt{x}(1+x^2)}$$

$$= \frac{x^3-1}{\sqrt{x}(x+1)(x^2+1)} + \frac{1}{\sqrt{x}(x^2+1)}$$

$$= \frac{x^3-1+x+1}{\sqrt{x}(x+1)(x^2+1)}$$

$$= \frac{x^3+x}{\sqrt{x}(x+1)(x^2+1)}$$

$$= \frac{x(x^2+1)}{x^{\frac{1}{2}}(x+1)(x^2+1)}$$

$$= \frac{x\cancel{(x^2+1)}}{x^{\frac{1}{2}}(x+1)\cancel{(x^2+1)}}$$

$$= \frac{x}{x^{\frac{1}{2}}(x+1)}$$

$$= \frac{x \cdot x^{-\frac{1}{2}}}{x+1}$$

$$= \frac{x^{\frac{1}{2}}}{x+1}$$

$$= \boxed{\frac{\sqrt{x}}{x+1}}$$

95. $$(r^2-x^2)^{\frac{1}{2}} - x^2(r^2-x^2)^{-\frac{1}{2}} + r^2 \cdot \frac{(r^2-x^2)^{\frac{1}{2}} + x^2(r^2-x^2)^{-\frac{1}{2}}}{(r^2-x^2)\left[1 + \left(\frac{\sqrt{r^2-x^2}}{x}\right)^{-2}\right]}$$

Solución.

$$(r^2-x^2)^{\frac{1}{2}} - x^2(r^2-x^2)^{-\frac{1}{2}} + r^2 \cdot \frac{(r^2-x^2)^{\frac{1}{2}} + x^2(r^2-x^2)^{-\frac{1}{2}}}{(r^2-x^2)\left[1 + \left(\frac{\sqrt{r^2-x^2}}{x}\right)^{-2}\right]} =$$

$$= \overbrace{(r^2 - x^2)^{\frac{1}{2}} - x^2(r^2 - x^2)^{-\frac{1}{2}}}^{A} + \overbrace{r^2 \cdot \frac{(r^2 - x^2)^{\frac{1}{2}} + x^2(r^2 - x^2)^{-\frac{1}{2}}}{(r^2 - x^2)\left[1 + \left(\dfrac{\sqrt{r^2 - x^2}}{x}\right)^{-2}\right]}}^{B}$$

$$A = (r^2 - x^2)^{\frac{1}{2}} - x^2(r^2 - x^2)^{-\frac{1}{2}}$$

$$= (r^2 - x^2)^{\frac{1}{2}} - \frac{x^2}{(r^2 - x^2)^{\frac{1}{2}}}$$

$$= \sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}$$

$$= \frac{(\sqrt{r^2 - x^2})^2 - x^2}{\sqrt{r^2 - x^2}}$$

$$= \frac{r^2 - x^2 - x^2}{\sqrt{r^2 - x^2}}$$

$$\boxed{A = \frac{r^2 - 2x^2}{\sqrt{r^2 - x^2}}}$$

$$B = r^2 \cdot \frac{(r^2 - x^2)^{\frac{1}{2}} + x^2(r^2 - x^2)^{-\frac{1}{2}}}{(r^2 - x^2)\left[1 + \left(\dfrac{\sqrt{r^2 - x^2}}{x}\right)^{-2}\right]}$$

$$= r^2 \cdot \frac{(r^2 - x^2)^{\frac{1}{2}} + \dfrac{x^2}{(r^2 - x^2)^{\frac{1}{2}}}}{(r^2 - x^2)\left[1 + \dfrac{1}{\left(\dfrac{\sqrt{r^2 - x^2}}{x}\right)^2}\right]}$$

$$= r^2 \cdot \frac{\sqrt{r^2 - x^2} + \dfrac{x^2}{\sqrt{r^2 - x^2}}}{(r^2 - x^2)\left[1 + \dfrac{1}{\dfrac{r^2 - x^2}{x^2}}\right]}$$

$$= r^2 \cdot \frac{\dfrac{(\sqrt{r^2 - x^2})^2 + x^2}{\sqrt{r^2 - x^2}}}{(r^2 - x^2)\left[1 + \dfrac{x^2}{r^2 - x^2}\right]}$$

$$= r^2 \cdot \frac{\dfrac{r^2 - x^2 + x^2}{\sqrt{r^2 - x^2}}}{(r^2 - x^2)\left[\dfrac{r^2 - x^2 + x^2}{r^2 - x^2}\right]}$$

$$= r^2 \cdot \frac{\dfrac{r^2}{\sqrt{r^2 - x^2}}}{\cancel{(r^2 - x^2)}\left[\dfrac{r^2}{\cancel{r^2 - x^2}}\right]}$$

$$= r^2 \cdot \frac{\frac{r^2}{\sqrt{r^2-x^2}}}{r^2}$$

$$= r^2 \cdot \frac{r^2}{r^2\sqrt{r^2-x^2}}$$

$$= r^2 \cdot \frac{\cancel{r^2}}{\cancel{r^2}\sqrt{r^2-x^2}}$$

$$= r^2 \cdot \frac{1}{\sqrt{r^2-x^2}}$$

$$\boxed{B = \frac{r^2}{\sqrt{r^2-x^2}}}$$

reemplazando:

$$(r^2-x^2)^{\frac{1}{2}} - x^2(r^2-x^2)^{-\frac{1}{2}} + r^2 \cdot \frac{(r^2-x^2)^{\frac{1}{2}} + x^2(r^2-x^2)^{-\frac{1}{2}}}{(r^2-x^2)\left[1+\left(\frac{\sqrt{r^2-x^2}}{x}\right)^{-2}\right]} = \frac{r^2-2x^2}{\sqrt{r^2-x^2}} + \frac{r^2}{\sqrt{r^2-x^2}}$$

$$= \frac{r^2-2x^2+r^2}{\sqrt{r^2-x^2}}$$

$$= \frac{2r^2-2x^2}{(r^2-x^2)^{\frac{1}{2}}}$$

$$= 2(r^2-x^2)(r^2-x^2)^{-\frac{1}{2}}$$

$$= 2(r^2-x^2)^{\frac{1}{2}}$$

$$= \boxed{2\sqrt{r^2-x^2}}$$

96. $(p^{\frac{1}{2}}+q^{\frac{1}{2}})^{-2}(p^{-1}+q^{-1}) + \frac{2}{(p^{\frac{1}{2}}+q^{\frac{1}{2}})^3}(p^{-\frac{1}{2}}+q^{-\frac{1}{2}})$

Solución.

$$(p^{\frac{1}{2}}+q^{\frac{1}{2}})^{-2}(p^{-1}+q^{-1}) + \frac{2}{(p^{\frac{1}{2}}+q^{\frac{1}{2}})^3}(p^{-\frac{1}{2}}+q^{-\frac{1}{2}}) = \overbrace{(p^{\frac{1}{2}}+q^{\frac{1}{2}})^{-2}(p^{-1}+q^{-1})}^{A} + \overbrace{\frac{2}{(p^{\frac{1}{2}}+q^{\frac{1}{2}})^3}(p^{-\frac{1}{2}}+q^{-\frac{1}{2}})}^{B}$$

$$A = (p^{\frac{1}{2}}+q^{\frac{1}{2}})^{-2}(p^{-1}+q^{-1})$$

$$= \frac{1}{(p^{\frac{1}{2}}+q^{\frac{1}{2}})^2}\left(\frac{1}{p}+\frac{1}{q}\right)$$

$$= \frac{1}{(\sqrt{p}+\sqrt{q})^2}\left(\frac{q+p}{pq}\right)$$

$$= \frac{q+p}{pq(\sqrt{p}+\sqrt{q})^2}$$

$$\boxed{A = \frac{p+q}{pq(\sqrt{p}+\sqrt{q})^2}}$$

$$B = \frac{2}{(p^{\frac{1}{2}}+q^{\frac{1}{2}})^3}(p^{-\frac{1}{2}}+q^{-\frac{1}{2}})$$

$$= \frac{2}{(\sqrt{p}+\sqrt{q})^3}\left(\frac{1}{p^{\frac{1}{2}}}+\frac{1}{q^{\frac{1}{2}}}\right)$$

$$= \frac{2}{(\sqrt{p}+\sqrt{q})^3}\left(\frac{1}{\sqrt{p}}+\frac{1}{\sqrt{q}}\right)$$

$$= \frac{2}{(\sqrt{p}+\sqrt{q})^3}\left(\frac{\sqrt{q}+\sqrt{p}}{\sqrt{p}\sqrt{q}}\right)$$

$$= \frac{2(\sqrt{q}+\sqrt{p})}{\sqrt{p}\sqrt{q}(\sqrt{p}+\sqrt{q})^3}$$

$$= \frac{2(\sqrt{p}+\sqrt{q})}{\sqrt{pq}(\sqrt{p}+\sqrt{q})^3}$$

$$= \frac{2\cancel{(\sqrt{p}+\sqrt{q})}}{\sqrt{pq}(\sqrt{p}+\sqrt{q})^{\cancel{3}}}$$

$$\boxed{B = \frac{2}{\sqrt{pq}(\sqrt{p}+\sqrt{q})^2}}$$

reemplazando:

$$(p^{\frac{1}{2}}+q^{\frac{1}{2}})^{-2}(p^{-1}+q^{-1}) + \frac{2}{(p^{\frac{1}{2}}+q^{\frac{1}{2}})^3}(p^{-\frac{1}{2}}+q^{-\frac{1}{2}}) = \frac{p+q}{pq(\sqrt{p}+\sqrt{q})^2} + \frac{2}{\sqrt{pq}(\sqrt{p}+\sqrt{q})^2}$$

$$= \frac{p+q+2\sqrt{pq}}{pq(\sqrt{p}+\sqrt{q})^2}$$

$$= \frac{p+2\sqrt{pq}+q}{pq(\sqrt{p}+\sqrt{q})^2}$$

$$= \frac{\sqrt{p^2}+2\sqrt{p}\sqrt{q}+\sqrt{q^2}}{pq(\sqrt{p}+\sqrt{q})^2}$$

$$= \frac{(\sqrt{p}+\sqrt{q})^2}{pq(\sqrt{p}+\sqrt{q})^2}$$

$$= \frac{\cancel{(\sqrt{p}+\sqrt{q})^2}}{pq\cancel{(\sqrt{p}+\sqrt{q})^2}}$$

$$= \boxed{\frac{1}{pq}}$$

97. $\left[\dfrac{(a+\sqrt[3]{a^2x}):(x+\sqrt[3]{ax^2})-1}{\sqrt[3]{a}-\sqrt[3]{x}} - \dfrac{1}{\sqrt[3]{x}}\right]^6$

Solución.

$$\left[\frac{(a+\sqrt[3]{a^2x}):(x+\sqrt[3]{ax^2})-1}{\sqrt[3]{a}-\sqrt[3]{x}} - \frac{1}{\sqrt[3]{x}}\right]^6 = \left[\overbrace{\frac{(a+\sqrt[3]{a^2x}):(x+\sqrt[3]{ax^2})-1}{\sqrt[3]{a}-\sqrt[3]{x}}}^{A} - \frac{1}{\sqrt[3]{x}}\right]^6$$

$$A = \frac{(a+\sqrt[3]{a^2x}):(x+\sqrt[3]{ax^2})-1}{\sqrt[3]{a}-\sqrt[3]{x}}$$

$$= \frac{\dfrac{a+\sqrt[3]{a^2x}}{x+\sqrt[3]{ax^2}}-1}{\sqrt[3]{a}-\sqrt[3]{x}}$$

$$= \frac{\dfrac{\sqrt[3]{a^3}+\sqrt[3]{a^2x}}{\sqrt[3]{x^3}+\sqrt[3]{ax^2}}-1}{\sqrt[3]{a}-\sqrt[3]{x}}$$

$$= \frac{\dfrac{\sqrt[3]{a^2}(\sqrt[3]{a}+\sqrt[3]{x})}{\sqrt[3]{x^2}(\sqrt[3]{x}+\sqrt[3]{a})}-1}{\sqrt[3]{a}-\sqrt[3]{x}}$$

$$= \frac{\dfrac{\sqrt[3]{a^2}\cancel{(\sqrt[3]{x}+\sqrt[3]{a})}}{\sqrt[3]{x^2}\cancel{(\sqrt[3]{x}+\sqrt[3]{a})}}-1}{\sqrt[3]{a}-\sqrt[3]{x}}$$

$$= \frac{\dfrac{\sqrt[3]{a^2}}{\sqrt[3]{x^2}}-1}{\sqrt[3]{a}-\sqrt[3]{x}}$$

$$= \frac{\dfrac{\sqrt[3]{a^2}-\sqrt[3]{x^2}}{\sqrt[3]{x^2}}}{\sqrt[3]{a}-\sqrt[3]{x}}$$

$$= \frac{\sqrt[3]{a^2}-\sqrt[3]{x^2}}{\sqrt[3]{x^2}(\sqrt[3]{a}-\sqrt[3]{x})}$$

$$= \frac{(\sqrt[3]{a})^2-(\sqrt[3]{x})^2}{\sqrt[3]{x^2}(\sqrt[3]{a}-\sqrt[3]{x})}$$

$$= \frac{(\sqrt[3]{a}-\sqrt[3]{x})(\sqrt[3]{a}+\sqrt[3]{x})}{\sqrt[3]{x^2}(\sqrt[3]{a}-\sqrt[3]{x})}$$

$$= \frac{\cancel{(\sqrt[3]{a}-\sqrt[3]{x})}(\sqrt[3]{a}+\sqrt[3]{x})}{\sqrt[3]{x^2}\cancel{(\sqrt[3]{a}-\sqrt[3]{x})}q}$$

$$\boxed{B = \frac{\sqrt[3]{a}+\sqrt[3]{x}}{\sqrt[3]{x^2}}}$$

reemplazando:

$$\left[\frac{(a+\sqrt[3]{a^2x}):(x+\sqrt[3]{ax^2})-1}{\sqrt[3]{a}-\sqrt[3]{x}}-\frac{1}{\sqrt[3]{x}}\right]^6 = \left[\frac{\sqrt[3]{a}+\sqrt[3]{x}}{\sqrt[3]{x^2}}-\frac{1}{\sqrt[3]{x}}\right]^6$$

$$= \left[\frac{\sqrt[3]{a}+\sqrt[3]{x}-\sqrt[3]{x}}{\sqrt[3]{x^2}}\right]^6$$

$$= \left[\frac{\sqrt[3]{a}}{\sqrt[3]{x^2}}\right]^6$$

$$= \left[\sqrt[3]{\frac{a}{x^2}}\right]^6$$

$$= \left[\frac{a}{x^2}\right]^2$$

$$= \boxed{\frac{a^2}{x^4}}$$

98. $$\left[\frac{(\sqrt{a}+1)^2-\dfrac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}{(\sqrt{a}+1)^3-a\sqrt{a}+2}\right]^{-3}$$

Solución.

$$\left[\frac{(\sqrt{a}+1)^2-\dfrac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}{(\sqrt{a}+1)^3-a\sqrt{a}+2}\right]^{-3}=\left[\frac{\overbrace{(\sqrt{a}+1)^2-\dfrac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}^{A}}{\underbrace{(\sqrt{a}+1)^3-a\sqrt{a}+2}_{B}}\right]^{-3}$$

$$\begin{aligned}
A &= (\sqrt{a}+1)^2-\frac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}\\
&= \left((\sqrt{a})^2+2\sqrt{a}+1\right)-\frac{\sqrt{a^2}-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}\\
&= (a+2\sqrt{a}+1)-\frac{\sqrt{a}(\sqrt{a}-\sqrt{x})}{\sqrt{a}-\sqrt{x}}\\
&= (a+2\sqrt{a}+1)-\frac{\sqrt{a}\cancel{(\sqrt{a}-\sqrt{x})}}{\cancel{\sqrt{a}-\sqrt{x}}q}\\
&= a+2\sqrt{a}+1-\sqrt{a}\\
&\boxed{A=a+\sqrt{a}+1}\\
B &= (\sqrt{a}+1)^3-a\sqrt{a}+2\\
&= (\sqrt{a})^3+3(\sqrt{a})^2\cdot 1+3\sqrt{a}\cdot 1^2+1^3-a\sqrt{a}+2\\
&= \sqrt{a^3}+3a+3\sqrt{a}+1-a\sqrt{a}+2\\
&= a\sqrt{a}+3a+3\sqrt{a}-a\sqrt{a}+3\\
&= 3a+3\sqrt{a}+3\\
&\boxed{B=3(a+\sqrt{a}+1)}
\end{aligned}$$

reemplazando:

$$\begin{aligned}
\left[\frac{(\sqrt{a}+1)^2-\dfrac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}{(\sqrt{a}+1)^3-a\sqrt{a}+2}\right]^{-3} &= \left[\frac{a+\sqrt{a}+1}{3(a+\sqrt{a}+1)}\right]^{-3}\\
&= \left[\frac{\cancel{a+\sqrt{a}+1}}{3\cancel{(a+\sqrt{a}+1)}}\right]^{-3}\\
&= \left[\frac{1}{3}\right]^{-3}\\
&= [3^{-1}]^{-3}\\
&= 3^3\\
&= \boxed{27}
\end{aligned}$$

99. $\left[\dfrac{4a-9a^{-1}}{2a^{\frac{1}{2}}-3a^{-\frac{1}{2}}}+\dfrac{a-4+3a^{-1}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}}\right]^2$

Solución.

$$\left[\frac{4a-9a^{-1}}{2a^{\frac{1}{2}}-3a^{-\frac{1}{2}}}+\frac{a-4+3a^{-1}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}}\right]^2=\left[\overbrace{\frac{4a-9a^{-1}}{2a^{\frac{1}{2}}-3a^{-\frac{1}{2}}}}^{A}+\underbrace{\frac{a-4+3a^{-1}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}}}_{B}\right]^2$$

$$A = \frac{4a - 9a^{-1}}{2a^{\frac{1}{2}} - 3a^{-\frac{1}{2}}}$$

$$= \frac{4a - \dfrac{9}{a}}{2a^{\frac{1}{2}} - \dfrac{3}{a^{\frac{1}{2}}}}$$

$$= \frac{\dfrac{4a^2 - 9}{a}}{\dfrac{2(a^{\frac{1}{2}})^2 - 3}{a^{\frac{1}{2}}}}$$

$$= \frac{\dfrac{(2a-3)(2a+3)}{a}}{\dfrac{2a-3}{a^{\frac{1}{2}}}}$$

$$= \frac{(2a-3)(2a+3)a^{\frac{1}{2}}}{a(2a-3)}$$

$$= \frac{\cancel{(2a-3)}(2a+3)a^{\frac{1}{2}}}{a\cancel{(2a-3)}}$$

$$= \frac{(2a+3)a^{\frac{1}{2}}}{a}$$

$$= \frac{2a+3}{a \cdot a^{-\frac{1}{2}}}$$

$$= \frac{2a+3}{a^{\frac{1}{2}}}$$

$$\boxed{A = \frac{2a+3}{\sqrt{a}}}$$

$$B = \frac{a - 4 + 3a^{-1}}{a^{\frac{1}{2}} - a^{-\frac{1}{2}}}$$

$$= \frac{a - 4 + \dfrac{3}{a}}{a^{\frac{1}{2}} - \dfrac{1}{a^{\frac{1}{2}}}}$$

$$= \frac{\dfrac{a^2 - 4a + 3}{a}}{\dfrac{(a^{\frac{1}{2}})^2 - 1}{a^{\frac{1}{2}}}}$$

$$= \frac{\dfrac{(a-3)(a-1)}{a}}{\dfrac{a-1}{a^{\frac{1}{2}}}}$$

$$= \frac{(a-3)(a-1)a^{\frac{1}{2}}}{a(a-1)}$$

$$= \frac{(a-3)\cancel{(a-1)}a^{\frac{1}{2}}}{a\cancel{(a-1)}}$$

$$= \frac{(a-3)a^{\frac{1}{2}}}{a}$$

$$= \frac{a-3}{a \cdot a^{-\frac{1}{2}}}$$

$$= \frac{a-3}{a^{\frac{1}{2}}}$$

$$\boxed{B = \frac{a-3}{\sqrt{a}}}$$

reemplazando:

$$\left[\frac{4a-9a^{-1}}{2a^{\frac{1}{2}}-3a^{-\frac{1}{2}}} + \frac{a-4+3a^{-1}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}}\right]^2 = \left[\frac{2a+3}{\sqrt{a}} + \frac{a-3}{\sqrt{a}}\right]^2$$

$$= \left[\frac{2a+3+a-3}{\sqrt{a}}\right]^2$$

$$= \left[\frac{3a}{\sqrt{a}}\right]^2$$

$$= \frac{9a^2}{a}$$

$$= \frac{9a^{\not{2}}}{\not{a}}$$

$$= \boxed{9a}$$

100. $$\left[(a-b)\sqrt{\frac{a+b}{a-b}} + a - b\right]\left[(a-b)\left(\sqrt{\frac{a+b}{a-b}} - 1\right)\right]$$

Solución.

$$\left[(a-b)\sqrt{\frac{a+b}{a-b}} + a - b\right]\left[(a-b)\left(\sqrt{\frac{a+b}{a-b}} - 1\right)\right] = \overbrace{\left[(a-b)\sqrt{\frac{a+b}{a-b}} + a - b\right]}^{A}\overbrace{\left[(a-b)\left(\sqrt{\frac{a+b}{a-b}} - 1\right)\right]}^{B}$$

$$A = \left[(a-b)\sqrt{\frac{a+b}{a-b}} + a - b\right]$$

$$= \left[(a-b)\sqrt{\frac{a+b}{a-b}} + (a-b)\right]$$

$$\boxed{A = (a-b)\left[\sqrt{\frac{a+b}{a-b}} + 1\right]}$$

$$B = \left[(a-b)\left(\sqrt{\frac{a+b}{a-b}} - 1\right)\right]$$

$$\boxed{B = (a-b)\left[\sqrt{\frac{a+b}{a-b}} - 1\right]}$$

reemplazando:

$$\left[(a-b)\sqrt{\frac{a+b}{a-b}} + a - b\right]\left[(a-b)\left(\sqrt{\frac{a+b}{a-b}} - 1\right)\right] = (a-b)\left[\sqrt{\frac{a+b}{a-b}} + 1\right] \cdot (a-b)\left[\sqrt{\frac{a+b}{a-b}} - 1\right]$$

$$= (a-b)^2\left[\sqrt{\frac{a+b}{a-b}} + 1\right]\left[\sqrt{\frac{a+b}{a-b}} - 1\right]$$

$$= (a-b)^2\left[\left(\sqrt{\frac{a+b}{a-b}}\right)^2 - 1^2\right]$$

$$= (a-b)^2\left[\frac{a+b}{a-b} - 1\right]$$

$$= (a-b)^2\left[\frac{a+b-a+b}{a-b}\right]$$

$$= (a-b)^2\left[\frac{2b}{a-b}\right]$$

$$= \frac{2b(a-b)^2}{a-b}$$

$$= \frac{2b(a-b)^{\not 2}}{\cancel{a-b}}$$

$$= \boxed{2b(a-b)}$$

101. $\left(\sqrt{ab} - \dfrac{ab}{a+\sqrt{ab}}\right) : \dfrac{\sqrt[4]{ab}-\sqrt{b}}{a-b}$

Solución:

$$\left(\sqrt{ab} - \frac{ab}{a+\sqrt{ab}}\right) : \frac{\sqrt[4]{ab}-\sqrt{b}}{a-b} = \overbrace{\left(\sqrt{ab} - \frac{ab}{a+\sqrt{ab}}\right)}^{A} : \overbrace{\frac{\sqrt[4]{ab}-\sqrt{b}}{a-b}}^{B}$$

$$A = \left(\sqrt{ab} - \frac{ab}{a+\sqrt{ab}}\right)$$

$$= \left(\sqrt{ab} - \frac{\sqrt{(ab)^2}}{a+\sqrt{ab}}\right)$$

$$= \sqrt{ab}\left(1 - \frac{\sqrt{ab}}{a+\sqrt{ab}}\right)$$

$$= \sqrt{ab}\left(\frac{a+\sqrt{ab}-\sqrt{ab}}{a+\sqrt{ab}}\right)$$

$$= \sqrt{ab}\left(\frac{a}{a+\sqrt{ab}}\right)$$

$$= \frac{a\sqrt{ab}}{a+\sqrt{ab}}$$

$$= \frac{a\sqrt{ab}}{\sqrt{a^2}+\sqrt{ab}}$$

$$= \frac{a\sqrt{a}\sqrt{b}}{\sqrt{a}(\sqrt{a}+\sqrt{b})}$$

$$= \frac{a\cancel{\sqrt{a}}\sqrt{b}}{\cancel{\sqrt{a}}(\sqrt{a}+\sqrt{b})}$$

$$\boxed{A = \frac{a\sqrt{b}}{\sqrt{a}+\sqrt{b}}}$$

$$B=\frac{\sqrt[4]{ab}-\sqrt{b}}{a-b}$$
$$=\frac{\sqrt[4]{ab}-\sqrt[2\cdot 2]{b^2}}{(\sqrt{a}\,)^2-(\sqrt{b}\,)^2}$$
$$=\frac{\sqrt[4]{ab}-\sqrt[4]{b^2}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}$$
$$=\frac{\sqrt[4]{b}(\sqrt[4]{a}-\sqrt[4]{b}\,)}{\big((\sqrt[2\cdot 2]{a})^2-(\sqrt[2\cdot 2]{b})^2\big)(\sqrt{a}+\sqrt{b})}$$
$$=\frac{\sqrt[4]{b}(\sqrt[4]{a}-\sqrt[4]{b}\,)}{\big((\sqrt[4]{a}\,)^2-(\sqrt[4]{b}\,)^2\big)(\sqrt{a}+\sqrt{b})}$$
$$=\frac{\sqrt[4]{b}(\sqrt[4]{a}-\sqrt[4]{b}\,)}{(\sqrt[4]{a}-\sqrt[4]{b})(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})}$$
$$=\frac{\sqrt[4]{b}\cancel{(\sqrt[4]{a}-\sqrt[4]{b}\,)}}{\cancel{(\sqrt[4]{a}-\sqrt[4]{b})}(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})}$$
$$\boxed{B=\frac{\sqrt[4]{b}}{(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})}}$$

reemplazando:

$$\left(\sqrt{ab}-\frac{ab}{a+\sqrt{ab}}\right):\frac{\sqrt[4]{ab}-\sqrt{b}}{a-b}=\frac{a\sqrt{b}}{\sqrt{a}+\sqrt{b}}:\frac{\sqrt[4]{b}}{(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})}$$
$$=\frac{a\sqrt{b}}{\sqrt{a}+\sqrt{b}}\cdot\frac{(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})}{\sqrt[4]{b}}$$
$$=\frac{a\sqrt{b}(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})}{\sqrt[4]{b}(\sqrt{a}+\sqrt{b})}$$
$$=\frac{a\sqrt{b}(\sqrt[4]{a}+\sqrt[4]{b})\cancel{(\sqrt{a}+\sqrt{b})}}{\sqrt[4]{b}\cancel{(\sqrt{a}+\sqrt{b})}}$$
$$=\frac{a\sqrt{b}(\sqrt[4]{a}+\sqrt[4]{b})}{\sqrt[4]{b}}$$
$$=\frac{ab^{\frac{1}{2}}(\sqrt[4]{a}+\sqrt[4]{b})}{b^{\frac{1}{4}}}$$
$$=ab^{\frac{1}{2}}\cdot b^{-\frac{1}{4}}(\sqrt[4]{a}+\sqrt[4]{b})$$
$$=ab^{\frac{1}{4}}(\sqrt[4]{a}+\sqrt[4]{b})$$
$$=\boxed{a\sqrt[4]{b}(\sqrt[4]{a}+\sqrt[4]{b})}$$

102. $(a+b^{\frac{3}{2}}:\sqrt{a})^{\frac{2}{3}}\left(\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}}+\dfrac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^{-\frac{2}{3}}$

Solución.

$$(a+b^{\frac{3}{2}}:\sqrt{a})^{\frac{2}{3}}\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^{-\frac{2}{3}}=\overbrace{(a+b^{\frac{3}{2}}:\sqrt{a})^{\frac{2}{3}}}^{A}\overbrace{\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^{-\frac{2}{3}}}^{B}$$

$$A=(a+b^{\frac{3}{2}}:\sqrt{a})^{\frac{2}{3}}$$

$$= \left(a + \frac{b^{\frac{3}{2}}}{\sqrt{a}}\right)^{\frac{2}{3}}$$

$$= \left(a + \frac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)^{\frac{2}{3}}$$

$$= \left(\frac{a^{\frac{3}{2}} + b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)^{\frac{2}{3}}$$

$$= \left(\frac{\sqrt{a^3} + \sqrt{b^3}}{\sqrt{a}}\right)^{\frac{2}{3}}$$

$$= \left(\frac{(\sqrt{a})^3 + (\sqrt{b})^3}{\sqrt{a}}\right)^{\frac{2}{3}}$$

$$= \left(\frac{(\sqrt{a} + \sqrt{b})\big((\sqrt{a})^2 - \sqrt{a}\sqrt{b} + (\sqrt{b})^2\big)}{\sqrt{a}}\right)^{\frac{2}{3}}$$

$$\boxed{A = \left(\frac{(\sqrt{a} + \sqrt{b})(a - \sqrt{ab} + b)}{\sqrt{a}}\right)^{\frac{2}{3}}}$$

$$B = \left(\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a}} + \frac{\sqrt{b}}{\sqrt{a} - \sqrt{b}}\right)^{-\frac{2}{3}}$$

$$= \left(\frac{(\sqrt{a} - \sqrt{b})^2 + \sqrt{a}\sqrt{b}}{\sqrt{a}(\sqrt{a} - \sqrt{b})}\right)^{-\frac{2}{3}}$$

$$= \left(\frac{(\sqrt{a})^2 - 2\sqrt{a}\sqrt{b} + (\sqrt{b})^2 + \sqrt{ab}}{\sqrt{a}(\sqrt{a} - \sqrt{b})}\right)^{-\frac{2}{3}}$$

$$= \left(\frac{a - 2\sqrt{ab} + b + \sqrt{ab}}{\sqrt{a}(\sqrt{a} - \sqrt{b})}\right)^{-\frac{2}{3}}$$

$$\boxed{B = \left(\frac{a - \sqrt{ab} + b}{\sqrt{a}(\sqrt{a} - \sqrt{b})}\right)^{-\frac{2}{3}}}$$

reemplazando:

$$(a + b^{\frac{3}{2}} : \sqrt{a})^{\frac{2}{3}} \left(\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a}} + \frac{\sqrt{b}}{\sqrt{a} - \sqrt{b}}\right)^{-\frac{2}{3}} = \left(\frac{(\sqrt{a} + \sqrt{b})(a - \sqrt{ab} + b)}{\sqrt{a}}\right)^{\frac{2}{3}} \left(\frac{a - \sqrt{ab} + b}{\sqrt{a}(\sqrt{a} - \sqrt{b})}\right)^{-\frac{2}{3}}$$

$$= \frac{\left(\frac{(\sqrt{a} + \sqrt{b})(a - \sqrt{ab} + b)}{\sqrt{a}}\right)^{\frac{2}{3}}}{\left(\frac{a - \sqrt{ab} + b}{\sqrt{a}(\sqrt{a} - \sqrt{b})}\right)^{\frac{2}{3}}}$$

$$= \left(\frac{\frac{(\sqrt{a} + \sqrt{b})(a - \sqrt{ab} + b)}{\sqrt{a}}}{\frac{a - \sqrt{ab} + b}{\sqrt{a}(\sqrt{a} - \sqrt{b})}}\right)^{\frac{2}{3}}$$

$$= \left(\frac{\sqrt{a}(\sqrt{a}+\sqrt{b})(a-\sqrt{ab}+b)(\sqrt{a}-\sqrt{b})}{\sqrt{a}(a-\sqrt{ab}+b)} \right)^{\frac{2}{3}}$$

$$= \left(\frac{\cancel{\sqrt{a}}(\sqrt{a}+\sqrt{b})\cancel{(a-\sqrt{ab}+b)}(\sqrt{a}-\sqrt{b})}{\cancel{\sqrt{a}}\cancel{(a-\sqrt{ab}+b)}} \right)^{\frac{2}{3}}$$

$$= \left((\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) \right)^{\frac{2}{3}}$$

$$= \left((\sqrt{a})^2-(\sqrt{b})^2 \right)^{\frac{2}{3}}$$

$$= (a-b)^{\frac{2}{3}}$$

$$= \boxed{\sqrt{(a-b)^3}}$$

103. $\left[\frac{1}{x^{\frac{1}{2}}-4a^{-\frac{1}{2}}} + \frac{2\sqrt[3]{x}}{x\sqrt[3]{x}-4\sqrt[3]{x}} \right]^{-2} - \sqrt{x^2+8x+16}$

Solución.

$$\left[\frac{1}{x^{\frac{1}{2}}-4a^{-\frac{1}{2}}} + \frac{2\sqrt[3]{x}}{x\sqrt[3]{x}-4\sqrt[3]{x}} \right]^{-2} - \sqrt{x^2+8x+16} = \overbrace{\left[\frac{1}{x^{\frac{1}{2}}-4a^{-\frac{1}{2}}} + \frac{2\sqrt[3]{x}}{x\sqrt[3]{x}-4\sqrt[3]{x}} \right]^{-2}}^{A} - \overbrace{\sqrt{x^2+8x+16}}^{B}$$

$$A = \left[\frac{1}{x^{2}-4a^{-\frac{1}{2}}} + \frac{2\sqrt[3]{x}}{x\sqrt[3]{x}-4\sqrt[3]{x}} \right]^{-2}$$

$$= \left[\frac{1}{x^{\frac{1}{2}}-\frac{4}{x^{\frac{1}{2}}}} + \frac{2\sqrt[3]{x}}{\sqrt[3]{x}(x-4)} \right]^{-2}$$

$$= \left[\frac{1}{\frac{x^{\frac{1}{2}}\cdot x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}} + \frac{2\cancel{\sqrt[3]{x}}}{\cancel{\sqrt[3]{x}}(x-4)} \right]^{-2}$$

$$= \left[\frac{1}{\frac{x-4}{\sqrt{x}}} + \frac{2}{x-4} \right]^{-2}$$

$$= \left[\frac{\sqrt{x}}{x-4} + \frac{2}{x-4} \right]^{-2}$$

$$= \left[\frac{\sqrt{x}+2}{x-4} \right]^{-2}$$

$$= \left[\frac{\sqrt{x}+2}{(\sqrt{x})^2-4} \right]^{-2}$$

$$= \left[\frac{\sqrt{x}+2}{(\sqrt{x}-2)(\sqrt{x}+2)} \right]^{-2}$$

$$= \left[\frac{\cancel{\sqrt{x}+2}}{(\sqrt{x}-2)\cancel{(\sqrt{x}+2)}} \right]^{-2}$$

$$= \left[\frac{1}{\sqrt{x}-2} \right]^{-2}$$

$$= \frac{1}{\left[\dfrac{1}{\sqrt{x}-2}\right]^2}$$
$$= \left[\frac{1}{\sqrt{x}-2}\right]^{-2}$$
$$= \frac{1}{\dfrac{1}{[\sqrt{x}-2]^2}}$$
$$= \frac{[\sqrt{x}-2]^2}{1}$$
$$= [\sqrt{x}-2]^2$$
$$= (\sqrt{x})^2 - 4\sqrt{x} + 4$$
$$\boxed{A = x - 4\sqrt{x} + 4}$$

$$B = \sqrt{x^2+8x+16}$$
$$= \sqrt{(x+4)^2}$$
$$\boxed{B = x+4}$$

reemplazando:

$$\left[\frac{1}{x^{\frac{1}{2}} - 4a^{-\frac{1}{2}}} + \frac{2\sqrt[3]{x}}{x\sqrt[3]{x} - 4\sqrt[3]{x}}\right]^{-2} - \sqrt{x^2+8x+16} = x - 4\sqrt{x} + 4 - (x+4)$$
$$= x - 4\sqrt{x} + 4 - x - 4$$
$$= \boxed{-4\sqrt{x}}$$

104. $x^3\left[\dfrac{(\sqrt[4]{x}+\sqrt[4]{y})^2+(\sqrt[4]{x}-\sqrt[4]{y})^2}{x+\sqrt{xy}}\right]^5\sqrt[3]{x\sqrt{x}}$

Solución.

$$x^3\left[\frac{(\sqrt[4]{x}+\sqrt[4]{y})^2+(\sqrt[4]{x}-\sqrt[4]{y})^2}{x+\sqrt{xy}}\right]^5\sqrt[3]{x\sqrt{x}} = \overbrace{\left[\frac{(\sqrt[4]{x}+\sqrt[4]{y})^2+(\sqrt[4]{x}-\sqrt[4]{y})^2}{x+\sqrt{xy}}\right]^5}^{A}\overbrace{\sqrt[3]{x\sqrt{x}}}^{B}$$

$$A = \left[\frac{(\sqrt[4]{x}+\sqrt[4]{y})^2+(\sqrt[4]{x}-\sqrt[4]{y})^2}{x+\sqrt{xy}}\right]^5$$
$$= \left[\frac{(\sqrt[4]{x})^2+2\sqrt[4]{x}\sqrt[4]{y}+(\sqrt[4]{y})^2+(\sqrt[4]{x})^2-2\sqrt[4]{x}\sqrt[4]{y}+(\sqrt[4]{y})^2}{\sqrt{x^2}+\sqrt{xy}}\right]^5$$
$$= \left[\frac{\sqrt{x}+2\sqrt[4]{xy}+\sqrt{y}+\sqrt{x}-2\sqrt[4]{xy}+\sqrt{y}}{\sqrt{x}(\sqrt{x}+\sqrt{y})}\right]^5$$
$$= \left[\frac{2\sqrt{x}+2\sqrt{y}}{\sqrt{x}(\sqrt{x}+\sqrt{y})}\right]^5$$
$$= \left[\frac{2(\sqrt{x}+\sqrt{y})}{\sqrt{x}(\sqrt{x}+\sqrt{y})}\right]^5$$
$$= \left[\frac{2\cancel{(\sqrt{x}+\sqrt{y})}}{\sqrt{x}\cancel{(\sqrt{x}+\sqrt{y})}}\right]^5$$

$$= \left[\frac{2}{\sqrt{x}}\right]^5$$

$$= \left[\frac{2}{x^{\frac{1}{2}}}\right]^5$$

$$= [2 \cdot x^{-\frac{1}{2}}]^5$$

$$\boxed{A = 32x^{-\frac{5}{2}}}$$

$$B = \sqrt[3]{x\sqrt{x}}$$

[

$$= (x\sqrt{x})^{\frac{1}{3}}$$

$$= (x \cdot x^{\frac{1}{2}})^{\frac{1}{3}}$$

$$= (x^{\frac{3}{2}})^{\frac{1}{3}}$$

$$\boxed{B = x^{\frac{1}{2}}}$$

reemplazando:

$$x^3\left[\frac{(\sqrt[4]{x}+\sqrt[4]{y})^2+(\sqrt[4]{x}-\sqrt[4]{y})^2}{x+\sqrt{xy}}\right]^5\sqrt[3]{x\sqrt{x}} = x^3 \cdot 32x^{-\frac{5}{2}} \cdot x^{\frac{1}{2}}$$

$$= \boxed{32x}$$

105. $\left(\frac{\sqrt[4]{ax^3}-\sqrt[4]{a^3x}}{\sqrt{a}-\sqrt{x}}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}\sqrt{1+2\sqrt{\frac{a}{x}}+\frac{a}{x}}$

Solución.

$$\left(\frac{\sqrt[4]{ax^3}-\sqrt[4]{a^3x}}{\sqrt{a}-\sqrt{x}}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}\sqrt{1+2\sqrt{\frac{a}{x}}+\frac{a}{x}} = \overbrace{\left(\frac{\sqrt[4]{ax^3}-\sqrt[4]{a^3x}}{\sqrt{a}-\sqrt{x}}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}}^{A}\overbrace{\sqrt{1+2\sqrt{\frac{a}{x}}+\frac{a}{x}}}^{B}$$

$$A = \left(\frac{\sqrt[4]{ax^3}-\sqrt[4]{a^3x}}{\sqrt{a}-\sqrt{x}}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}$$

$$= \left(\frac{\sqrt[4]{ax}(\sqrt[4]{x^2}-\sqrt[4]{a^2})}{-\sqrt{x}+\sqrt{a}}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}$$

$$= \left(\frac{\sqrt[4]{ax}(\sqrt{x}-\sqrt{a})}{-(\sqrt{x}-\sqrt{a})}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}$$

$$= \left(-\frac{\sqrt[4]{ax}\cancel{(\sqrt{x}-\sqrt{a})}}{\cancel{(\sqrt{x}-\sqrt{a})}}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}$$

$$= \left(-\sqrt[4]{ax}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}$$

$$= \left(\frac{-\sqrt[4]{(ax)^2}+1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}$$

$$= \left(\frac{-\sqrt{ax}+1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}$$

$$= \left(\frac{1}{\sqrt[4]{ax}}\right)^{-2} = \left(\frac{1}{(ax)^{\frac{1}{4}}}\right)^{-2}$$

$$= ((ax)^{-\frac{1}{4}})^{-2} = (ax)^{\frac{1}{2}}$$

$$\boxed{A = \sqrt{ax}}$$

$$B = \sqrt{1 + 2\sqrt{\frac{a}{x}} + \frac{a}{x}}$$

$$= \sqrt{1 + 2 \cdot \frac{\sqrt{a}}{\sqrt{x}} + \frac{a}{x}}$$

$$= \sqrt{1 + \frac{2\sqrt{a}}{\sqrt{x}} + \frac{a}{x}}$$

$$= \sqrt{\frac{x + 2\sqrt{a}\sqrt{x} + a}{x}}$$

$$= \sqrt{\frac{(x+a)^2}{x}}$$

$$= \frac{\sqrt{(x+a)^2}}{\sqrt{x}}$$

$$\boxed{B = \frac{x+a}{\sqrt{x}}}$$

reemplazando:

$$\left(\frac{\sqrt[4]{ax^3} - \sqrt[4]{a^3x}}{\sqrt{a} - \sqrt{x}} + \frac{1 + \sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2} \sqrt{1 + 2\sqrt{\frac{a}{x}} + \frac{a}{x}} = \sqrt{ax} \cdot \frac{x+a}{\sqrt{x}}$$

$$= \frac{\sqrt{ax}(x+a)}{\sqrt{x}}$$

$$= \frac{\sqrt{a}\sqrt{x}(x+a)}{\sqrt{x}}$$

$$= \frac{\sqrt{a}\cancel{\sqrt{x}}(x+a)}{\cancel{\sqrt{x}}}$$

$$= \boxed{\sqrt{a}(x+a)}$$

106. $\dfrac{(a-b^2)\sqrt{3} - b\sqrt{3}\sqrt[3]{-8b^2}}{\sqrt{2(a-b^2)^2 + (2b\sqrt{2a})^2}} \cdot \dfrac{\sqrt{2a} - \sqrt{2c}}{\sqrt{\frac{3}{a}} - \sqrt{\frac{3}{c}}}$

Solución.

$$\frac{(a-b^2)\sqrt{3} - b\sqrt{3}\sqrt[3]{-8b^2}}{\sqrt{2(a-b^2)^2 + (2b\sqrt{2a})^2}} \cdot \frac{\sqrt{2a} - \sqrt{2c}}{\sqrt{\frac{3}{a}} - \sqrt{\frac{3}{c}}} = \overbrace{\frac{(a-b^2)\sqrt{3} - b\sqrt{3}\sqrt[3]{-8b^2}}{\sqrt{2(a-b^2)^2 + (2b\sqrt{2a})^2}}}^{A} \cdot \overbrace{\frac{\sqrt{2a} - \sqrt{2c}}{\sqrt{\frac{3}{a}} - \sqrt{\frac{3}{c}}}}^{B}$$

$$A = \frac{(a-b^2)\sqrt{3} - b\sqrt{3}\sqrt[3]{-8b^2}}{\sqrt{2(a-b^2)^2 + (2b\sqrt{2a})^2}}$$

$$= \frac{\sqrt{3}a - \sqrt{3}b^2 - b\sqrt{3}(-2b)}{\sqrt{2(a^2 - 2ab^2 + b^4) + 4b^2 \cdot 2a}}$$

$$= \frac{\sqrt{3}a - \sqrt{3}b^2 + 2\sqrt{3}b^2}{\sqrt{2a^2 - 4ab^2 + 2b^4 + 8ab^2}}$$

$$= \frac{\sqrt{3}a + \sqrt{3}b^2}{\sqrt{2a^2 + 4ab^2 + 2b^4}}$$

$$= \frac{\sqrt{3}(a + b^2)}{\sqrt{2(a^2 + 2ab^2 + b^4)}}$$

$$= \frac{\sqrt{3}(a + b^2)}{\sqrt{2(a + b^2)^2}}$$

$$= \frac{\sqrt{3}(a + b^2)}{\sqrt{2}(a + b^2)}$$

$$= \frac{\sqrt{3}\cancel{(a + b^2)}}{\sqrt{2}\cancel{(a + b^2)}}$$

$$\boxed{A = \frac{\sqrt{3}}{\sqrt{2}}}$$

$$B = \frac{\sqrt{2a} - \sqrt{2c}}{\sqrt{\frac{3}{a}} - \sqrt{\frac{3}{c}}}$$

$$= \frac{\sqrt{2}(\sqrt{a} - \sqrt{c})}{\frac{\sqrt{3}}{\sqrt{a}} - \frac{\sqrt{3}}{\sqrt{c}}}$$

$$= \frac{\sqrt{2}(-\sqrt{c} + \sqrt{a})}{\sqrt{3}\left(\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{c}}\right)}$$

$$= \frac{-\sqrt{2}(\sqrt{c} - \sqrt{a})}{\sqrt{3}\left(\frac{\sqrt{c} - \sqrt{a}}{\sqrt{c}\sqrt{a}}\right)}$$

$$= \frac{-\sqrt{2}(\sqrt{c} - \sqrt{a})}{\frac{\sqrt{3}(\sqrt{c} - \sqrt{a})}{\sqrt{ac}}}$$

$$= \frac{-\sqrt{2}\sqrt{ac}(\sqrt{c} - \sqrt{a})}{\sqrt{3}(\sqrt{c} - \sqrt{a})}$$

$$= \frac{-\sqrt{2}\sqrt{ac}\cancel{(\sqrt{c} - \sqrt{a})}}{\sqrt{3}\cancel{(\sqrt{c} - \sqrt{a})}}$$

$$\boxed{B = \frac{-\sqrt{2}\sqrt{ac}}{\sqrt{3}}}$$

reemplazando:

$$\frac{(a - b^2)\sqrt{3} - b\sqrt{3}\sqrt[3]{-8b^2}}{\sqrt{2(a - b^2)^2 + (2b\sqrt{2a})^2}} \cdot \frac{\sqrt{2a} - \sqrt{2c}}{\sqrt{\frac{3}{a}} - \sqrt{\frac{3}{c}}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{-\sqrt{2}\sqrt{ac}}{\sqrt{3}}$$

$$= \frac{-\sqrt{2}\sqrt{3}\sqrt{ac}}{\sqrt{2}\sqrt{3}}$$

$$= \frac{-\sqrt{6}\sqrt{ac}}{\sqrt{6}}$$

$$= \frac{-\cancel{\sqrt{6}}\sqrt{ac}}{\cancel{\sqrt{6}}}$$

$$= \boxed{-\sqrt{ac}}$$

107. $\left\{\sqrt{1+\left[(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{1}{2}}x^{-\frac{1}{3}}\right]^2}\right\}^{-6} - \frac{1}{a^2}\sqrt{(a^2-x^2)^2+4a^2x^2}$

Solución.

$\left\{\sqrt{1+\left[(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{1}{2}}x^{-\frac{1}{3}}\right]^2}\right\}^{-6} - \frac{1}{a^2}\sqrt{(a^2-x^2)^2+4a^2x^2} =$

$$= \overbrace{\left\{\sqrt{1+\left[(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{1}{2}}x^{-\frac{1}{3}}\right]^2}\right\}^{-6}}^{A} - \overbrace{\frac{1}{a^2}\sqrt{(a^2-x^2)^2+4a^2x^2}}^{B}$$

$$A = \left\{\sqrt{1+[(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{1}{2}}x^{-\frac{1}{3}}]^2}\right\}^{-6}$$

$$= \left\{\sqrt{1+(a^{\frac{2}{3}}-x^{\frac{2}{3}})x^{-\frac{2}{3}}}\right\}^{-6}$$

$$= \left\{\sqrt{1+\frac{a^{\frac{2}{3}}-x^{\frac{2}{3}}}{x^{\frac{2}{3}}}}\right\}^{-6}$$

$$= \left\{\sqrt{\frac{x^{\frac{2}{3}}+a^{\frac{2}{3}}-x^{\frac{2}{3}}}{x^{\frac{2}{3}}}}\right\}^{-6}$$

$$= \left\{\sqrt{\frac{a^{\frac{2}{3}}}{x^{\frac{2}{3}}}}\right\}^{-6}$$

$$= \{\sqrt{a^{\frac{2}{3}}x^{-\frac{2}{3}}}\}^{-6}$$

$$= \{\sqrt{(a^{\frac{1}{3}}x^{-\frac{1}{3}})^2}\}^{-6}$$

$$= \{a^{\frac{1}{3}}x^{-\frac{1}{3}}\}^{-6}$$

$$= a^{-2}x^2$$

$$\boxed{A = \frac{x^2}{a^2}}$$

$$B = \frac{1}{a^2}\sqrt{(a^2-x^2)^2+4a^2x^2}$$

$$= \frac{1}{a^2}\sqrt{a^4-2a^2x^2+x^4+4a^2x^2}$$

$$= \frac{1}{a^2}\sqrt{a^4+2a^2x^2+x^4}$$

$$= \frac{1}{a^2}\sqrt{(a^2+x^2)^2}$$

$$= \frac{1}{a^2}(a^2+x^2)$$

$$\boxed{B = \frac{a^2+x^2}{a^2}}$$

reemplazando:

$$\left\{\sqrt{1+\left[(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{1}{2}}x^{-\frac{1}{3}}\right]^2}\right\}^{-6} - \frac{1}{a^2}\sqrt{(a^2-x^2)^2+4a^2x^2} = \frac{x^2}{a^2} - \frac{a^2+x^2}{a^2}$$

$$= \frac{x^2-(a^2+x^2)}{a^2}$$

$$= \frac{x^2-a^2-x^2}{a^2}$$

$$= \frac{-a^2}{a^2}$$

$$= -\frac{a^2}{a^2}$$

$$= \boxed{-1}$$

108. $\left[(\sqrt[4]{x}-\sqrt[4]{a})^{-1}+(\sqrt[4]{x}+\sqrt[4]{a})^{-1}\right]^{-2} : \dfrac{x-a}{4\sqrt{x}-4\sqrt{a}}$

Solución

$$\left[(\sqrt[4]{x}-\sqrt[4]{a})^{-1}+(\sqrt[4]{x}+\sqrt[4]{a})^{-1}\right]^{-2} : \frac{x-a}{4\sqrt{x}-4\sqrt{a}} = \overbrace{\left[(\sqrt[4]{x}-\sqrt[4]{a})^{-1}+(\sqrt[4]{x}+\sqrt[4]{a})^{-1}\right]^{-2}}^{A} : \overbrace{\frac{x-a}{4\sqrt{x}-4\sqrt{a}}}^{B}$$

$$A = \left[(\sqrt[4]{x}-\sqrt[4]{a})^{-1}+(\sqrt[4]{x}+\sqrt[4]{a})^{-1}\right]^{-2}$$

$$= \left[\frac{1}{\sqrt[4]{x}-\sqrt[4]{a}}+\frac{1}{\sqrt[4]{x}+\sqrt[4]{a}}\right]^{-2}$$

$$= \left[\frac{\sqrt[4]{x}+\sqrt[4]{a}+\sqrt[4]{x}-\sqrt[4]{a}}{(\sqrt[4]{x}-\sqrt[4]{a})(\sqrt[4]{x}+\sqrt[4]{a})}\right]^{-2}$$

$$= \left[\frac{2\sqrt[4]{x}}{\sqrt[4]{x^2}-\sqrt[4]{a^2}}\right]^{-2}$$

$$= \left[\frac{2\sqrt[4]{x}}{\sqrt{x}-\sqrt{a}}\right]^{-2}$$

$$= \frac{1}{\left[\dfrac{2\sqrt[4]{x}}{\sqrt{x}-\sqrt{a}}\right]^{2}}$$

$$= \frac{1}{\dfrac{4\sqrt[4]{x^2}}{(\sqrt{x}-\sqrt{a})^2}}$$

$$= \frac{1}{\dfrac{4\sqrt{x}}{(\sqrt{x}-\sqrt{a})^2}}$$

$$\boxed{A = \frac{(\sqrt{x}-\sqrt{a})^2}{4\sqrt{x}}}$$

$$B = \frac{x-a}{4\sqrt{x}-4\sqrt{a}}$$

$$= \frac{\sqrt{x^2} - \sqrt{a^2}}{4(\sqrt{x} - \sqrt{a})}$$

$$= \frac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{4(\sqrt{x} - \sqrt{a})}$$

$$= \frac{\cancel{(\sqrt{x} - \sqrt{a})}(\sqrt{x} + \sqrt{a})}{4\cancel{(\sqrt{x} - \sqrt{a})}}$$

$$\boxed{B = \frac{\sqrt{x} - \sqrt{a}}{4}}$$

reemplazando:

$$\left[(\sqrt[4]{x} - \sqrt[4]{a})^{-1} + (\sqrt[4]{x} + \sqrt[4]{a})^{-1}\right]^{-2} : \frac{x-a}{4\sqrt{x} - 4\sqrt{a}} = \frac{(\sqrt{x} - \sqrt{a})^2}{4\sqrt{x}} : \frac{\sqrt{x} - \sqrt{a}}{4}$$

$$= \frac{(\sqrt{x} - \sqrt{a})^2}{4\sqrt{x}} \cdot \frac{4}{\sqrt{x} - \sqrt{a}}$$

$$= \frac{4(\sqrt{x} - \sqrt{a})^2}{4\sqrt{x}(\sqrt{x} - \sqrt{a})}$$

$$= \frac{\cancel{4}(\sqrt{x} - \sqrt{a})^{\cancel{2}}}{\cancel{4}\sqrt{x}\cancel{(\sqrt{x} - \sqrt{a})}}$$

$$= \boxed{\frac{\sqrt{x} - \sqrt{a}}{\sqrt{x}}}$$

109. $\left[\dfrac{\sqrt[6]{a^2x} + \sqrt{x}}{\sqrt[3]{x} + \sqrt[3]{a}} + \sqrt[6]{x}\right]^3 + 4(x+1) + (\sqrt[3]{x\sqrt{x}} + 1)^2$

Solución.

$$\left[\frac{\sqrt[6]{a^2x} + \sqrt{x}}{\sqrt[3]{x} + \sqrt[3]{a}} + \sqrt[6]{x}\right]^3 + 4(x+1) + (\sqrt[3]{x\sqrt{x}} + 1)^2 = \overbrace{\left[\frac{\sqrt[6]{a^2x} + \sqrt{x}}{\sqrt[3]{x} + \sqrt[3]{a}} + \sqrt[6]{x}\right]^3}^{A} + 4(x+1) + \overbrace{(\sqrt[3]{x\sqrt{x}} + 1)^2}^{B}$$

$$A = \left[\frac{\sqrt[6]{a^2x} + \sqrt{x}}{\sqrt[3]{x} + \sqrt[3]{a}} + \sqrt[6]{x}\right]^3$$

$$= \left[\frac{\sqrt[6]{a^2x} + \sqrt[2\cdot 3]{x^3}}{\sqrt[3]{a} + \sqrt[3]{x}} + \sqrt[6]{x}\right]^3$$

$$= \left[\frac{\sqrt[6]{a^2x} + \sqrt[6]{x^3}}{\sqrt[3]{a} + \sqrt[3]{x}} + \sqrt[6]{x}\right]^3$$

$$= \left[\frac{\sqrt[6]{x}(\sqrt[6]{a^2} + \sqrt[6]{x^2})}{\sqrt[3]{a} + \sqrt[3]{x}} + \sqrt[6]{x}\right]^3$$

$$= \left[\frac{\sqrt[6]{x}(\sqrt[3]{a} + \sqrt[3]{x})}{\sqrt[3]{a} + \sqrt[3]{x}} + \sqrt[6]{x}\right]^3$$

$$= \left[\frac{\sqrt[6]{x}\cancel{(\sqrt[3]{a} + \sqrt[3]{x})}}{\cancel{\sqrt[3]{a} + \sqrt[3]{x}}} + \sqrt[6]{x}\right]^3$$

$$= [\sqrt[6]{x} + \sqrt[6]{x}]^3$$

$$= [2\sqrt[6]{x}]^3$$

$$= 8\sqrt[6]{x^3}$$

$$\boxed{A = 8\sqrt{x}}$$

$$B = (\sqrt[3]{x\sqrt{x}} + 1)^2$$

$$= \left((x\sqrt{x})^{\frac{1}{3}} + 1\right)^2$$

$$= \left((x \cdot x^{\frac{1}{2}})^{\frac{1}{3}} + 1\right)^2$$

$$= \left((x^{\frac{3}{2}})^{\frac{1}{3}} + 1\right)^2$$

$$= (x^{\frac{1}{2}} + 1)^2$$

$$= (\sqrt{x} + 1)^2$$

$$= \sqrt{x^2} + 2\sqrt{x} + 1$$

$$\boxed{B = x + 2\sqrt{x} + 1}$$

reemplazando:

$$\left[\frac{\sqrt[6]{a^2x} + \sqrt{x}}{\sqrt[3]{x} + \sqrt[3]{a}} + \sqrt[6]{x}\right]^3 + 4(x+1) + (\sqrt[3]{x\sqrt{x}} + 1)^2 = 8\sqrt{x} + 4(x+1) + x + 2\sqrt{x} + 1$$

$$= 8\sqrt{x} + 4x + 4 + x + 2\sqrt{x} + 1$$

$$= 5x + 10\sqrt{x} + 5$$

$$= 5(x + 2\sqrt{x} + 1)$$

$$= 5(\sqrt{x^2} + 2\sqrt{x} + 1)$$

$$= \boxed{5(\sqrt{x} + 1)^2}$$

110. $\left[\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}} - 2x^{-\frac{1}{3}}} - \frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}} - x^{\frac{1}{3}}}\right]^{-1} - \left(\frac{1-2x}{3x-2}\right)^{-1}$

Solución.

$$\left[\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}} - 2x^{-\frac{1}{3}}} - \frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}} - x^{\frac{1}{3}}}\right]^{-1} - \left(\frac{1-2x}{3x-2}\right)^{-1} = \overbrace{\left[\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}} - 2x^{-\frac{1}{3}}} - \frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}} - x^{\frac{1}{3}}}\right]^{-1}}^{A} - \overbrace{\left(\frac{1-2x}{3x-2}\right)^{-1}}^{B}$$

$$A = \left[\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}} - 2x^{-\frac{1}{3}}} - \frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}} - x^{\frac{1}{3}}}\right]^{-1}$$

$$= \left[\frac{\frac{3}{x^{\frac{1}{3}}}}{x^{\frac{2}{3}} - \frac{2}{x^{\frac{1}{3}}}} - \frac{x^{\frac{1}{3}}}{x \cdot x^{\frac{1}{3}} - x^{\frac{1}{3}}}\right]^{-1}$$

$$= \left[\frac{\frac{3}{x^{\frac{1}{3}}}}{\frac{x^{\frac{2}{3}} \cdot x^{\frac{1}{3}} - 2}{x^{\frac{1}{3}}}} - \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}(x-1)}\right]^{-1}$$

$$= \left[\frac{\frac{3}{x^{\frac{1}{3}}}}{\frac{x-2}{x^{\frac{1}{3}}}} - \frac{1}{x-1}\right]^{-1}$$

$$= \left[\frac{3x^{\frac{1}{3}}}{x^{\frac{1}{3}}(x-2)} - \frac{1}{x-1}\right]^{-1}$$

$$= \left[\frac{3\cancel{x^{\frac{1}{3}}}}{\cancel{x^{\frac{1}{3}}}(x-2)} - \frac{1}{x-1}\right]^{-1}$$

$$= \left[\frac{3}{x-2} - \frac{1}{x-1}\right]^{-1}$$

$$= \left[\frac{3(x-1)-(x-2)}{(x-2)(x-1)}\right]^{-1}$$

$$= \left[\frac{3x-3-x+2}{x^2-x-2x+2}\right]^{-1}$$

$$= \left[\frac{2x-1}{x^2-3x+2}\right]^{-1}$$

$$= \frac{1}{\frac{2x-1}{x^2-3x+2}}$$

$$\boxed{A = \frac{x^2-3x+2}{2x-1}}$$

$$B = \left(\frac{1-2x}{3x-2}\right)^{-1}$$

$$= \frac{1}{\frac{1-2x}{3x-2}}$$

$$= \frac{3x-2}{1-2x}$$

$$= \frac{3x-2}{-2x+1}$$

$$\boxed{B = \frac{3x-2}{-(2x-1)}}$$

reemplazando:

$$\left[\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}}-2x^{-\frac{1}{3}}} - \frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}-x^{\frac{1}{3}}}\right]^{-1} - \left(\frac{1-2x}{3x-2}\right)^{-1} = \frac{x^2-3x+2}{2x-1} - \frac{3x-2}{-(2x-1)}$$

$$= \frac{x^2-3x+2}{2x-1} + \frac{3x-2}{2x-1}$$

$$= \frac{x^2-3x+2+3x-2}{2x-1}$$

$$= \boxed{\frac{x^2}{2x-1}}$$

111. $\sqrt[-\frac{1}{2}]{\sqrt{a}}\left[\sqrt{a^2+a\sqrt{a^2-b^2}} - \sqrt{a^2-a\sqrt{a^2-b^2}}\right]^2$

Solución.

$$\sqrt[-\frac{1}{2}]{\sqrt{a}}\left[\sqrt{a^2+a\sqrt{a^2-b^2}}-\sqrt{a^2-a\sqrt{a^2-b^2}}\right]^2=\overbrace{\sqrt[-\frac{1}{2}]{\sqrt{a}}}^{A}\overbrace{\left[\sqrt{a^2+a\sqrt{a^2-b^2}}-\sqrt{a^2-a\sqrt{a^2-b^2}}\right]^2}^{B}$$

$$\begin{aligned}A &= \sqrt[-\frac{1}{2}]{\sqrt{a}}\\ &= (\sqrt{a})^{\frac{1}{-\frac{1}{2}}}\\ &= (\sqrt{a})^{-2}\\ &= (a^{\frac{1}{2}})^{-2}\\ &= a^{-1}\\ &\boxed{A=\frac{1}{a}}\end{aligned}$$

$$\begin{aligned}B &= \left[\sqrt{a^2+a\sqrt{a^2-b^2}}-\sqrt{a^2-a\sqrt{a^2-b^2}}\right]^2\\ &= (\sqrt{a^2+a\sqrt{a^2-b^2}})^2-2\sqrt{a^2+a\sqrt{a^2-b^2}}\cdot\sqrt{a^2-a\sqrt{a^2-b^2}}+(\sqrt{a^2-a\sqrt{a^2-b^2}})^2\\ &= a^2+a\sqrt{a^2-b^2}-2\sqrt{(a^2+a\sqrt{a^2-b^2})(a^2-a\sqrt{a^2-b^2})}+a^2-a\sqrt{a^2-b^2}\\ &= 2a^2-2\sqrt{(a^2)^2-(a\sqrt{a^2-b^2})^2}\\ &= 2a^2-2\sqrt{a^4-a^2(a^2-b^2)}\\ &= 2a^2-2\sqrt{a^4-a^4+a^2b^2}\\ &= 2a^2-2\sqrt{a^2b^2}\\ &= 2a^2-2\sqrt{(ab)^2}\\ &= 2a^2-2ab\\ &= \boxed{2a(a-b)}\end{aligned}$$

reemplazando:

$$\begin{aligned}\sqrt[-\frac{1}{2}]{\sqrt{a}}\left[\sqrt{a^2+a\sqrt{a^2-b^2}}-\sqrt{a^2-a\sqrt{a^2-b^2}}\right]^2 &= \frac{1}{a}\cdot 2a(a-b)\\ &= \frac{2a(a-b)}{a}\\ &= \frac{2\not{a}(a-b)}{\not{a}}\\ &= \boxed{2(a-b)}\end{aligned}$$

112. $\left[\frac{(\sqrt[3]{x}-\sqrt[3]{a})^3+2x+a}{(\sqrt[3]{x}-\sqrt[3]{a})^3-x-2a}\right]^3+\sqrt{(a^3+3a^2x+3ax^2+x^3)^{\frac{2}{3}}}:a$

Solución.

$$\left[\frac{(\sqrt[3]{x}-\sqrt[3]{a})^3+2x+a}{(\sqrt[3]{x}-\sqrt[3]{a})^3-x-2a}\right]^3+\sqrt{(a^3+3a^2x+3ax^2+x^3)^{\frac{2}{3}}}:a=$$

$$=\overbrace{\left[\frac{(\sqrt[3]{x}-\sqrt[3]{a})^3+2x+a}{(\sqrt[3]{x}-\sqrt[3]{a})^3-x-2a}\right]^3}^{A}+\overbrace{\sqrt{(a^3+3a^2x+3ax^2+x^3)^{\frac{2}{3}}}:a}^{B}$$

$$A=\left[\frac{(\sqrt[3]{x}-\sqrt[3]{a})^3+2x+a}{(\sqrt[3]{x}-\sqrt[3]{a})^3-x-2a}\right]^3$$

$$= \left[\frac{(\sqrt[3]{x})^3 - 3(\sqrt[3]{x})^2\sqrt[3]{a} + 3\sqrt[3]{x}(\sqrt[3]{a})^2 - (\sqrt[3]{a})^3 + 2x + a}{(\sqrt[3]{x})^3 - 3(\sqrt[3]{x})^2\sqrt[3]{a} + 3\sqrt[3]{x}(\sqrt[3]{a})^2 - (\sqrt[3]{a})^3 - x - 2a}\right]^3$$

$$= \left[\frac{x - 3\sqrt[3]{x^2}\sqrt[3]{a} + 3\sqrt[3]{x}\sqrt[3]{a^2} - a + 2x + a}{x - 3\sqrt[3]{x^2}\sqrt[3]{a} + 3\sqrt[3]{x}\sqrt[3]{a^2} - a - x - 2a}\right]^3$$

$$= \left[\frac{3x - 3\sqrt[3]{x^2}\sqrt[3]{a} + 3\sqrt[3]{x}\sqrt[3]{a^2}}{-3\sqrt[3]{x^2}\sqrt[3]{a} + 3\sqrt[3]{x}\sqrt[3]{a^2} - 3a}\right]^3$$

$$= \left[\frac{3\sqrt[3]{x^3} - 3\sqrt[3]{x^2}\sqrt[3]{a} + 3\sqrt[3]{x}\sqrt[3]{a^2}}{-3\sqrt[3]{x^2}\sqrt[3]{a} + 3\sqrt[3]{x}\sqrt[3]{a^2} - 3\sqrt[3]{a^3}}\right]^3$$

$$= \left[\frac{3\sqrt[3]{x}(\sqrt[3]{x^2} - 3\sqrt[3]{x}\sqrt[3]{a} + 3\sqrt[3]{a^2})}{-3\sqrt[3]{a}(\sqrt[3]{x^2} - \sqrt[3]{x}\sqrt[3]{a} + \sqrt[3]{a^2})}\right]^3$$

$$= \left[-\frac{\cancel{3}\sqrt[3]{x}\cancel{(\sqrt[3]{x^2} - 3\sqrt[3]{x}\sqrt[3]{a} + 3\sqrt[3]{a^2})}}{\cancel{3}\sqrt[3]{a}\cancel{(\sqrt[3]{x^2} - \sqrt[3]{x}\sqrt[3]{a} + \sqrt[3]{a^2})}}\right]^3$$

$$= \left[-\frac{\sqrt[3]{x}}{\sqrt[3]{a}}\right]^3$$

$$= \left[-\frac{x^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]^3$$

$$= \left[-\left(\frac{x}{a}\right)^{\frac{1}{3}}\right]^3$$

$$\boxed{A = -\frac{x}{a}}$$

$$B = \sqrt{(a^3 + 3a^2x + 3ax^2 + x^3)^{\frac{2}{3}}} : a$$

$$= \sqrt{\left((a+x)^3\right)^{\frac{2}{3}}} \cdot \frac{1}{a}$$

$$= \sqrt{(a+x)^2} \cdot \frac{1}{a}$$

$$= (a+x)\frac{1}{a}$$

$$\boxed{B = \frac{a+x}{a}}$$

reemplazando:

$$\left[\frac{(\sqrt[3]{x} - \sqrt[3]{a})^3 + 2x + a}{(\sqrt[3]{x} - \sqrt[3]{a})^3 - x - 2a}\right]^3 + \sqrt{(a^3 + 3a^2x + 3ax^2 + x^3)^{\frac{2}{3}}} : a = -\frac{x}{a} + \frac{a+x}{a}$$

$$= \frac{-x + a + x}{a}$$

$$= \frac{a}{a}$$

$$= \boxed{1}$$

113. $\left[\frac{(\sqrt{a} + \sqrt{b})^2 - (2\sqrt{b})^2}{a - b} - (a^{\frac{1}{2}} - b^{\frac{1}{2}})(a^{\frac{1}{2}} + b^{\frac{1}{2}})^{-1}\right] : \frac{(4b)^{\frac{3}{2}}}{a^{\frac{1}{2}} + b^{\frac{1}{2}}}$

Solución.

$$\left[\frac{(\sqrt{a}+\sqrt{b})^2-(2\sqrt{b})^2}{a-b}-(a^{\frac{1}{2}}-b^{\frac{1}{2}})(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-1}\right]:\frac{(4b)^{\frac{3}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}=$$

$$=\overbrace{\left[\frac{(\sqrt{a}+\sqrt{b})^2-(2\sqrt{b})^2}{a-b}-(a^{\frac{1}{2}}-b^{\frac{1}{2}})(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-1}\right]}^{A}:\overbrace{\frac{(4b)^{\frac{3}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}}^{B}$$

$$A=\left[\frac{(\sqrt{a}+\sqrt{b})^2-(2\sqrt{b})^2}{a-b}-(a^{\frac{1}{2}}-b^{\frac{1}{2}})(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-1}\right]$$

$$=\left[\frac{(\sqrt{a}+\sqrt{b}-2\sqrt{b})(\sqrt{a}+\sqrt{b}+2\sqrt{b})}{(\sqrt{a})^2-(\sqrt{b})^2}-\frac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}\right]$$

$$=\left[\frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}+3\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}-\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right]$$

$$=\left[\frac{\cancel{(\sqrt{a}-\sqrt{b})}(\sqrt{a}+3\sqrt{b})}{\cancel{(\sqrt{a}-\sqrt{b})}(\sqrt{a}+\sqrt{b})}-\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right]$$

$$=\left[\frac{\sqrt{a}+3\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right]$$

$$=\frac{\sqrt{a}+3\sqrt{b}-\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$\boxed{A=\frac{4\sqrt{b}}{\sqrt{a}+\sqrt{b}}}$$

$$B=\frac{(4b)^{\frac{3}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}$$

$$=\frac{\sqrt{(4b)^3}}{\sqrt{a}+\sqrt{b}}$$

$$=\frac{4b\sqrt{4b}}{\sqrt{a}+\sqrt{b}}$$

$$=\frac{4b\cdot 2\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$\boxed{B=\frac{8b\sqrt{b}}{\sqrt{a}+\sqrt{b}}}$$

reemplazando:

$$\left[\frac{(\sqrt{a}+\sqrt{b})^2-(2\sqrt{b})^2}{a-b}-(a^{\frac{1}{2}}-b^{\frac{1}{2}})(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-1}\right]:\frac{(4b)^{\frac{3}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}=\frac{4\sqrt{b}}{\sqrt{a}+\sqrt{b}}:\frac{8b\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$=\frac{4\sqrt{b}}{\sqrt{a}+\sqrt{b}}\cdot\frac{\sqrt{a}+\sqrt{b}}{8b\sqrt{b}}$$

$$=\frac{4\sqrt{b}(\sqrt{a}+\sqrt{b})}{8b\sqrt{b}(\sqrt{a}+\sqrt{b})}$$

$$=\frac{4\cancel{\sqrt{b}}\cancel{(\sqrt{a}+\sqrt{b})}}{8b\cancel{\sqrt{b}}\cancel{(\sqrt{a}+\sqrt{b})}}$$

$$= \boxed{\frac{1}{2b}}$$

114. $\left(\frac{a-4b}{a+(ab)^{\frac{1}{2}}-6b} - \frac{a-9b}{a+6(ab)^{\frac{1}{2}}+9b}\right)\frac{b^{-\frac{1}{2}}}{a^{\frac{1}{2}}-3b^{\frac{1}{2}}}$

Solución.

$$\left(\frac{a-4b}{a+(ab)^{\frac{1}{2}}-6b} - \frac{a-9b}{a+6(ab)^{\frac{1}{2}}+9b}\right)\frac{b^{-\frac{1}{2}}}{a^{\frac{1}{2}}-3b^{\frac{1}{2}}} = \overbrace{\left(\frac{a-4b}{a+(ab)^{\frac{1}{2}}-6b} - \frac{a-9b}{a+6(ab)^{\frac{1}{2}}+9b}\right)}^{A}\overbrace{\frac{b^{-\frac{1}{2}}}{a^{\frac{1}{2}}-3b^{\frac{1}{2}}}}^{B}$$

$$\begin{aligned}
A &= \left(\frac{a-4b}{a+(ab)^{\frac{1}{2}}-6b} - \frac{a-9b}{a+6(ab)^{\frac{1}{2}}+9b}\right)\\
&= \left(\frac{(\sqrt{a})^2-(2\sqrt{b})^2}{(\sqrt{a})^2+\sqrt{ab}-6(\sqrt{b})^2} - \frac{(\sqrt{a})^2-(3\sqrt{b})^2}{(\sqrt{a})^2+6\sqrt{ab}+(3\sqrt{b})^2}\right)\\
&= \left(\frac{(\sqrt{a}-2\sqrt{b})(\sqrt{a}+2\sqrt{b})}{(\sqrt{a}+3\sqrt{b})(\sqrt{a}-2\sqrt{b})} - \frac{(\sqrt{a}-3\sqrt{b})(\sqrt{a}+3\sqrt{b})}{(\sqrt{a}+3\sqrt{b})^2}\right)\\
&= \left(\frac{\cancel{(\sqrt{a}-2\sqrt{b})}(\sqrt{a}+2\sqrt{b})}{(\sqrt{a}+3\sqrt{b})\cancel{(\sqrt{a}-2\sqrt{b})}} - \frac{(\sqrt{a}-3\sqrt{b})\cancel{(\sqrt{a}+3\sqrt{b})}}{(\sqrt{a}+3\sqrt{b})^{\cancel{2}}}\right)\\
&= \left(\frac{\sqrt{a}+2\sqrt{b}}{\sqrt{a}+3\sqrt{b}} - \frac{\sqrt{a}-3\sqrt{b}}{\sqrt{a}+3\sqrt{b}}\right)\\
&= \frac{\sqrt{a}+2\sqrt{b}-\sqrt{a}+3\sqrt{b}}{\sqrt{a}+3\sqrt{b}}
\end{aligned}$$

$$\boxed{A = \frac{5\sqrt{b}}{\sqrt{a}+3\sqrt{b}}}$$

$$\begin{aligned}
B &= \frac{b^{-\frac{1}{2}}}{a^{\frac{1}{2}}-3b^{\frac{1}{2}}}\\
&= \frac{1}{b^{\frac{1}{2}}(a^{\frac{1}{2}}-3b^{\frac{1}{2}})}
\end{aligned}$$

$$\boxed{B = \frac{1}{\sqrt{b}(\sqrt{a}-3\sqrt{b})}}$$

reemplazando:

$$\begin{aligned}
\left(\frac{a-4b}{a+(ab)^{\frac{1}{2}}-6b} - \frac{a-9b}{a+6(ab)^{\frac{1}{2}}+9b}\right)\frac{b^{-\frac{1}{2}}}{a^{\frac{1}{2}}-3b^{\frac{1}{2}}} &= \frac{5\sqrt{b}}{\sqrt{a}+3\sqrt{b}}\cdot\frac{1}{\sqrt{b}(\sqrt{a}-3\sqrt{b})}\\
&= \frac{5\sqrt{b}}{\sqrt{b}(\sqrt{a}+3\sqrt{b})(\sqrt{a}-3\sqrt{b})}\\
&= \frac{5\cancel{\sqrt{b}}}{\cancel{\sqrt{b}}(\sqrt{a})^2-(3\sqrt{b})^2}\\
&= \boxed{\frac{5}{a-9b}}
\end{aligned}$$

115. $$\frac{\left(\dfrac{a-b}{\sqrt{a}+\sqrt{b}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3a^2+3b\sqrt{ab}}+\frac{\sqrt{ab}-a}{a\sqrt{a}-b\sqrt{a}}$$

Solución.

$$\frac{\left(\dfrac{a-b}{\sqrt{a}+\sqrt{b}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3a^2+3b\sqrt{ab}}+\frac{\sqrt{ab}-a}{a\sqrt{a}-b\sqrt{a}}=\overbrace{\frac{\left(\dfrac{a-b}{\sqrt{a}+\sqrt{b}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3a^2+3b\sqrt{ab}}}^{A}+\overbrace{\frac{\sqrt{ab}-a}{a\sqrt{a}-b\sqrt{a}}}^{B}$$

$$\begin{aligned}
A&=\frac{\left(\dfrac{a-b}{\sqrt{a}+\sqrt{b}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3a^2+3b\sqrt{ab}}\\
&=\frac{\left(\dfrac{\sqrt{a^2}-\sqrt{b^2}}{\sqrt{a}+\sqrt{b}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3\sqrt{(a^2)^2}+3\sqrt{b^2}\sqrt{ab}}\\
&=\frac{\left(\dfrac{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}{\sqrt{a}+\sqrt{b}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3\sqrt{a^4}+3\sqrt{ab^3}}\\
&=\frac{\left(\dfrac{(\sqrt{a}-\sqrt{b})\cancel{(\sqrt{a}+\sqrt{b})}}{\cancel{\sqrt{a}+\sqrt{b}}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3\sqrt{a}(\sqrt{a^3}+\sqrt{b^3}\,)}\\
&=\frac{(\sqrt{a}-\sqrt{b})^3+2a\sqrt{a}+b\sqrt{b}}{3\sqrt{a}(\sqrt{a^3}+\sqrt{b^3})}\\
&=\frac{\sqrt{a^3}-3\sqrt{a^2}\sqrt{b}+3\sqrt{a}\sqrt{b^2}-\sqrt{b^3}+2a\sqrt{a}+b\sqrt{b}}{3\sqrt{a}(\sqrt{a}+\sqrt{b})(\sqrt{a^2}-\sqrt{a}\sqrt{b}+\sqrt{b^2})}\\
&=\frac{a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b-b\sqrt{b}+2a\sqrt{a}+b\sqrt{b}}{3\sqrt{a}(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}\\
&=\frac{3a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b}{3\sqrt{a}(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}\\
&=\frac{3\sqrt{a}(a-\sqrt{a}\sqrt{b}+b)}{3\sqrt{a}(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}\\
&=\frac{\cancel{3\sqrt{a}}\cancel{(a-\sqrt{a}\sqrt{b}+b)}}{\cancel{3\sqrt{a}}(\sqrt{a}+\sqrt{b})\cancel{(a-\sqrt{a}\sqrt{b}+b)}}
\end{aligned}$$

$$\boxed{A=\frac{1}{\sqrt{a}+\sqrt{b}}}$$

$$\begin{aligned}
B&=\frac{\sqrt{ab}-a}{a\sqrt{a}-b\sqrt{a}}\\
&=\frac{\sqrt{ab}-\sqrt{a^2}}{\sqrt{a}(a-b)}\\
&=\frac{\sqrt{a}(\sqrt{b}-\sqrt{a})}{\sqrt{a}(\sqrt{a^2}-\sqrt{b^2})}
\end{aligned}$$

$$= \frac{-\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}$$

$$= \frac{-(\sqrt{a}-\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}$$

$$= \frac{-\cancel{(\sqrt{a}-\sqrt{b})}}{\cancel{(\sqrt{a}-\sqrt{b})}(\sqrt{a}+\sqrt{b})}$$

$$\boxed{B = \frac{-1}{\sqrt{a}+\sqrt{b}}}$$

reemplazando:

$$\frac{\left(\frac{a-b}{\sqrt{a}+\sqrt{b}}\right)^3 + 2a\sqrt{a}+b\sqrt{b}}{3a^2+3b\sqrt{ab}} + \frac{\sqrt{ab}-a}{a\sqrt{a}-b\sqrt{a}} = \frac{1}{\sqrt{a}+\sqrt{b}} + \frac{-1}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{a}+\sqrt{b}}$$

$$= \boxed{0}$$

116. $\dfrac{(\sqrt{a}-\sqrt{b})^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}} + \dfrac{3\sqrt{ab}-3b}{a-b}$

Solución.

$$\frac{(\sqrt{a}-\sqrt{b})^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}} + \frac{3\sqrt{ab}-3b}{a-b} = \overbrace{\frac{(\sqrt{a}-\sqrt{b})^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}}}^{A} + \overbrace{\frac{3\sqrt{ab}-3b}{a-b}}^{B}$$

$$A = \frac{(\sqrt{a}-\sqrt{b})^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}}$$

$$= \frac{\sqrt{a^3}-3\sqrt{a^2}\sqrt{b}+3\sqrt{a}\sqrt{b^2}-\sqrt{b^3}+2a^2\cdot\frac{1}{\sqrt{a}}+b\sqrt{b}}{\sqrt{a^2}\sqrt{a}+\sqrt{b^2}\sqrt{b}}$$

$$= \frac{a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b-b\sqrt{b}+2a^2\cdot\frac{1}{a^{\frac{1}{2}}}+b\sqrt{b}}{\sqrt{a^3}+\sqrt{b^3}}$$

$$= \frac{a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b+2a^2\cdot a^{-\frac{1}{2}}}{(\sqrt{a}+\sqrt{b})(\sqrt{a^2}-\sqrt{a}\sqrt{b}+\sqrt{b^2})}$$

$$= \frac{a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b+2a^{\frac{3}{2}}}{(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}$$

$$= \frac{a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b+2\sqrt{a^3}}{(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}$$

$$= \frac{a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b+2a\sqrt{a}}{(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}$$

$$= \frac{3a\sqrt{a}-3a\sqrt{b}+3\sqrt{a}b}{(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}$$

$$= \frac{3\sqrt{a}(a-\sqrt{a}\sqrt{b}+b)}{(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}$$

$$= \frac{3\sqrt{a}(a-\sqrt{a}\sqrt{b}+b)}{(\sqrt{a}+\sqrt{b})(a-\sqrt{a}\sqrt{b}+b)}$$

$$\boxed{A = \frac{3\sqrt{a}}{\sqrt{a}+\sqrt{b}}}$$

$$B = \frac{3\sqrt{ab}-3b}{a-b}$$

$$= \frac{3\sqrt{ab}-3\sqrt{b^2}}{\sqrt{a^2}-\sqrt{b^2}}$$

$$= \frac{3\sqrt{b}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}$$

$$= \frac{3\sqrt{b}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}$$

$$\boxed{B = \frac{3\sqrt{b}}{\sqrt{a}+\sqrt{b}}}$$

reemplazando:

$$\frac{(\sqrt{a}-\sqrt{b})^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}} + \frac{3\sqrt{ab}-3b}{a-b} = \frac{3\sqrt{a}}{\sqrt{a}+\sqrt{b}} + \frac{3\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{3\sqrt{a}+3\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{3(\sqrt{a}+\sqrt{b})}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{3(\sqrt{a}+\sqrt{b})}{\sqrt{a}+\sqrt{b}}$$

$$= \boxed{3}$$

117. $\left[\frac{1}{(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}}-b^{\frac{3}{2}}}\right)^{-1}\right](ab)^{-\frac{1}{2}}$

Solución.

$$\left[\frac{1}{(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}}-b^{\frac{3}{2}}}\right)^{-1}\right](ab)^{-\frac{1}{2}} = \overbrace{\left[\frac{1}{(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}}-b^{\frac{3}{2}}}\right)^{-1}\right]}^{A}\overbrace{(ab)^{-\frac{1}{2}}}^{B}$$

$$A = \left[\frac{1}{(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}}-b^{\frac{3}{2}}}\right)^{-1}\right]$$

$$= \left[(a^{\frac{1}{2}}+b^{\frac{1}{2}})^2 - \left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a^3}-\sqrt{b^3}}\right)^{-1}\right]$$

$$= \left[(\sqrt{a}+\sqrt{b})^2 - \left(\frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a^2}+\sqrt{a}\sqrt{b}+\sqrt{b^2})}\right)^{-1}\right]$$

$$= \left[\sqrt{a^2} + 2\sqrt{a}\sqrt{b} + \sqrt{b^2} - \left(\frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a^2}+\sqrt{a}\sqrt{b}+\sqrt{b^2})}\right)^{-1}\right]$$

$$= \left[a + 2\sqrt{a}\sqrt{a} + b - \left(\frac{1}{a+\sqrt{a}\sqrt{b}+b}\right)^{-1}\right]$$

$$= \left[a + 2\sqrt{a}\sqrt{a} + b - \frac{1}{\dfrac{1}{a+\sqrt{a}\sqrt{b}+b}}\right]$$

$$= \left[a + 2\sqrt{a}\sqrt{b} + b - (a+\sqrt{a}\sqrt{b}+b)\right]$$

$$= [a + 2\sqrt{a}\sqrt{b} + b - a - \sqrt{a}\sqrt{b} - b]$$

$$= \sqrt{a}\sqrt{b}$$

$$\boxed{A = \sqrt{ab}}$$

$$B = (ab)^{-\frac{1}{2}}$$

$$= \frac{1}{(ab)^{\frac{1}{2}}}$$

$$\boxed{B = \frac{1}{\sqrt{ab}}}$$

reemplazando:

$$\left[\frac{1}{(a^{\frac{1}{2}}+b^{\frac{1}{2}})^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}}-b^{\frac{3}{2}}}\right)^{-1}\right](ab)^{-\frac{1}{2}} = \frac{1}{\sqrt{ab}}\cdot\sqrt{ab}$$

$$= \frac{\sqrt{ab}}{\sqrt{ab}}$$

$$= \boxed{1}$$

118. $$\left[\frac{\dfrac{1}{a}-a}{\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}-1\right)} + \sqrt[3]{a}\right]^{-3}$$

Solución.

$$\left[\frac{\dfrac{1}{a}-a}{\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}-1\right)} + \sqrt[3]{a}\right]^{-3} = \left[\overbrace{\frac{\dfrac{1}{a}-a}{\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}-1\right)}}^{A} + \sqrt[3]{a}\right]^{-3}$$

$$A = \frac{\dfrac{1}{a}-a}{\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}-1\right)}$$

$$= \frac{\dfrac{1-a}{a}}{\left(\sqrt[3]{a}+\dfrac{1}{\sqrt[3]{a}}+1\right)\left(\sqrt[3]{a}+\dfrac{1}{\sqrt[3]{a}}-1\right)}$$

$$= \frac{\dfrac{(1-a)(1+a)}{a}}{\left(\dfrac{\sqrt[3]{a^2}+1+\sqrt[3]{a}}{\sqrt[3]{a}}\right)\left(\dfrac{\sqrt[3]{a^2}+1-\sqrt[3]{a}}{\sqrt[3]{a}}\right)}$$

$$= \frac{\dfrac{(1-\sqrt[3]{a^3})(1+\sqrt[3]{a^3})}{a}}{\left(\dfrac{1+\sqrt[3]{a}+\sqrt[3]{a^2}}{\sqrt[3]{a}}\right)\left(\dfrac{1-\sqrt[3]{a}+\sqrt[3]{a^2}}{\sqrt[3]{a}}\right)}$$

$$= \frac{\dfrac{(1-\sqrt[3]{a})(1+\sqrt[3]{a}+\sqrt[3]{a^2})(1+\sqrt[3]{x})(1-\sqrt[3]{a}+\sqrt[3]{a^2})}{a}}{\dfrac{(1+\sqrt[3]{a}+\sqrt[3]{a^2})(1-\sqrt[3]{a}+\sqrt[3]{a^2})}{\sqrt[3]{a^2}}}$$

$$= \frac{(1-\sqrt[3]{a})(1+\sqrt[3]{a}+\sqrt[3]{a^2})(1+\sqrt[3]{x})(1-\sqrt[3]{a}+\sqrt[3]{a^2})\sqrt[3]{a^2}}{a(1+\sqrt[3]{a}+\sqrt[3]{a^2})(1-\sqrt[3]{a}+\sqrt[3]{a^2})}$$

$$= \frac{(1-\sqrt[3]{a})\cancel{(1+\sqrt[3]{a}+\sqrt[3]{a^2})}(1+\sqrt[3]{x})\cancel{(1-\sqrt[3]{a}+\sqrt[3]{a^2})}\sqrt[3]{a^2}}{a\cancel{(1+\sqrt[3]{a}+\sqrt[3]{a^2})}\cancel{(1-\sqrt[3]{a}+\sqrt[3]{a^2})}}$$

$$= \frac{(1-\sqrt[3]{a})(1+\sqrt[3]{x})\sqrt[3]{a^2}}{a}$$

$$= \frac{(1-\sqrt[3]{a^2})a^{\frac{2}{3}}}{a}$$

$$= \frac{1-\sqrt[3]{a^2}}{a\cdot a^{-\frac{2}{3}}}$$

$$= \frac{1-\sqrt[3]{a^2}}{a^{\frac{1}{3}}}$$

$$\boxed{A = \frac{1-\sqrt[3]{a^2}}{\sqrt[3]{a}}}$$

reemplazando:

$$\left[\frac{\dfrac{1}{a}-a}{\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{\dfrac{1}{a}}-1\right)}+\sqrt[3]{a}\right]^{-3} = \left[\frac{1-\sqrt[3]{a^2}}{\sqrt[3]{a}}+\sqrt[3]{a}\right]^{-3}$$

$$= \left[\frac{1-\sqrt[3]{a^2}+\sqrt[3]{a^2}}{\sqrt[3]{a}}\right]^{-3}$$

$$= \left[\frac{1}{\sqrt[3]{a}}\right]^{-3}$$

$$= \left[\frac{1}{a^{\frac{1}{3}}}\right]^{-3}$$

$$= \left[a^{-\frac{1}{3}}\right]^{-3}$$

$$= \boxed{a}$$

119. $\left[\dfrac{a\sqrt[3]{a}+\sqrt[3]{a^2}}{a+\sqrt[3]{a}}-\sqrt[3]{x}\right]\left[(\sqrt[3]{a}-\sqrt[3]{x})^2+3(\sqrt[3]{a}+\sqrt[3]{x})^2\right]$

Solución.

$$\left[\frac{a\sqrt[3]{a}+\sqrt[3]{a^2}}{a+\sqrt[3]{a}}-\sqrt[3]{x}\right]\left[(\sqrt[3]{a}-\sqrt[3]{x})^2+3(\sqrt[3]{a}+\sqrt[3]{x})^2\right]=\overbrace{\left[\frac{a\sqrt[3]{a}+\sqrt[3]{a^2}}{a+\sqrt[3]{a}}-\sqrt[3]{x}\right]}^{A}\overbrace{\left[(\sqrt[3]{a}-\sqrt[3]{x})^2+3(\sqrt[3]{a}+\sqrt[3]{x})^2\right]}^{B}$$

$$A=\left[\frac{a\sqrt[3]{a}+\sqrt[3]{a^2}}{a+\sqrt[3]{a}}-\sqrt[3]{x}\right]$$

$$=\left[\frac{\sqrt[3]{a}(a+\sqrt[3]{a})}{a+\sqrt[3]{a}}-\sqrt[3]{x}\right]$$

$$=\left[\frac{\sqrt[3]{a}\cancel{(a+\sqrt[3]{a})}}{\cancel{a+\sqrt[3]{a}}}-\sqrt[3]{x}\right]$$

$$\boxed{A=\left[\sqrt[3]{a}-\sqrt[3]{x}\right]}$$

$$B=\left[(\sqrt[3]{a}-\sqrt[3]{x})^2+3(\sqrt[3]{a}+\sqrt[3]{x})^2\right]$$

$$=\left[\sqrt[3]{a^2}-2\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2}+3(\sqrt[3]{a^2}+2\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2})\right]$$

$$=\left[\sqrt[3]{a^2}-2\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2}+3\sqrt[3]{a^2}+6\sqrt[3]{a}\sqrt[3]{x}+3\sqrt[3]{x^2}\right]$$

$$=\left[4\sqrt[3]{a^2}+4\sqrt[3]{a}\sqrt[3]{x}+4\sqrt[3]{x^2}\right]$$

$$\boxed{B=4[\sqrt[3]{a^2}+\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2}]}$$

reemplazando:

$$\left[\frac{a\sqrt[3]{a}+\sqrt[3]{a^2}}{a+\sqrt[3]{a}}-\sqrt[3]{x}\right]\left[(\sqrt[3]{a}-\sqrt[3]{x})^2+3(\sqrt[3]{a}+\sqrt[3]{x})^2\right]=\left[\sqrt[3]{a}-\sqrt[3]{x}\right]\cdot 4\left[\sqrt[3]{a^2}+\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2}\right]$$

$$=4\left[\sqrt[3]{a}-\sqrt[3]{x}\right]\left[\sqrt[3]{a^2}+\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2}\right]$$

$$=4\left[\sqrt[3]{a^3}-\sqrt[3]{x^3}\right]$$

$$=\boxed{4[a-x]}$$

120. $\left[\left(\dfrac{a^2-b\sqrt{a}}{\sqrt{a}-\sqrt[3]{b}}+a\sqrt[3]{b}\right):(a+\sqrt[6]{a^3b^2})-\sqrt[3]{b}\right]^2$

Solución.

$$\left[\left(\frac{a^2-b\sqrt{a}}{\sqrt{a}-\sqrt[3]{b}}+a\sqrt[3]{b}\right):(a+\sqrt[6]{a^3b^2})-\sqrt[3]{b}\right]^2=\left[\overbrace{\left(\frac{a^2-b\sqrt{a}}{\sqrt{a}-\sqrt[3]{b}}+a\sqrt[3]{b}\right)}^{A}:\underbrace{(a+\sqrt[6]{a^3b^2})-\sqrt[3]{b}}_{B}\right]^2$$

$$A=\left(\frac{a^2-b\sqrt{a}}{\sqrt{a}-\sqrt[3]{b}}+a\sqrt[3]{b}\right)$$

$$=\left(\frac{\sqrt{(a^2)^2}-b\sqrt{a}}{\sqrt{a}-\sqrt[3]{b}}+a\sqrt[3]{b}\right)$$

$$=\left(\frac{\sqrt{a^4}-b\sqrt{a}}{\sqrt{a}-\sqrt[3]{b}}+a\sqrt[3]{b}\right)$$

$$= \left(\frac{\sqrt{a}(\sqrt{a^3} - b)}{\sqrt{a} - \sqrt[3]{b}} + a\sqrt[3]{b} \right)$$

$$= \left(\frac{\sqrt{a}(\sqrt{a^3} - \sqrt[3]{b^3})}{\sqrt{a} - \sqrt[3]{b}} + a\sqrt[3]{b} \right)$$

$$= \left(\frac{\sqrt{a}(\sqrt{a} - \sqrt[3]{b})(\sqrt{a^2} + \sqrt{a}\sqrt[3]{b} + \sqrt[3]{b^2})}{\sqrt{a} - \sqrt[3]{b}} + a\sqrt[3]{b} \right)$$

$$= \left(\frac{\sqrt{a}\cancel{(\sqrt{a} - \sqrt[3]{b})}(\sqrt{a^2} + \sqrt{a}\sqrt[3]{b} + \sqrt[3]{b^2})}{\cancel{\sqrt{a} - \sqrt[3]{b}}} + a\sqrt[3]{b} \right)$$

$$= \sqrt{a}(\sqrt{a^2} + \sqrt{a}\sqrt[3]{b} + \sqrt[3]{b^2}) + a\sqrt[3]{b}$$

$$= \sqrt{a}(a + \sqrt{a}\sqrt[3]{b} + \sqrt[3]{b^2}) + \sqrt{a^2}\sqrt[3]{b}$$

$$= \sqrt{a}(\sqrt{a^2} + \sqrt{a}\sqrt[3]{b} + \sqrt[3]{b^2} + \sqrt{a}\sqrt[3]{b})$$

$$= \sqrt{a}(\sqrt{a^2} + 2\sqrt{a}\sqrt[3]{b} + \sqrt[3]{b^2})$$

$$\boxed{A = \sqrt{a}(\sqrt{a} + \sqrt[3]{b})^2}$$

$$B = (a + \sqrt[6]{a^3b^2})$$

$$= (a + \sqrt[6]{a^3}\sqrt[6]{b^2})$$

$$= (\sqrt{a^2} + \sqrt{a}\sqrt[3]{b})$$

$$\boxed{B = \sqrt{a}(\sqrt{a} + \sqrt[3]{b})}$$

reemplazando:

$$\left[\left(\frac{a^2 - b\sqrt{a}}{\sqrt{a} - \sqrt[3]{b}} + a\sqrt[3]{b}\right) : (a + \sqrt[6]{a^3b^2}) - \sqrt[3]{b}\right]^2 = [\sqrt{a}(\sqrt{a} + \sqrt[3]{b})^2 : \sqrt{a}(\sqrt{a} + \sqrt[3]{b}) - \sqrt[3]{b}]^2$$

$$= \left[\sqrt{a}(\sqrt{a} + \sqrt[3]{b})^2 \cdot \frac{1}{\sqrt{a}(\sqrt{a} + \sqrt[3]{b})} - \sqrt[3]{b}\right]^2$$

$$= \left[\frac{\sqrt{a}(\sqrt{a} + \sqrt[3]{b})^2}{\sqrt{a}(\sqrt{a} + \sqrt[3]{b})} - \sqrt[3]{b}\right]^2$$

$$= \left[\frac{\cancel{\sqrt{a}}(\sqrt{a} + \sqrt[3]{b})^{\cancel{2}}}{\cancel{\sqrt{a}}\cancel{(\sqrt{a} + \sqrt[3]{b})}} - \sqrt[3]{b}\right]^2$$

$$= [\sqrt{a} + \sqrt[3]{b} - \sqrt[3]{b}]^2$$

$$= [\sqrt{a}]^2$$

$$= \boxed{a}$$

121. $\left[\frac{a^2\sqrt[4]{x} + x\sqrt{a}}{a\sqrt[4]{x} + \sqrt{ax}} - \sqrt{a^2 + x + 2a\sqrt{x}}\right]^4$

Solución

$$\left[\frac{a^2\sqrt[4]{x} + x\sqrt{a}}{a\sqrt[4]{x} + \sqrt{ax}} - \sqrt{a^2 + x + 2a\sqrt{x}}\right]^4 = \left[\overbrace{\frac{a^2\sqrt[4]{x} + x\sqrt{a}}{a\sqrt[4]{x} + \sqrt{ax}}}^{A} - \underbrace{\sqrt{a^2 + x + 2a\sqrt{x}}}_{B}\right]^4$$

$$\begin{aligned}
A &= \frac{a^2\sqrt[4]{x}+x\sqrt{a}}{a\sqrt[4]{x}+\sqrt{ax}}\\
&= \frac{\sqrt{a^4}\sqrt[4]{x}+\sqrt[4]{x^4}\sqrt{a}}{\sqrt{a^2}\sqrt[4]{x}+\sqrt{ax}}\\
&= \frac{\sqrt{a}\sqrt[4]{x}(\sqrt{a^3}+\sqrt[4]{x^3})}{\sqrt{a}(\sqrt{a}\sqrt[4]{x}+\sqrt{x})}\\
&= \frac{\sqrt[4]{x}(\sqrt{a^3}+\sqrt[4]{x^3})}{\sqrt{a}\sqrt[4]{x}+\sqrt{x}}\\
&= \frac{\sqrt[4]{x}(\sqrt{a}+\sqrt[4]{x})(\sqrt{a^2}-\sqrt{a}\sqrt[4]{x}+\sqrt[4]{x^2})}{\sqrt{a}\sqrt[4]{x}+\sqrt[4]{x^2}}\\
&= \frac{\sqrt[4]{x}(\sqrt{a}+\sqrt[4]{x})(a-\sqrt{a}\sqrt[4]{x}+\sqrt{x})}{\sqrt[4]{x}(\sqrt{a}+\sqrt[4]{x})}\\
&= \frac{\cancel{\sqrt[4]{x}}\cancel{(\sqrt{a}+\sqrt[4]{x})}(a-\sqrt{a}\sqrt[4]{x}+\sqrt{x})}{\cancel{\sqrt[4]{x}}\cancel{(\sqrt{a}+\sqrt[4]{x})}}
\end{aligned}$$

$$\boxed{A = a-\sqrt{a}\sqrt[4]{x}+\sqrt{x}}$$

$$\begin{aligned}
B &= \sqrt{a^2+x+2a\sqrt{x}}\\
&= \sqrt{a^2+2a\sqrt{x}+x}\\
&= \sqrt{a^2+2a\sqrt{x}+\sqrt{x^2}}\\
&= \sqrt{(a+\sqrt{x})^2}
\end{aligned}$$

$$\boxed{B = a+\sqrt{x}}$$

reemplazando:

$$\begin{aligned}
\left[\frac{a^2\sqrt[4]{x}+x\sqrt{a}}{a\sqrt[4]{x}+\sqrt{ax}}-\sqrt{a^2+x+2a\sqrt{x}}\right]^4 &= [a-\sqrt{a}\sqrt[4]{x}+\sqrt{x}-(a+\sqrt{x})]^4\\
&= [a-\sqrt{a}\sqrt[4]{x}+\sqrt{x}-a-\sqrt{x}]^4\\
&= [-\sqrt{a}\sqrt[4]{x}]^4\\
&= \sqrt{a^4}\sqrt[4]{x^4}\\
&= \boxed{a^2x}
\end{aligned}$$

122. $$\left[\frac{x\sqrt{x}-x}{\left(\frac{\sqrt[4]{x^3}-1}{\sqrt[4]{x}-1}-\sqrt{x}\right)\left(\frac{\sqrt[4]{x^3}+1}{\sqrt[4]{x}+1}-\sqrt{x}\right)}\right]^3$$

Solución.

$$\left[\frac{x\sqrt{x}-x}{\left(\frac{\sqrt[4]{x^3}-1}{\sqrt[4]{x}-1}-\sqrt{x}\right)\left(\frac{\sqrt[4]{x^3}+1}{\sqrt[4]{x}+1}-\sqrt{x}\right)}\right]^3 = \left[\frac{\overbrace{x\sqrt{x}-x}^{A}}{\underbrace{\left(\frac{\sqrt[4]{x^3}-1}{\sqrt[4]{x}-1}-\sqrt{x}\right)}_{B}\underbrace{\left(\frac{\sqrt[4]{x^3}+1}{\sqrt[4]{x}+1}-\sqrt{x}\right)}_{C}}\right]^3$$

$$A = x\sqrt{x} - x$$

$$\boxed{A = x(\sqrt{x} - 1)}$$

$$\begin{aligned}
B &= \frac{\sqrt[4]{x^3} - 1}{\sqrt[4]{x} - 1} - \sqrt{x} \\
&= \frac{(\sqrt[4]{x} - 1)(\sqrt[4]{x^2} + \sqrt[4]{x} + 1)}{\sqrt[4]{x} - 1} - \sqrt{x} \\
&= \frac{\cancel{(\sqrt[4]{x} - 1)}(\sqrt[4]{x^2} + \sqrt[4]{x} + 1)}{\cancel{\sqrt[4]{x} - 1}} - \sqrt{x} \\
&= \sqrt{x} + \sqrt[4]{x} + 1 - \sqrt{x}
\end{aligned}$$

$$\boxed{B = \sqrt[4]{x} + 1}$$

$$\begin{aligned}
C &= \frac{\sqrt[4]{x^3} + 1}{\sqrt[4]{x} + 1} - \sqrt{x} \\
&= \frac{(\sqrt[4]{x} + 1)(\sqrt[4]{x^2} - \sqrt[4]{x} + 1)}{\sqrt[4]{x} + 1} - \sqrt{x} \\
&= \frac{(\sqrt[4]{x} + 1)(\sqrt[4]{x^2} - \sqrt[4]{x} + 1)}{\sqrt[4]{x} + 1} - \sqrt{x} \\
&= \sqrt{x} - \sqrt[4]{x} + 1 - \sqrt{x} \\
&= \sqrt[4]{x} + 1 \\
&= -\sqrt[4]{x} + 1
\end{aligned}$$

$$\boxed{C = -(\sqrt[4]{x} - 1)}$$

reemplazando:

$$\begin{aligned}
\left[\frac{x\sqrt{x} - x}{\left(\frac{\sqrt[4]{x^3} - 1}{\sqrt[4]{x} - 1} - \sqrt{x}\right)\left(\frac{\sqrt[4]{x^3} + 1}{\sqrt[4]{x} + 1} - \sqrt{x}\right)}\right]^3 &= \left[\frac{x(\sqrt{x} - 1)}{(\sqrt[4]{x} + 1)\big(-(\sqrt[4]{x} - 1)\big)}\right]^3 \\
&= \left[\frac{x(\sqrt{x} - 1)}{-(\sqrt[4]{x} + 1)(\sqrt[4]{x} - 1)}\right]^3 \\
&= \left[-\frac{x(\sqrt{x} - 1)}{(\sqrt[4]{x^2} - 1)}\right]^3 \\
&= \left[-\frac{x(\sqrt{x} - 1)}{\sqrt{x} - 1}\right]^3 \\
&= \left[-\frac{x\cancel{(\sqrt{x} - 1)}}{\cancel{\sqrt{x} - 1}}\right]^3 \\
&= [-x]^3 \\
&= \boxed{-x^3}
\end{aligned}$$

123. $\sqrt{a}\left[\frac{a + \sqrt[4]{a^3b^2} + b\sqrt[4]{ab^2} + b^2}{(\sqrt[4]{a} + \sqrt{b})^2} - b\right]^{-1} + \frac{1}{a^{-\frac{1}{4}}b^{\frac{1}{2}} - 1}$

Solución.

$$\sqrt{a}\left[\frac{a+\sqrt[4]{a^3b^2}+b\sqrt[4]{ab^2}+b^2}{(\sqrt[4]{a}+\sqrt{b})^2}-b\right]^{-1}+\frac{1}{a^{-\frac{1}{4}}b^{\frac{1}{2}}-1}=\sqrt{a}\left[\overbrace{\frac{a+\sqrt[4]{a^3b^2}+b\sqrt[4]{ab^2}+b^2}{(\sqrt[4]{a}+\sqrt{b})^2}-b}^{A}\right]^{-1}+\overbrace{\frac{1}{a^{-\frac{1}{4}}b^{\frac{1}{2}}-1}}^{B}$$

$$\begin{aligned}
A &= \frac{a+\sqrt[4]{a^3b^2}+b\sqrt[4]{ab^2}+b^2}{(\sqrt[4]{a}+\sqrt{b})^2}-b\\
&= \frac{\sqrt[4]{a^4}+\sqrt[4]{a^3}\sqrt{b}+b(\sqrt[4]{ab^2}+b)}{(\sqrt[4]{a}+\sqrt{b})^2}-b\\
&= \frac{\sqrt[4]{a^3}(\sqrt[4]{a}+\sqrt{b})+b(\sqrt[4]{a}\sqrt{b}+\sqrt{b^2})}{(\sqrt[4]{a}+\sqrt{b})^2}-b\\
&= \frac{\sqrt[4]{a^3}(\sqrt[4]{a}+\sqrt{b})+b\sqrt{b}(\sqrt[4]{a}+\sqrt{b})}{(\sqrt[4]{a}+\sqrt{b})^2}-b\\
&= \frac{(\sqrt[4]{a}+\sqrt{b})(\sqrt[4]{a^3}+b\sqrt{b})}{(\sqrt[4]{a}+\sqrt{b})^2}-b\\
&= \frac{\cancel{(\sqrt[4]{a}+\sqrt{b})}(\sqrt[4]{a^3}+b\sqrt{b})}{(\sqrt[4]{a}+\sqrt{b})^{\cancel{2}}}-b\\
&= \frac{\sqrt[4]{a^3}+b\sqrt{b}}{\sqrt[4]{a}+\sqrt{b}}-b\\
&= \frac{\sqrt[4]{a^3}+\sqrt{b^2}\sqrt{b}}{\sqrt[4]{a}+\sqrt{b}}-b\\
&= \frac{\sqrt[4]{a^3}+\sqrt{b^3}}{\sqrt[4]{a}+\sqrt{b}}-b\\
&= \frac{(\sqrt[4]{a}+\sqrt{b})(\sqrt[4]{a^2}-\sqrt[4]{a}\sqrt{b}+\sqrt{b^2})}{\sqrt[4]{a}+\sqrt{b}}-b\\
&= \frac{\cancel{(\sqrt[4]{a}+\sqrt{b})}(\sqrt[4]{a^2}-\sqrt[4]{a}\sqrt{b}+\sqrt{b^2})}{\cancel{\sqrt[4]{a}+\sqrt{b}}}-b\\
&= \sqrt[4]{a^2}-\sqrt[4]{a}\sqrt{b}+\sqrt{b^2}-b\\
&= \sqrt{a}-\sqrt[4]{a}\sqrt{b}+b-b\\
&= \sqrt{a}-\sqrt[4]{a}\sqrt{b}\\
&= \sqrt[4]{a^2}-\sqrt[4]{a}\sqrt{b}
\end{aligned}$$

$$\boxed{A=\sqrt[4]{a}(\sqrt[4]{a}-\sqrt{b})}$$

$$\begin{aligned}
B &= \frac{1}{a^{-\frac{1}{4}}b^{\frac{1}{2}}-1}\\
&= \frac{1}{\frac{b^{\frac{1}{2}}}{a^{\frac{1}{4}}}-1}\\
&= \frac{1}{\frac{\sqrt{b}}{\sqrt[4]{a}}-1}
\end{aligned}$$

$$= \frac{1}{\dfrac{\sqrt{b}-\sqrt[4]{a}}{\sqrt[4]{a}}}$$

$$= \frac{\sqrt[4]{a}}{\sqrt{b}-\sqrt[4]{a}}$$

$$= \frac{\sqrt[4]{a}}{-\sqrt[4]{a}+\sqrt{b}}$$

$$\boxed{B = \frac{\sqrt[4]{a}}{-(\sqrt[4]{a}-\sqrt{b})}}$$

reemplazando:

$$\sqrt{a}\left[\frac{a+\sqrt[4]{a^3b^2}+b\sqrt[4]{ab^2}+b^2}{(\sqrt[4]{a}+\sqrt{b})^2}-b\right]^{-1}+\frac{1}{a^{-\frac{1}{4}}b^{\frac{1}{2}}-1} = \sqrt{a}\big[\sqrt[4]{a}(\sqrt[4]{a}-\sqrt{b})\big]^{-1}+\frac{\sqrt[4]{a}}{-(\sqrt[4]{a}-\sqrt{b})}$$

$$= \sqrt{a}\cdot\frac{1}{\sqrt[4]{a}(\sqrt[4]{a}-\sqrt{b})}-\frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt{b}}$$

$$= \frac{\sqrt{a}}{\sqrt[4]{a}(\sqrt[4]{a}-\sqrt{b})}-\frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt{b}}$$

$$= \frac{a^{\frac{1}{2}}}{a^{\frac{1}{4}}(\sqrt[4]{a}-\sqrt{b})}-\frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt{b}}$$

$$= \frac{a^{\frac{1}{2}}\cdot a^{-\frac{1}{4}}}{\sqrt[4]{a}-\sqrt{b}}-\frac{\sqrt[4]{a}}{\sqrt[4]{a}+\sqrt{b}}$$

$$= \frac{a^{\frac{1}{4}}}{\sqrt[4]{a}-\sqrt{b}}-\frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt{b}}$$

$$= \frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt{b}}-\frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt{b}}$$

$$= \boxed{0}$$

124. $$\frac{\dfrac{a+x}{\sqrt[3]{a^2}-\sqrt[3]{x^2}}+\dfrac{\sqrt[3]{ax^2}-\sqrt[3]{a^2x}}{\sqrt[3]{a^2}-2\sqrt[3]{ax}+\sqrt[3]{x^2}}}{\sqrt[6]{a}-\sqrt[6]{x}}-\sqrt[6]{x}$$

Solución.

$$\frac{\dfrac{a+x}{\sqrt[3]{a^2}-\sqrt[3]{x^2}}+\dfrac{\sqrt[3]{ax^2}-\sqrt[3]{a^2x}}{\sqrt[3]{a^2}-2\sqrt[3]{ax}+\sqrt[3]{x^2}}}{\sqrt[6]{a}-\sqrt[6]{x}}-\sqrt[6]{x} = \frac{\dfrac{\sqrt[3]{a^3}+\sqrt[3]{x^3}}{(\sqrt[3]{a}-\sqrt[3]{x})(\sqrt[3]{a}+\sqrt[3]{x})}+\dfrac{-\sqrt[3]{a^2x}+\sqrt[3]{ax^2}}{(\sqrt[3]{a}-\sqrt[3]{x})^2}}{\sqrt[6]{a}-\sqrt[6]{x}}-\sqrt[6]{x}$$

$$= \frac{\dfrac{(\sqrt[3]{a}+\sqrt[3]{x})(\sqrt[3]{a^2}-\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2})}{(\sqrt[3]{a}-\sqrt[3]{x})(\sqrt[3]{a}+\sqrt[3]{x})}+\dfrac{-\sqrt[3]{ax}(\sqrt[3]{a}-\sqrt[3]{x})}{(\sqrt[3]{a}-\sqrt[3]{x})^2}}{\sqrt[6]{a}-\sqrt[6]{x}}-\sqrt[6]{x}$$

$$= \frac{\dfrac{\cancel{(\sqrt[3]{a}+\sqrt[3]{x})}(\sqrt[3]{a^2}-\sqrt[3]{a}\sqrt[3]{x}+\sqrt[3]{x^2})}{(\sqrt[3]{a}-\sqrt[3]{x})\cancel{(\sqrt[3]{a}+\sqrt[3]{x})}}+\dfrac{-\sqrt[3]{ax}\cancel{(\sqrt[3]{a}-\sqrt[3]{x})}}{(\sqrt[3]{a}-\sqrt[3]{x})^{\cancel{2}}}}{\sqrt[6]{a}-\sqrt[6]{x}}-\sqrt[6]{x}$$

$$= \frac{\dfrac{\sqrt[3]{a^2}-\sqrt[3]{ax}+\sqrt[3]{x^2}}{\sqrt[3]{a}-\sqrt[3]{x}}-\dfrac{\sqrt[3]{ax}}{\sqrt[3]{a}-\sqrt[3]{x}}}{\sqrt[6]{a}-\sqrt[6]{x}}-\sqrt[6]{x}$$

$$= \frac{\dfrac{\sqrt[3]{a^2} - \sqrt[3]{ax} + \sqrt[3]{x^2} - \sqrt[3]{ax}}{\sqrt[3]{a} - \sqrt[3]{x}}}{\sqrt[6]{a} - \sqrt[6]{x}} - \sqrt[6]{x}$$

$$= \frac{\dfrac{\sqrt[3]{a^2} - 2\sqrt[3]{ax} + \sqrt[3]{x^2}}{\sqrt[3]{a} - \sqrt[3]{x}}}{\sqrt[6]{a} - \sqrt[6]{x}} - \sqrt[6]{x}$$

$$= \frac{\dfrac{(\sqrt[3]{a} - \sqrt[3]{x})^2}{\sqrt[3]{a} - \sqrt[3]{x}}}{\sqrt[6]{a} - \sqrt[6]{x}} - \sqrt[6]{x}$$

$$= \frac{\dfrac{(\sqrt[3]{a} - \sqrt[3]{x})^{\not 2}}{\cancel{\sqrt[3]{a} - \sqrt[3]{x}}}}{\sqrt[6]{a} - \sqrt[6]{x}} - \sqrt[6]{x}$$

$$= \frac{\sqrt[3]{a} - \sqrt[3]{x}}{\sqrt[6]{a} - \sqrt[6]{x}} - \sqrt[6]{x}$$

$$= \frac{\sqrt[6]{a^2} - \sqrt[6]{x^2}}{\sqrt[6]{a} - \sqrt[6]{x}} - \sqrt[6]{x}$$

$$= \frac{(\sqrt[6]{a} - \sqrt[6]{x})(\sqrt[6]{a} + \sqrt[6]{x})}{\sqrt[6]{a} - \sqrt[6]{x}} - \sqrt[6]{x}$$

$$= \frac{\cancel{(\sqrt[6]{a} - \sqrt[6]{x})}(\sqrt[6]{a} + \sqrt[6]{x})}{\cancel{\sqrt[6]{a} - \sqrt[6]{x}}} - \sqrt[6]{x}$$

$$= \sqrt[6]{a} + \sqrt[6]{x} - \sqrt[6]{x}$$

$$= \boxed{\sqrt[6]{a}}$$

125. $\dfrac{1}{a^{\frac{1}{4}} + a^{\frac{1}{8}} + 1} + \dfrac{1}{a^{\frac{1}{4}} - a^{\frac{1}{8}} + 1} - \dfrac{2a^{\frac{1}{4}} - 2}{a^{\frac{1}{2}} - a^{\frac{1}{4}} + 1}$

Solución.

Realizando operaciones en las dos primeras fraciones:

$$\frac{1}{a^{\frac{1}{4}} + a^{\frac{1}{8}} + 1} + \frac{1}{a^{\frac{1}{4}} - a^{\frac{1}{8}} + 1} = \frac{1}{a^{\frac{1}{4}} + 1 + a^{\frac{1}{8}}} + \frac{1}{a^{\frac{1}{4}} + 1 - a^{\frac{1}{8}}}$$

$$= \frac{a^{\frac{1}{4}} + 1 - a^{\frac{1}{8}} + a^{\frac{1}{4}} + 1 + a^{\frac{1}{8}}}{(a^{\frac{1}{4}} + 1 + a^{\frac{1}{8}})(a^{\frac{1}{4}} + 1 - a^{\frac{1}{8}})}$$

$$= \frac{2a^{\frac{1}{4}} + 2}{(a^{\frac{1}{4}} + 1)^2 - (a^{\frac{1}{8}})^2}$$

$$= \frac{2a^{\frac{1}{4}} + 2}{a^{\frac{1}{2}} + 2a^{\frac{1}{4}} + 1 - a^{\frac{1}{4}}}$$

$$= \boxed{\frac{2a^{\frac{1}{4}} + 2}{a^{\frac{1}{2}} + 1 + a^{\frac{1}{4}}}}$$

reemplazando:

$$\frac{1}{a^{\frac{1}{4}} + a^{\frac{1}{8}} + 1} + \frac{1}{a^{\frac{1}{4}} - a^{\frac{1}{8}} + 1} - \frac{2a^{\frac{1}{4}} - 2}{a^{\frac{1}{2}} - a^{\frac{1}{4}} + 1} = \frac{2a^{\frac{1}{4}} + 2}{a^{\frac{1}{2}} + 1 + a^{\frac{1}{4}}} - \frac{2a^{\frac{1}{4}} - 2}{a^{\frac{1}{2}} - a^{\frac{1}{4}} + 1}$$

$$= \frac{2\sqrt[4]{a} + 2}{\sqrt{a} + 1 + \sqrt[4]{a}} - \frac{2\sqrt[4]{a} - 2}{\sqrt{a} + 1 - \sqrt[4]{a}}$$

$$= \frac{2(\sqrt[4]{a}+1)}{\sqrt{a}+1+\sqrt[4]{a}} - \frac{2(\sqrt[4]{a}-1)}{\sqrt{a}+1-\sqrt[4]{a}}$$

$$= 2\left(\frac{\sqrt[4]{a}+1}{\sqrt{a}+1+\sqrt[4]{a}} - \frac{\sqrt[4]{a}-1}{\sqrt{a}+1-\sqrt[4]{a}}\right)$$

$$= 2\left(\frac{(\sqrt[4]{a}+1)(\sqrt{a}+1-\sqrt[4]{a}) - (\sqrt[4]{a}-1)(\sqrt{a}+1+\sqrt[4]{a})}{(\sqrt{a}+1+\sqrt[4]{a})(\sqrt{a}+1-\sqrt[4]{a})}\right)$$

$$= 2\left(\frac{(\sqrt[4]{a^3}+\sqrt[4]{a}-\sqrt{a}+\sqrt{a}+1-\sqrt[4]{a}) - (\sqrt[4]{a^3}+\sqrt[4]{a}+\sqrt{a}-\sqrt{a}-1-\sqrt[4]{a})}{(\sqrt{a}+1)^2-(\sqrt[4]{a})^2}\right)$$

$$= 2\left(\frac{(\sqrt[4]{a^3}+1)-(\sqrt[4]{a^3}-1)}{\sqrt{a^2}+2\sqrt{a}+1-\sqrt[4]{a^2}}\right)$$

$$= 2\left(\frac{\sqrt[4]{a^3}+1-\sqrt[4]{a^3}+1}{a+2\sqrt{a}+1-\sqrt{a}}\right)$$

$$= 2\left(\frac{2}{a+\sqrt{a}+1}\right)$$

$$= \boxed{\frac{4}{a+\sqrt{a}+1}}$$

126. $\dfrac{\sqrt{\sqrt{2}-1}\,\sqrt[4]{3+2\sqrt{2}} + \sqrt[3]{(x+12)\sqrt{x}-6x-8}}{\dfrac{x-\sqrt{x}}{\sqrt{x}-1} - \sqrt{\sqrt{2}+1}\,\sqrt[4]{3-2\sqrt{2}}}$

Solución.

$$\frac{\sqrt{\sqrt{2}-1}\,\sqrt[4]{3+2\sqrt{2}} + \sqrt[3]{(x+12)\sqrt{x}-6x-8}}{\dfrac{x-\sqrt{x}}{\sqrt{x}-1} - \sqrt{\sqrt{2}+1}\,\sqrt[4]{3-2\sqrt{2}}} = \frac{\overbrace{\sqrt{\sqrt{2}-1}\,\sqrt[4]{3+2\sqrt{2}}}^{A} + \overbrace{\sqrt[3]{(x+12)\sqrt{x}-6x-8}}^{B}}{\underbrace{\dfrac{x-\sqrt{x}}{\sqrt{x}-1}}_{C} - \underbrace{\sqrt{\sqrt{2}+1}\,\sqrt[4]{3-2\sqrt{2}}}_{D}}$$

$$\begin{aligned}
A &= \sqrt{\sqrt{2}-1}\,\sqrt[4]{3+2\sqrt{2}} \\
&= \sqrt[2\cdot 2]{(\sqrt{2}-1)^2}\,\sqrt[4]{3+2\sqrt{2}} \\
&= \sqrt[4]{(\sqrt{2}-1)^2}\,\sqrt[4]{3+2\sqrt{2}} \\
&= \sqrt[4]{2-2\sqrt{2}+1}\,\sqrt[4]{3+2\sqrt{2}} \\
&= \sqrt[4]{3-2\sqrt{2}}\,\sqrt[4]{3+2\sqrt{2}} \\
&= \sqrt[4]{(3-2\sqrt{2})(3+2\sqrt{2})} \\
&= \sqrt[4]{3^2-2^2\cdot 2} \\
&= \sqrt[4]{9-8} \\
&= \sqrt[4]{1}
\end{aligned}$$

$$\boxed{A=1}$$

$$\begin{aligned}
B &= \sqrt[3]{(x+12)\sqrt{x}-6x-8} \\
&= \sqrt[3]{x\sqrt{x}+12\sqrt{x}-6x-8} \\
&= \sqrt[3]{\sqrt{x^2}\sqrt{x}-6x+12\sqrt{x}-8}
\end{aligned}$$

$$= \sqrt[3]{\sqrt{x^3} - 3\cdot\sqrt{x^2}\cdot 2 + 3\cdot\sqrt{x}\cdot 2^2 - 2^3}$$
$$= \sqrt[3]{(\sqrt{x}-2)^3}$$
$$\boxed{B = \sqrt{x}-2}$$

$$C = \frac{x-\sqrt{x}}{\sqrt{x}-1}$$
$$= \frac{\sqrt{x^2}-\sqrt{x}}{\sqrt{x}-1}$$
$$= \frac{\sqrt{x}(\sqrt{x}-1)}{\sqrt{x}-1}$$
$$= \frac{\sqrt{x}\cancel{(\sqrt{x}-1)}}{\cancel{\sqrt{x}-1}}$$
$$\boxed{C = \sqrt{x}}$$

$$D = \sqrt{\sqrt{2}+1}\sqrt[4]{3-2\sqrt{2}}$$
$$= \sqrt[2\cdot 2]{(\sqrt{2}+1)^2}\sqrt[4]{3-2\sqrt{2}}$$
$$= \sqrt[4]{(\sqrt{2}+1)^2}\sqrt[4]{3-2\sqrt{2}}$$
$$= \sqrt[4]{2+2\sqrt{2}+1}\sqrt[4]{3-2\sqrt{2}}$$
$$= \sqrt[4]{3+2\sqrt{2}}\sqrt[4]{3-2\sqrt{2}}$$
$$= \sqrt[4]{(3+2\sqrt{2})(3-2\sqrt{2})}$$
$$= \sqrt[4]{3^2-2^2\cdot 2}$$
$$= \sqrt[4]{9-8}$$
$$= \sqrt[4]{1}$$
$$\boxed{D = 1}$$

reemplazando:

$$\frac{\sqrt{\sqrt{2}-1}\sqrt[4]{3+2\sqrt{2}} + \sqrt[3]{(x+12)\sqrt{x}-6x-8}}{\dfrac{x-\sqrt{x}}{\sqrt{x}-1} - \sqrt{\sqrt{2}+1}\sqrt[4]{3-2\sqrt{2}}} = \frac{1+\sqrt{x}-2}{\sqrt{x}-1}$$
$$= \frac{\sqrt{x}-1}{\sqrt{x}-1}$$
$$= \boxed{1}$$

127. $\dfrac{\sqrt{a^3b}\sqrt[3]{a^4} + \sqrt{a^4b^3} : \sqrt[6]{a}}{(b^2-ab-2a^2)\sqrt{ab}} - a^{-\frac{2}{3}}\left(\dfrac{3a^2}{3b-6a+2ab-b^2} : \dfrac{a+b}{3a-ab} - \dfrac{ab}{a+b}\right)$

Solución.

$$\frac{\sqrt{a^3b}\sqrt[3]{a^4} + \sqrt{a^4b^3} : \sqrt[6]{a}}{(b^2-ab-2a^2)\sqrt{ab}} - a^{-\frac{2}{3}}\left(\frac{3a^2}{3b-6a+2ab-b^2} : \frac{a+b}{3a-ab} - \frac{ab}{a+b}\right) =$$

$$= \overbrace{\frac{\sqrt{a^3b}\sqrt[3]{a^4}+\sqrt{a^4b^3}:\sqrt[6]{a}}{(b^2-ab-2a^2)\sqrt{ab}}}^{A} - \overbrace{a^{-\frac{2}{3}}\left(\frac{3a^2}{3b-6a+2ab-b^2}:\frac{a+b}{3a-ab}-\frac{ab}{a+b}\right)}^{B}$$

$$A = \frac{\sqrt{a^3b}\sqrt[3]{a^4}+\sqrt{a^4b^3}:\sqrt[6]{a}}{(b^2-ab-2a^2)\sqrt{ab}} = \frac{\overbrace{\sqrt{a^3b}\sqrt[3]{a^4}}^{A_1}+\overbrace{\sqrt{a^4b^3}:\sqrt[6]{a}}^{A_2}}{(b^2-ab-2a^2)\sqrt{ab}}$$

$$\begin{aligned} A_1 &= \sqrt{a^3b}\sqrt[3]{a^4} \\ &= \sqrt[2\cdot 3]{(a^3b)^3}\ \sqrt[3\cdot 2]{(a^4)^2} \\ &= \sqrt[6]{a^9b^3}\sqrt[6]{a^8} \\ &= \sqrt[6]{a^{17}b^3} \end{aligned}$$

$$\boxed{A_1 = a^2\sqrt[6]{a^5b^3}}$$

$$\begin{aligned} A_2 &= \sqrt{a^4b^3}:\sqrt[6]{a} \\ &= \sqrt[2\cdot 3]{(a^4b^3)^3}\cdot\frac{1}{\sqrt[6]{a}} \\ &= \sqrt[6]{a^{12}b^9}\cdot\frac{1}{\sqrt[6]{a}} \\ &= \frac{\sqrt[6]{a^{12}b^9}}{\sqrt[6]{a}} \\ &= \sqrt[6]{\frac{a^{12}b^9}{a}} \\ &= \sqrt[6]{a^{11}b^9} \end{aligned}$$

$$\boxed{A_2 = ab\sqrt[6]{a^5b^3}}$$

$$\begin{aligned} A &= \frac{\sqrt{a^3b}\sqrt[3]{a^4}+\sqrt{a^4b^3}:\sqrt[6]{a}}{(b^2-ab-2a^2)\sqrt{ab}} \\ &= \frac{a^2\sqrt[6]{a^5b^3}+ab\sqrt[6]{a^5b^3}}{(b-2a)(b+a)\sqrt{ab}} \\ &= \frac{a\sqrt[6]{a^5b^3}(a+b)}{(b-2a)(a+b)\sqrt{ab}} \\ &= \frac{a\sqrt[6]{a^2\cdot a^3b^3}\cancel{(a+b)}}{(b-2a)\cancel{(a+b)}\sqrt[2\cdot 3]{(ab)^3}} \\ &= \frac{a\sqrt[6]{a^2}\sqrt[6]{a^3b^3}}{(b-2a)\sqrt[6]{a^3b^3}} \\ &= \frac{a\sqrt[3]{a}\cancel{\sqrt[6]{a^3b^3}}}{(b-2a)\cancel{\sqrt[6]{a^3b^3}}} \end{aligned}$$

$$\boxed{A = \frac{a\sqrt[3]{a}}{b-2a}}$$

$$B = a^{-\frac{2}{3}}\left(\frac{3a^2}{3b-6a+2ab-b^2}:\frac{a+b}{3a-ab}-\frac{ab}{a+b}\right) = a^{-\frac{2}{3}}\left(\frac{3a^2}{\underbrace{3b-6a+2ab-b^2}_{B_1}}:\frac{a+b}{3a-ab}-\frac{ab}{a+b}\right)$$

$$
\begin{aligned}
B_1 &= 3b - 6a + 2ab - b^2 \\
&= 3b - b^2 - 6a + 2ab \\
&= b(3-b) - 2a(3-b)
\end{aligned}
$$

$$\boxed{B_1 = (3-b)(b-2a)}$$

$$
\begin{aligned}
B &= a^{-\frac{2}{3}}\left(\frac{3a^2}{3b-6a+2ab-b^2} : \frac{a+b}{3a-ab} - \frac{ab}{a+b}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^2}{(3-b)(b-2a)} \cdot \frac{3a-ab}{a+b} - \frac{ab}{a+b}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^2}{(3-b)(b-2a)} \cdot \frac{a(3-b)}{a+b} - \frac{ab}{a+b}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^3(3-b)}{(3-b)(b-2a)(a+b)} - \frac{ab}{a+b}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^3\cancel{(3-b)}}{\cancel{(3-b)}(b-2a)(a+b)} - \frac{ab}{a+b}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^3}{(b-2a)(a+b)} - \frac{ab}{a+b}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^3 - ab(b-2a)}{(b-2a)(a+b)}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^3 - ab^2 + 2a^2b}{(b-2a)(a+b)}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{3a^3 + 2a^2b - ab^2}{(b-2a)(a+b)}\right) \\
&= a^{-\frac{2}{3}}\left(\frac{a(3a^2 + 2ab - b^2)}{(b-2a)(a+b)}\right) \\
&= \frac{a^{-\frac{2}{3}} \cdot a(a+b)(3a-b)}{(b-2a)(a+b)} \\
&= \frac{a^{\frac{1}{3}}\cancel{(a+b)}(3a-b)}{(b-2a)\cancel{(a+b)}} \\
&= \frac{a^{\frac{1}{3}}(3a-b)}{b-2a}
\end{aligned}
$$

$$\boxed{B = \frac{\sqrt[3]{a}(3a-b)}{b-2a}}$$

reemplazando:

$$
\begin{aligned}
\frac{\sqrt{a^3b}\sqrt[3]{a^4} + \sqrt{a^4b^3} : \sqrt[6]{a}}{(b^2-ab-2a^2)\sqrt{ab}} - a^{-\frac{2}{3}}\left(\frac{3a^2}{3b-6a+2ab-b^2} : \frac{a+b}{3a-ab} - \frac{ab}{a+b}\right) &= \frac{a\sqrt[3]{a}}{b-2a} - \frac{\sqrt[3]{a}(3a-b)}{b-2a} \\
&= \frac{\sqrt[3]{a}}{b-2a}\big(a - (3a-b)\big) \\
&= \frac{\sqrt[3]{a}}{b-2a}(a - 3a + b) \\
&= \frac{\sqrt[3]{a}}{b-2a}(b-2a) \\
&= \frac{\sqrt[3]{a}(b-2a)}{b-2a}
\end{aligned}
$$

$$= \frac{\sqrt[3]{a}\cancel{(b-2a)}}{\cancel{b-2a}}$$

$$= \boxed{\sqrt[3]{a}}$$

128. $\left[\frac{10x^2+3ax}{4x^2-a^2}+\frac{bx-x^2-ax+ab}{2x+a}:(b-x)-2\right]\left[\frac{(a+2x)^{-\frac{1}{2}}+(2x-a)^{\frac{1}{2}}}{(4x^2-a^2)^{-\frac{1}{2}}+1}\right]^2$

Solución.

$$\left[\frac{10x^2+3ax}{4x^2-a^2}+\frac{bx-x^2-ax+ab}{2x+a}:(b-x)-2\right]\left[\frac{(a+2x)^{-\frac{1}{2}}+(2x-a)^{\frac{1}{2}}}{(4x^2-a^2)^{-\frac{1}{2}}+1}\right]^2$$

$$= \overbrace{\left[\frac{10x^2+3ax}{4x^2-a^2}+\frac{bx-x^2-ax+ab}{2x+a}:(b-x)-2\right]}^{A}\overbrace{\left[\frac{(a+2x)^{-\frac{1}{2}}+(2x-a)^{\frac{1}{2}}}{(4x^2-a^2)^{-\frac{1}{2}}+1}\right]^2}^{B}$$

$$A = \left[\frac{10x^2+3ax}{4x^2-a^2}+\frac{bx-x^2-ax+ab}{2x+a}:(b-x)-2\right]$$

$$4x^2-a^2 = (2x)^2-a^2$$
$$= \boxed{(2x-a)(2x+a)}$$

$$bx-x^2-ax+ab = bx-x^2+ab-ax$$
$$= x(b-x)+a(b-x)$$
$$= \boxed{(b-x)(x+a)}$$

$$A = \left[\frac{10x^2+3ax}{4x^2-a^2}+\frac{bx-x^2-ax+ab}{2x+a}:(b-x)-2\right]$$
$$= \left[\frac{10x^2+3ax}{(2x-a)(2x+a)}+\frac{(b-x)(x+a)}{2x+a}\cdot\frac{1}{b-x}-2\right]$$
$$= \left[\frac{10x^2+3ax}{(2x-a)(2x+a)}+\frac{(b-x)(x+a)}{(2x+a)(b-x)}-2\right]$$
$$= \left[\frac{10x^2+3ax}{(2x-a)(2x+a)}+\frac{\cancel{(b-x)}(x+a)}{(2x+a)\cancel{(b-x)}}-2\right]$$
$$= \left[\frac{10x^2+3ax}{(2x-a)(2x+a)}+\frac{x+a}{2x+a}-2\right]$$
$$= \frac{10x^2+3ax+(x+a)(2x-a)-2(2x-a)(2x+a)}{(2x-a)(2x+a)}$$
$$= \frac{10x^2+3ax+(2x^2-ax+2ax-a^2)-2(4x^2-a^2)}{(2x-a)(2x+a)}$$
$$= \frac{10x^2+3ax+2x^2+ax-a^2-8x^2+2a^2}{(2x-a)(2x+a)}$$
$$= \frac{4x^2+4ax+a^2}{(2x-a)(2x+a)}$$
$$= \frac{(2x+a)^2}{(2x-a)(2x+a)}$$
$$= \frac{(2x+a)^{\cancel{2}}}{(2x-a)\cancel{(2x+a)}}$$

$$\boxed{A=\frac{2x+a}{2x-a}}$$

$$\begin{aligned}
B&=\left[\frac{(a+2x)^{-\frac{1}{2}}+(2x-a)^{\frac{1}{2}}}{(4x^2-a^2)^{-\frac{1}{2}}+1}\right]^2\\
&=\left[\frac{\frac{1}{(a+2x)^{\frac{1}{2}}}+(2x-a)^{\frac{1}{2}}}{\frac{1}{(4x^2-a^2)^{\frac{1}{2}}}+1}\right]^2\\
&=\left[\frac{\frac{1}{\sqrt{2x+a}}+\sqrt{2x-a}}{\frac{1}{\sqrt{4x^2-a^2}}+1}\right]^2\\
&=\left[\frac{\frac{1+\sqrt{2x-a}\cdot\sqrt{2x+a}}{\sqrt{2x+a}}}{\frac{1+\sqrt{4x^2-a^2}}{\sqrt{4x^2-a^2}}}\right]^2\\
&=\left[\frac{\frac{1+\sqrt{4x^2-a^2}}{\sqrt{2x+a}}}{\frac{1+\sqrt{4x^2-a^2}}{\sqrt{(2x-a)(2x+a)}}}\right]^2\\
&=\left[\frac{\left(1+\sqrt{4x^2-a^2}\right)\sqrt{(2x-a)(2x+a)}}{\sqrt{2x+a}\left(1+\sqrt{4x^2-a^2}\right)}\right]^2\\
&=\left[\frac{\sqrt{2x-a}\sqrt{2x+a}}{\sqrt{2x+a}}\right]^2\\
&=\left[\frac{\sqrt{2x-a}\cancel{\sqrt{2x+a}}}{\cancel{\sqrt{2x+a}}}\right]^2\\
&=\left[\sqrt{2x-a}\right]^2
\end{aligned}$$

$$\boxed{B=2x-a}$$

reemplazando:

$$\begin{aligned}
\left[\frac{10x^2+3ax}{4x^2-a^2}+\frac{bx-x^2-ax+ab}{2x+a}:(b-x)-2\right]\left[\frac{(a+2x)^{-\frac{1}{2}}+(2x-a)^{\frac{1}{2}}}{(4x^2-a^2)^{-\frac{1}{2}}+1}\right]^2&=\frac{2x+a}{2x-a}\cdot(2x-a)\\
&=\frac{(2x+a)(2x-a)}{2x-a}\\
&=\frac{(2x+a)\cancel{(2x-a)}}{\cancel{2x-a}}\\
&=\boxed{2x+a}
\end{aligned}$$

129. $\left[\frac{x+4}{2x^2-2x-4}+\frac{x+2}{2(x^2+3x+2)}\right]\sqrt{2}x-\left(\sqrt{2}+\sqrt{x}-\frac{x+6}{\sqrt{x}+\sqrt{2}}\right):(x^{\frac{1}{2}}-2^{\frac{1}{2}})^2$

Solución.

$$\left[\frac{x+4}{2x^2-2x-4}+\frac{x+2}{2(x^2+3x+2)}\right]\sqrt{2}x-\left(\sqrt{2}+\sqrt{x}-\frac{x+6}{\sqrt{x}+\sqrt{2}}\right):(x^{\frac{1}{2}}-2^{\frac{1}{2}})^2=$$

$$= \overbrace{\left[\frac{x+4}{2x^2-2x-4}+\frac{x+2}{2(x^2+3x+2)}\right]\sqrt{2}x}^{A} - \overbrace{\left(\sqrt{2}+\sqrt{x}-\frac{x+6}{\sqrt{x}+\sqrt{2}}\right):(x^{\frac{1}{2}}-2^{\frac{1}{2}})^2}^{B}$$

$$A = \left[\frac{x+4}{2x^2-2x-4}+\frac{x+2}{2(x^2+3x+2)}\right]\sqrt{2}x$$

$$= \left[\frac{x+4}{2(x^2-x-2)}+\frac{x+2}{2(x+2)(x+1)}\right]\sqrt{2}x$$

$$= \left[\frac{x+4}{2(x-2)(x+1)}+\frac{1}{2(x+1)}\right]\sqrt{2}x$$

$$= \left[\frac{x+4+x-2}{2(x-2)(x+1)}\right]\sqrt{2}x$$

$$= \left[\frac{2x+2}{2(x-2)(x+1)}\right]\sqrt{2}x$$

$$= \left[\frac{2(x+1)}{2(x-2)(x+1)}\right]\sqrt{2}x$$

$$= \left[\frac{2\cancel{(x+1)}}{2(x-2)\cancel{(x+1)}}\right]\sqrt{2}x$$

$$= \left[\frac{1}{x-2}\right]\sqrt{2}x$$

$$\boxed{A = \frac{\sqrt{2}x}{x-2}}$$

$$B = \left(\sqrt{2}+\sqrt{x}-\frac{x+6}{\sqrt{x}+\sqrt{2}}\right):(x^{\frac{1}{2}}-2^{\frac{1}{2}})^2$$

$$= \left(\left(\sqrt{x}+\sqrt{2}\right)-\frac{x+6}{\sqrt{x}+\sqrt{2}}\right):\left(\sqrt{x}-\sqrt{2}\right)^2$$

$$= \left(\frac{\left(\sqrt{x}+\sqrt{2}\right)^2-(x+6)}{\sqrt{x}+\sqrt{2}}\right)\cdot\frac{1}{\left(\sqrt{x}-\sqrt{2}\right)^2}$$

$$= \left(\frac{\sqrt{x^2}+2\sqrt{x}\sqrt{2}+\sqrt{2^2}-x-6}{\sqrt{x}+\sqrt{2}}\right)\cdot\frac{1}{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}-\sqrt{2}\right)}$$

$$= \left(\frac{x+2\sqrt{x}\sqrt{2}+2-x-6}{\sqrt{x}+\sqrt{2}}\right)\cdot\frac{1}{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}-\sqrt{2}\right)}$$

$$= \left(\frac{2\sqrt{2}\sqrt{x}-4}{\sqrt{x}+\sqrt{2}}\right)\cdot\frac{1}{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}-\sqrt{2}\right)}$$

$$= \frac{2\sqrt{2}\sqrt{x}-4}{(\sqrt{x}+\sqrt{2})\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}-\sqrt{2}\right)}$$

$$= \frac{2\left(\sqrt{2}\sqrt{x}-2\right)}{\left(\sqrt{x^2}-\sqrt{2^2}\right)\left(\sqrt{x}-\sqrt{2}\right)}$$

$$= \frac{2\left(\sqrt{2}\sqrt{x}-\sqrt{2^2}\right)}{(x-2)\left(\sqrt{x}-\sqrt{2}\right)}$$

$$= \frac{2\sqrt{2}\left(\sqrt{x}-\sqrt{2}\right)}{(x-2)\left(\sqrt{x}-\sqrt{2}\right)}$$

$$= \frac{2\sqrt{2}\cancel{(\sqrt{x}-\sqrt{2})}}{(x-2)\cancel{(\sqrt{x}-\sqrt{2})}}$$

$$\boxed{B = \frac{2\sqrt{2}}{x-2}}$$

reemplazando:

$$\left[\frac{x+4}{2x^2-2x-4} + \frac{x+2}{2(x^2+3x+2)}\right]\sqrt{2}x - \left(\sqrt{2}+\sqrt{x}-\frac{x+6}{\sqrt{x}+\sqrt{2}}\right) : (x^{\frac{1}{2}} - 2^{\frac{1}{2}})^2 = \frac{\sqrt{2}x}{x-2} - \frac{2\sqrt{2}}{x-2}$$

$$= \frac{\sqrt{2}x - 2\sqrt{2}}{x-2}$$

$$= \frac{\sqrt{2}(x-2)}{x-2}$$

$$= \frac{\sqrt{2}\cancel{(x-2)}}{\cancel{x-2}}$$

$$= \boxed{\sqrt{2}}$$

130. $\dfrac{(1-x^2)^{-\frac{1}{2}}+1}{(1+x)^{-\frac{1}{2}}+(1-x)^{\frac{1}{2}}} : \dfrac{\sqrt{1-x}}{x-2} + (x+1)\left(\dfrac{1}{x+1}+\dfrac{4}{x^2-4x}-\dfrac{6}{3x^2-3x-4}\right)$

Solución.

$$\frac{(1-x^2)^{-\frac{1}{2}}+1}{(1+x)^{-\frac{1}{2}}+(1-x)^{\frac{1}{2}}} : \frac{\sqrt{1-x}}{x-2} + (x+1)\left(\frac{1}{x+1}+\frac{4}{x^2-4x}-\frac{6}{3x^2-3x-4}\right) =$$

$$= \overbrace{\frac{(1-x^2)^{-\frac{1}{2}}+1}{(1+x)^{-\frac{1}{2}}+(1-x)^{\frac{1}{2}}} : \frac{\sqrt{1-x}}{x-2}}^{A} + \overbrace{(x+1)\left(\frac{1}{x+1}+\frac{4}{x^2-4x}-\frac{6}{3x^2-3x-4}\right)}^{B}$$

$$A = \frac{(1-x^2)^{-\frac{1}{2}}+1}{(1+x)^{-\frac{1}{2}}+(1-x)^{\frac{1}{2}}} : \frac{\sqrt{1-x}}{x-2}$$

$$= \frac{\dfrac{1}{(1-x^2)^{\frac{1}{2}}}+1}{\dfrac{1}{(1+x)^{\frac{1}{2}}}+(1-x)^{\frac{1}{2}}} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{\dfrac{1}{\sqrt{1-x^2}}+1}{\dfrac{1}{\sqrt{1+x}}+\sqrt{1-x}} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{\dfrac{1+\sqrt{1-x^2}}{\sqrt{1-x^2}}}{\dfrac{1+\sqrt{1-x}\sqrt{1+x}}{\sqrt{1+x}}} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{\dfrac{1+\sqrt{1-x^2}}{\sqrt{(1-x)(1+x)}}}{\dfrac{1+\sqrt{(1-x)(1+x)}}{\sqrt{1+x}}} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{\dfrac{1+\sqrt{1-x^2}}{\sqrt{1-x}\sqrt{1+x}}}{\dfrac{1+\sqrt{1-x^2}}{\sqrt{1+x}}} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{(1+\sqrt{1-x^2}\,)\sqrt{1+x}}{\sqrt{1-x}\sqrt{1+x}(1+\sqrt{1-x^2}\,)} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{\cancel{(1+\sqrt{1-x^2}\,)}\cancel{\sqrt{1+x}}}{\sqrt{1-x}\cancel{\sqrt{1+x}}\cancel{(1+\sqrt{1-x^2}\,)}} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{x-2}{\sqrt{1-x}}$$

$$= \frac{x-2}{(\sqrt{1-x}\,)^2}$$

$$= \frac{x-2}{1-x}$$

$$= \frac{x-2}{-x+1}$$

$$= \frac{x-2}{-(x-1)}$$

$$\boxed{A = -\frac{x-2}{x-1}}$$

$$B = (x+1)\left(\frac{1}{x+1} + \frac{4}{x^2-4x} - \frac{6}{3x^2-3x-4}\right)$$

$$= (x+1)\left(\frac{1}{x+1} + \frac{4}{x(x-4)} - \frac{6}{(x-4)(x+1)}\right)$$

$$= (x+1)\left(\frac{x(x-4)+4(x+1)-5x}{x(x-4)(x+1)}\right)$$

$$= (x+1)\left(\frac{x^2-4x+4x+4-5x}{x(x-4)(x+1)}\right)$$

$$= (x+1)\left(\frac{x^2-5x+4}{x(x-4)(x+1)}\right)$$

$$= (x+1)\left(\frac{(x-4)(x-1)}{x(x-4)(x+1)}\right)$$

$$= (x+1)\left(\frac{\cancel{(x-4)}(x-1)}{x\cancel{(x-4)}(x+1)}\right)$$

$$= (x+1)\left(\frac{x-1}{x(x+1)}\right)$$

$$= \frac{(x+1)(x-1)}{x(x+1)}$$

$$= \frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}}$$

$$\boxed{B = \frac{x-1}{x}}$$

reemplazando:

$$\frac{(1-x^2)^{-\frac{1}{2}}+1}{(1+x)^{-\frac{1}{2}}+(1-x)^{\frac{1}{2}}}:\frac{\sqrt{1-x}}{x-2}+(x+1)\left(\frac{1}{x+1}+\frac{4}{x^2-4x}-\frac{6}{3x^2-3x-4}\right)=-\frac{x-2}{x-1}+\frac{x-1}{x}$$

$$=\frac{-x(x-2)+(x-1)^2}{x(x-1)}$$

$$=\frac{-x^2+2x+x^2-2x+1}{x(x-1)}$$

$$=\boxed{\frac{1}{x(x-1)}}$$

131. $\dfrac{a^2\sqrt{ab^{-1}}\,\sqrt[3]{b^2\sqrt{ab}}-2\sqrt{a^3b}\,\sqrt[6]{ab^5}}{(a^2-ab-2b^2)\sqrt[3]{a^5b}}-\dfrac{a-3}{a+2b}\left[\dfrac{a+2b}{a^2+ab-3a-3b}-(a-1)(a^2-4a+3)^{-1}\right]$

Solución.

$$\frac{a^2\sqrt{ab^{-1}}\,\sqrt[3]{b^2\sqrt{ab}}-2\sqrt{a^3b}\,\sqrt[6]{ab^5}}{(a^2-ab-2b^2)\sqrt[3]{a^5b}}-\frac{a-3}{a+2b}\left[\frac{a+2b}{a^2+ab-3a-3b}-(a-1)(a^2-4a+3)^{-1}\right]=$$

$$=\overbrace{\frac{a^2\sqrt{ab^{-1}}\,\sqrt[3]{b^2\sqrt{ab}}-2\sqrt{a^3b}\,\sqrt[6]{ab^5}}{(a^2-ab-2b^2)\sqrt[3]{a^5b}}}^{A}-\overbrace{\frac{a-3}{a+2b}\left[\frac{a+2b}{a^2+ab-3a-3b}-(a-1)(a^2-4a+3)^{-1}\right]}^{B}$$

$$A=\frac{a^2\sqrt{ab^{-1}}\,\sqrt[3]{b^2\sqrt{ab}}-2\sqrt{a^3b}\,\sqrt[6]{ab^5}}{(a^2-ab-2b^2)\sqrt[3]{a^5b}}$$

$$a^2\sqrt{ab^{-1}}\,\sqrt[3]{b^2\sqrt{ab}}=a^2\sqrt[2\cdot 3]{(ab^{-1})^3}\,\sqrt[3\cdot 2]{(b^2\sqrt{ab}\,)^2}$$

$$=a^2\sqrt[6]{a^3b^{-3}}\,\sqrt[6]{b^4\cdot ab}$$

$$=a^2\sqrt[6]{a^3b^{-3}}\,\sqrt[6]{ab^5}$$

$$=a^2\sqrt[6]{a^3b^{-3}\cdot ab^5}$$

$$=a^2\sqrt[6]{a^4b^2}$$

$$=\boxed{a^2\sqrt[3]{a^2b}}$$

$$\sqrt{a^3b}\,\sqrt[6]{ab^5}=a\sqrt{ab}\,\sqrt[6]{ab^5}$$

$$=a\sqrt[2\cdot 3]{(ab)^3}\,\sqrt[6]{ab^5}$$

$$=a\sqrt[6]{a^3b^3}\,\sqrt[6]{ab^5}$$

$$=a\sqrt[6]{a^3b^3\cdot ab^5}$$

$$=a\sqrt[6]{a^4b^8}$$

$$=ab\sqrt[6]{a^4b^2}$$

$$=\boxed{ab\sqrt[3]{a^2b}}$$

$$(a^2-ab-2b^2)\sqrt[3]{a^5b}=(a-2b)(a+b)a\sqrt[3]{a^2b}$$

$$=\boxed{a(a-2b)(a+b)\sqrt[3]{a^2b}}$$

$$A=\frac{a^2\sqrt{ab^{-1}}\,\sqrt[3]{b^2\sqrt{ab}}-2\sqrt{a^3b}\,\sqrt[6]{ab^5}}{(a^2-ab-2b^2)\sqrt[3]{a^5b}}$$

$$=\frac{a^2\sqrt[3]{a^2b}-2ab\sqrt[3]{a^2b}}{a(a-2b)(a+b)\sqrt[3]{a^2b}}$$

$$= \frac{a\sqrt[3]{a^2b}(a-2b)}{a(a-2b)(a+b)\sqrt[3]{a^2b}}$$

$$= \frac{a\sqrt[3]{a^2b}(a-2b)}{a(a-2b)(a+b)\sqrt[3]{a^2b}}$$

$$= \frac{\cancel{a}\cancel{\sqrt[3]{a^2b}}\cancel{(a-2b)}}{\cancel{a}\cancel{(a-2b)}(a+b)\cancel{\sqrt[3]{a^2b}}}$$

$$\boxed{A = \frac{1}{a+b}}$$

$$B = \frac{a-3}{a+2b}\left[\frac{a+2b}{a^2+ab-3a-3b} - (a-1)(a^2-4a+3)^{-1}\right]$$

$$\begin{aligned} a^2+ab-3a-3b &= a^2-3a+ab-3b \\ &= a(a-3)+b(a-3) \\ &= \boxed{(a-3)(a+b)} \end{aligned}$$

$$\begin{aligned} (a-1)(a^2-4a+3)^{-1} &= \frac{a-1}{a^2-4a+3} \\ &= \frac{a-1}{(a-3)(a-1)} \\ &= \frac{\cancel{a-1}}{(a-3)\cancel{(a-1)}} \\ &= \boxed{\frac{1}{a-3}} \end{aligned}$$

$$\begin{aligned} B &= \frac{a-3}{a+2b}\left[\frac{a+2b}{a^2+ab-3a-3b} - (a-1)(a^2-4a+3)^{-1}\right] \\ &= \frac{a-3}{a+2b}\left[\frac{a+2b}{(a-3)(a+b)} - \frac{1}{a-3}\right] \\ &= \frac{a-3}{a+2b}\left[\frac{a+2b-(a+b)}{(a-3)(a+b)}\right] \\ &= \frac{a-3}{a+2b}\left[\frac{a+2b-a-b}{(a-3)(a+b)}\right] \\ &= \frac{a-3}{a+2b}\left[\frac{b}{(a-3)(a+b)}\right] \\ &= \frac{b(a-3)}{(a+2b)(a-3)(a+b)} \\ &= \frac{b\cancel{(a-3)}}{(a+2b)\cancel{(a-3)}(a+b)} \end{aligned}$$

$$\boxed{B = \frac{b}{(a+2b)(a+b)}}$$

reemplazando:

$$\begin{aligned} \frac{a^2\sqrt{ab^{-1}}\sqrt[3]{b^2\sqrt{ab}} - 2\sqrt{a^3b}\sqrt[6]{ab^5}}{(a^2-ab-2b^2)\sqrt[3]{a^5b}} - \frac{a-3}{a+2b}\left[\frac{a+2b}{a^2+ab-3a-3b} - (a-1)(a^2-4a+3)^{-1}\right] &= \frac{1}{a+b} - \frac{b}{(a+2b)(a+b)} \\ &= \frac{a+2b-b}{(a+2b)(a+b)} \end{aligned}$$

$$= \frac{a+b}{(a+2b)(a+b)}$$

$$= \frac{\cancel{a+b}}{(a+2b)\cancel{(a+b)}}$$

$$= \boxed{\frac{1}{a+b}}$$

132. $\dfrac{\sqrt{a\sqrt{ab}}-(ab)^{\frac{3}{4}}:\sqrt{a}}{(a^2-b^2)a^{-1}}\left(\sqrt[4]{\dfrac{a}{b}}+\sqrt[4]{\dfrac{b}{a}}\right)+\dfrac{b}{a}\left(\dfrac{2a-2b}{a-4b}+\dfrac{a+3b}{2a+2b}-\dfrac{a^2-21ab}{2a^2-6ab-8b^2}\right)$

Solución.

$$\frac{\sqrt{a\sqrt{ab}}-(ab)^{\frac{3}{4}}:\sqrt{a}}{(a^2-b^2)a^{-1}}\left(\sqrt[4]{\frac{a}{b}}+\sqrt[4]{\frac{b}{a}}\right)+\frac{b}{a}\left(\frac{2a-2b}{a-4b}+\frac{a+3b}{2a+2b}-\frac{a^2-21ab}{2a^2-6ab-8b^2}\right)=$$

$$=\overbrace{\frac{\sqrt{a\sqrt{ab}}-(ab)^{\frac{3}{4}}:\sqrt{a}}{(a^2-b^2)a^{-1}}\left(\sqrt[4]{\frac{a}{b}}+\sqrt[4]{\frac{b}{a}}\right)}^{A}+\overbrace{\frac{b}{a}\left(\frac{2a-2b}{a-4b}+\frac{a+3b}{2a+2b}-\frac{a^2-21ab}{2a^2-6ab-8b^2}\right)}^{B}$$

$$A=\frac{\sqrt{a\sqrt{ab}}-(ab)^{\frac{3}{4}}:\sqrt{a}}{(a^2-b^2)a^{-1}}\left(\sqrt[4]{\frac{a}{b}}+\sqrt[4]{\frac{b}{a}}\right)$$

$$\sqrt{a\sqrt{ab}}=\sqrt[2\cdot 2]{(a\sqrt{ab})^2}$$

$$=\sqrt[4]{a^2\cdot ab}$$

$$=\boxed{\sqrt[4]{a^3b}}$$

$$(ab)^{\frac{3}{4}}:\sqrt{a}=\sqrt[4]{(ab)^3}\cdot\frac{1}{\sqrt{a}}$$

$$=\sqrt[4]{a^3b^3}\cdot\frac{1}{\sqrt[2\cdot 2]{a^2}}$$

$$=\frac{\sqrt[4]{a^3b^3}}{\sqrt[4]{a^2}}$$

$$=\sqrt[4]{\frac{a^3b^3}{a^2}}$$

$$=\boxed{\sqrt[4]{ab^3}}$$

$$(a^2-b^2)a^{-1}=\frac{a^2-b^2}{a}$$

$$=\frac{(a-b)(a+b)}{a}$$

$$=\frac{(\sqrt{a^2}-\sqrt{b^2}\,)(a+b)}{a}$$

$$=\boxed{\frac{(\sqrt{a}-\sqrt{b}\,)(\sqrt{a}+\sqrt{b}\,)(a+b)}{a}}$$

$$\left(\sqrt[4]{\frac{a}{b}}+\sqrt[4]{\frac{b}{a}}\right)=\left(\frac{\sqrt[4]{a}}{\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)$$

$$=\frac{\sqrt[4]{a^2}+\sqrt[4]{b^2}}{\sqrt[4]{a}\sqrt[4]{b}}$$

$$= \boxed{\frac{\sqrt{a}+\sqrt{b}}{\sqrt[4]{ab}}}$$

$$A = \frac{\sqrt{a\sqrt{ab}} - (ab)^{\frac{3}{4}} : \sqrt{a}}{(a^2-b^2)a^{-1}}\left(\sqrt[4]{\frac{a}{b}} + \sqrt[4]{\frac{b}{a}}\right)$$

$$= \frac{\sqrt[4]{a^3b} - \sqrt[4]{ab^3}}{\frac{(\sqrt{a}-\sqrt{b}\,)(\sqrt{a}+\sqrt{b}\,)(a+b)}{a}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt[4]{ab}}$$

$$= \frac{\sqrt[4]{ab}(\sqrt[4]{a^2} - \sqrt[4]{b^2}\,)}{\frac{(\sqrt{a}-\sqrt{b}\,)(\sqrt{a}+\sqrt{b}\,)(a+b)}{a}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt[4]{ab}}$$

$$= \frac{\sqrt[4]{ab}(\sqrt{a}-\sqrt{b}\,)}{\frac{(\sqrt{a}-\sqrt{b}\,)(\sqrt{a}+\sqrt{b}\,)(a+b)}{a}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt[4]{ab}}$$

$$= \frac{\sqrt[4]{ab}\cancel{(\sqrt{a}-\sqrt{b}\,)}}{\frac{\cancel{(\sqrt{a}-\sqrt{b}\,)}(\sqrt{a}+\sqrt{b}\,)(a+b)}{a}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt[4]{ab}}$$

$$= \frac{\sqrt[4]{ab}}{\frac{(\sqrt{a}+\sqrt{b}\,)(a+b)}{a}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt[4]{ab}}$$

$$= \frac{a\sqrt[4]{ab}}{(\sqrt{a}+\sqrt{b}\,)(a+b)} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt[4]{ab}}$$

$$= \frac{a\sqrt[4]{ab}(\sqrt{a}+\sqrt{b}\,)}{(\sqrt{a}+\sqrt{b}\,)(a+b)\sqrt[4]{ab}}$$

$$= \frac{a\cancel{\sqrt[4]{ab}}\cancel{(\sqrt{a}+\sqrt{b}\,)}}{\cancel{(\sqrt{a}+\sqrt{b}\,)}(a+b)\cancel{\sqrt[4]{ab}}}$$

$$\boxed{A = \frac{a}{a+b}}$$

$$B = \frac{b}{a}\left(\frac{2a-2b}{a-4b} + \frac{a+3b}{2a+2b} - \frac{a^2-21ab}{2a^2-6ab-8b^2}\right)$$

$$2a^2-6ab-8b^2 = 2(a^2-3ab-4b^2)$$

$$= \boxed{2(a-4b)(a+b)}$$

$$B = \frac{b}{a}\left(\frac{2a-2b}{a-4b} + \frac{a+3b}{2a+2b} - \frac{a^2-21ab}{2a^2-6ab-8b^2}\right)$$

$$= \frac{b}{a}\left(\frac{2a-2b}{a-4b} + \frac{a+3b}{2(a+b)} - \frac{a^2+21ab}{2(a-4b)(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{4(a+b)^2 + (a+3b)(a-4b) - (a^2+21ab)}{2(a-4b)(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{4(a^2+2ab+b^2) + (a^2-4ab+3ab-12b^2) - a^2 - 21ab}{2(a-4b)(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{4a^2+4ab+4b^2+a^2-ab-12b^2-a^2-21ab}{2(a-4b)(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{4a^2-14ab-8b^2}{2(a-4b)(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{2(2a^2-7ab-4b^2)}{2(a-4b)(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{(a-4b)(2a+b)}{(a-4b)(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{\cancel{(a-4b)}(2a+b)}{\cancel{(a-4b)}(a+b)}\right)$$

$$= \frac{b}{a}\left(\frac{2a+b}{a+b}\right)$$

$$\boxed{B = \frac{b(2a+b)}{a(a+b)}}$$

reemplazando:

$$\frac{\sqrt{a\sqrt{ab}}-(ab)^{\frac{3}{4}}:\sqrt{a}}{(a^2-b^2)a^{-1}}\left(\sqrt[4]{\frac{a}{b}}+\sqrt[4]{\frac{b}{a}}\right)+\frac{b}{a}\left(\frac{2a-2b}{a-4b}+\frac{a+3b}{2a+2b}-\frac{a^2-21ab}{2a^2-6ab-8b^2}\right) = \frac{a}{a+b}+\frac{b(2a+b)}{a(a+b)}$$

$$= \frac{a^2+b(2a+b)}{a(a+b)}$$

$$= \frac{a^2+2ab+b^2}{a(a+b)}$$

$$= \frac{(a+b)^{\cancel{2}}}{a\cancel{(a+b)}}$$

$$= \boxed{\frac{a+b}{a}}$$

133. $$\left[\frac{(\sqrt[3]{ab^2\sqrt{b}}-\sqrt[3]{ab}\sqrt{a})^2}{ab\sqrt[6]{ab}}+4\right]:\frac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}-\sqrt{b}}+\frac{b^2-4a^2}{4a}\left(\frac{1}{b^2+3ab+2a^2}-\frac{3}{2a^2+ab-b^2}\right)$$

Solución.

$$\left[\frac{(\sqrt[3]{ab^2\sqrt{b}}-\sqrt[3]{ab}\sqrt{a})^2}{ab\sqrt[6]{ab}}+4\right]:\frac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}-\sqrt{b}}+\frac{b^2-4a^2}{4a}\left(\frac{1}{b^2+3ab+2a^2}-\frac{3}{2a^2+ab-b^2}\right)$$

$$= \underbrace{\overbrace{\left[\frac{(\sqrt[3]{ab^2\sqrt{b}}-\sqrt[3]{ab}\sqrt{a})^2}{ab\sqrt[6]{ab}}+4\right]}^{A}:\overbrace{\frac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}-\sqrt{b}}}^{B}}_{C}+\overbrace{\frac{b^2-4a^2}{4a}\left(\frac{1}{b^2+3ab+2a^2}-\frac{3}{2a^2+ab-b^2}\right)}^{D}$$

$$A = \left[\frac{(\sqrt[3]{ab^2\sqrt{b}}-\sqrt[3]{ab}\sqrt{a})^2}{ab\sqrt[6]{ab}}+4\right]$$

$$\sqrt[3]{ab^2\sqrt{b}} = \sqrt[3\cdot 2]{(ab^2\sqrt{b})^2}$$

$$= \sqrt[6]{a^2b^4\cdot b}$$

$$= \boxed{\sqrt[6]{a^2b^5}}$$

$$\sqrt[3]{ab}\sqrt{a} = \sqrt[3\cdot 2]{(ab)^2}\,\sqrt[2\cdot 3]{a^3}$$

$$= \sqrt[6]{a^2b^2}\,\sqrt[6]{a^3}$$

$$= \sqrt[6]{a^2b^2\cdot a^3}$$

$$= \boxed{\sqrt[6]{a^5b^2}}$$

$$
\begin{aligned}
ab\sqrt[6]{ab} &= \sqrt[6]{(ab)^6}\sqrt[6]{ab} \\
&= \sqrt[6]{a^6b^6}\sqrt[6]{ab} \\
&= \sqrt[6]{a^6b^6 \cdot ab} \\
&= \boxed{\sqrt[6]{a^7b^7}}
\end{aligned}
$$

$$
\begin{aligned}
A &= \left[\frac{(\sqrt[3]{ab^2\sqrt{b}} - \sqrt[3]{ab}\sqrt{a}\,)^2}{ab\sqrt[6]{ab}} + 4\right] \\
&= \frac{(\sqrt[6]{a^2b^5} - \sqrt[6]{a^5b^2}\,)^2}{\sqrt[6]{a^7b^7}} + 4 \\
&= \frac{\sqrt[6]{(a^2b^5)^2} - 2\sqrt[6]{a^2b^5}\sqrt[6]{a^5b^2} + \sqrt[6]{(a^5b^2)^2}}{\sqrt[6]{a^7b^7}} + 4 \\
&= \frac{\sqrt[6]{(a^2b^5)^2} - 2\sqrt[6]{a^7b^7} + \sqrt[6]{(a^5b^2)^2}}{\sqrt[6]{a^7b^7}} + 4 \\
&= \frac{\sqrt[6]{(a^2b^5)^2} - 2\sqrt[6]{a^7b^7} + \sqrt[6]{(a^5b^2)^2} + 4\sqrt[6]{a^7b^7}}{\sqrt[6]{a^7b^7}} \\
&= \frac{\sqrt[6]{(a^2b^5)^2} + 2\sqrt[6]{a^7b^7} + \sqrt[6]{(a^5b^2)^2}}{\sqrt[6]{a^7b^7}} \\
&= \frac{(\sqrt[6]{a^2b^2}(\sqrt[6]{b^3} + \sqrt[6]{a^3})\,)^2}{\sqrt[6]{a^7b^7}} \\
&= \frac{\sqrt[6]{(a^2b^2)^2}(\sqrt[6]{b^3} + \sqrt[6]{a^3}\,)^2}{\sqrt[6]{a^7b^7}} \\
&= \frac{\sqrt[6]{a^4b^4}(\sqrt{b} + \sqrt{a}\,)^2}{\sqrt[6]{a^7b^7}} \\
&= \frac{(\sqrt{b} + \sqrt{a}\,)^2}{\sqrt[6]{a^3b^3}} \\
&= \frac{(\sqrt{b} + \sqrt{a}\,)^2}{\sqrt[6]{(ab)^3}}
\end{aligned}
$$

$$
\boxed{A = \frac{(\sqrt{b} + \sqrt{a}\,)^2}{\sqrt{ab}}}
$$

$$
\begin{aligned}
B &= \frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{a} - \sqrt{b}} \\
&= \frac{\sqrt{a^2}\sqrt{b} + \sqrt{b^2}\sqrt{a}}{\sqrt{a} - \sqrt{b}} \\
&= \frac{\sqrt{a^2b} + \sqrt{ab^2}}{\sqrt{a} - \sqrt{b}}
\end{aligned}
$$

$$
\boxed{B = \frac{\sqrt{ab}(\sqrt{a} + \sqrt{b}\,)}{\sqrt{a} - \sqrt{b}}}
$$

$$
\begin{aligned}
C &= \left[\frac{(\sqrt[3]{ab^2\sqrt{b}} - \sqrt[3]{ab}\sqrt{a}\,)^2}{ab\sqrt[6]{ab}} + 4\right] : \frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{a} - \sqrt{b}} \\
&= \frac{(\sqrt{a} + \sqrt{b}\,)^2}{\sqrt{ab}} : \frac{\sqrt{ab}(\sqrt{a} + \sqrt{b}\,)}{\sqrt{a} - \sqrt{b}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(\sqrt{a}+\sqrt{b}\,)^2}{\sqrt{ab}}\cdot\frac{\sqrt{a}-\sqrt{b}}{\sqrt{ab}(\sqrt{a}+\sqrt{b}\,)}\\
&= \frac{(\sqrt{a}+\sqrt{b}\,)^2(\sqrt{a}-\sqrt{b})}{(\sqrt{ab})^2(\sqrt{a}+\sqrt{b})}\\
&= \frac{(\sqrt{a}+\sqrt{b}\,)^{\cancel{2}}(\sqrt{a}-\sqrt{b})}{ab\cancel{\sqrt{a}+\sqrt{b})}}\\
&= \frac{(\sqrt{a}+\sqrt{b}\,)(\sqrt{a}-\sqrt{b})}{ab}\\
&= \frac{\sqrt{a^2}-\sqrt{b^2}}{ab}
\end{aligned}
$$

$$\boxed{C=\frac{a-b}{ab}}$$

$$
\begin{aligned}
D &= \frac{b^2-4a^2}{4a}\left(\frac{1}{b^2+3ab+2a^2}-\frac{3}{2a^2+ab-b^2}\right)\\
&= \frac{-4a^2+b^2}{4a}\left(\frac{1}{(2a+b)(a+b)}-\frac{3}{(a+b)(2a-b)}\right)\\
&= \frac{-(4a^2+b^2)}{4a}\left(\frac{(2a-b)-3(2a+b)}{(a+b)(2a+b)(2a-b)}\right)\\
&= \frac{-(4a^2+b^2)}{4a}\left(\frac{2a-b-6a-3b}{(a+b)(4a^2-b^2)}\right)\\
&= \frac{-(4a^2+b^2)}{4a}\left(\frac{-4a-4b}{(a+b)(4a^2-b^2)}\right)\\
&= \frac{-(4a^2+b^2)}{4a}\left(\frac{-4(a+b)}{(a+b)(4a^2-b^2)}\right)\\
&= \frac{-(4a^2+b^2)}{4a}\left(\frac{-4\cancel{(a+b)}}{\cancel{(a+b)}(4a^2-b^2)}\right)\\
&= \frac{-(4a^2+b^2)}{4a}\left(\frac{-4}{4a^2-b^2}\right)\\
&= \frac{4(4a^2+b^2)}{4a(4a^2+b^2)}\\
&= \frac{\cancel{4}\cancel{(4a^2+b^2)}}{\cancel{4}a\cancel{(4a^2+b^2)}}
\end{aligned}
$$

$$\boxed{D=\frac{1}{a}}$$

reemplazando:

$$
\begin{aligned}
\left[\frac{(\sqrt[3]{ab^2\sqrt{b}}-\sqrt[3]{ab}\sqrt{a}\,)^2}{ab\sqrt[6]{ab}}+4\right]:\frac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}-\sqrt{b}}+\frac{b^2-4a^2}{4a}\left(\frac{1}{b^2+3ab+2a^2}-\frac{3}{2a^2+ab-b^2}\right) &= \frac{a-b}{ab}+\frac{1}{a}\\
&= \frac{a-b+b}{ab}\\
&= \frac{a}{ab}\\
&= \frac{\cancel{a}}{\cancel{a}b}
\end{aligned}
$$

$$= \boxed{\frac{1}{b}}$$

134. $\dfrac{4(2ab)^{\frac{3}{4}}(a+2b)^{-1}}{\sqrt{a}-\sqrt{2b}} : \dfrac{\sqrt{2b\sqrt{2ab}}+\sqrt[4]{2a^3b}}{\sqrt{2ab}} - 6\left(\dfrac{a}{6a-48b} - \dfrac{2b}{3a-6b} - \dfrac{8b^2}{a^2-10ab+16b^2}\right)$

Solución.

$$\frac{4(2ab)^{\frac{3}{4}}(a+2b)^{-1}}{\sqrt{a}-\sqrt{2b}} : \frac{\sqrt{2b\sqrt{2ab}}+\sqrt[4]{2a^3b}}{\sqrt{2ab}} - 6\left(\frac{a}{6a-48b} - \frac{2b}{3a-6b} - \frac{8b^2}{a^2-10ab+16b^2}\right) =$$

$$= \overbrace{\frac{4(2ab)^{\frac{3}{4}}(a+2b)^{-1}}{\sqrt{a}-\sqrt{2b}} : \frac{\sqrt{2b\sqrt{2ab}}+\sqrt[4]{2a^3b}}{\sqrt{2ab}}}^{A} - \overbrace{6\left(\frac{a}{6a-48b} - \frac{2b}{3a-6b} - \frac{8b^2}{a^2-10ab+16b^2}\right)}^{B}$$

$$A = \frac{4(2ab)^{\frac{3}{4}}(a+2b)^{-1}}{\sqrt{a}-\sqrt{2b}} : \frac{\sqrt{2b\sqrt{2ab}}+\sqrt[4]{2a^3b}}{\sqrt{2ab}}$$

$$4(2ab)^{\frac{3}{4}}(a+2b)^{-1} = \frac{4(2ab)^{\frac{3}{4}}}{a+2b}$$

$$= \frac{4\sqrt[4]{(2ab)^3}}{a+2b}$$

$$= \boxed{\frac{4\sqrt[4]{8a^3b^3}}{a+2b}}$$

$$\sqrt{2b\sqrt{2ab}} = \sqrt[2\cdot 2]{(2b\sqrt{2ab})^2}$$

$$= \sqrt[4]{4b^2\cdot 2ab}$$

$$= \boxed{\sqrt[4]{8ab^3}}$$

$$\sqrt{2ab} = \sqrt[2\cdot 2]{(2ab)^2}$$

$$= \boxed{\sqrt[4]{4a^2b^2}}$$

$$A = \frac{4(2ab)^{\frac{3}{4}}(a+2b)^{-1}}{\sqrt{a}-\sqrt{2b}} : \frac{\sqrt{2b\sqrt{2ab}}+\sqrt[4]{2a^3b}}{\sqrt{2ab}}$$

$$= \frac{\frac{4\sqrt[4]{8a^3b^3}}{a+2b}}{\sqrt{a}-\sqrt{2b}} : \frac{\sqrt[4]{8ab^3}+\sqrt[4]{2a^3b}}{\sqrt[4]{4a^2b^2}}$$

$$= \frac{4\sqrt[4]{8a^3b^3}}{(a+2b)(\sqrt{a}-\sqrt{2b})} : \frac{\sqrt[4]{2ab}(\sqrt[4]{4b^2}+\sqrt[4]{a^2})}{\sqrt[4]{4a^2b^2}}$$

$$= \frac{4\sqrt[4]{8a^3b^3}}{(a+2b)(\sqrt{a}-\sqrt{2b})} : \frac{\sqrt{2b}+\sqrt{a}}{\sqrt[4]{2ab}}$$

$$= \frac{4\sqrt[4]{8a^3b^3}}{(a+2b)(\sqrt{a}-\sqrt{2b})} \cdot \frac{\sqrt[4]{2ab}}{\sqrt{2b}+\sqrt{a}}$$

$$= \frac{4\sqrt[4]{16a^4b^4}}{(a+2b)(\sqrt{a}-\sqrt{2b})(\sqrt{a}+\sqrt{2b})}$$

$$= \frac{4\cdot 2ab}{(a+2b)(\sqrt{a^2}-\sqrt{(2b)^2})}$$

$$\boxed{A = \frac{8ab}{(a+2b)(a-2b)}}$$

$$\begin{aligned}
B &= 6\left(\frac{a}{6a-48b} - \frac{2b}{3a-6b} - \frac{8b^2}{a^2-10ab+16b^2}\right)\\
&= 6\left(\frac{a}{6(a-8b)} - \frac{2b}{3(a-2b)} - \frac{8b^2}{(a-8b)(a-2b)}\right)\\
&= 6\left(\frac{a(a-2b)-4b(a-8b)-48b^2}{6(a-8b)(a-2b)}\right)\\
&= 6\left(\frac{a^2-2ab-4ab-+32b^2-48b^2}{6(a-8b)(a-2b)}\right)\\
&= 6\left(\frac{a^2-6ab-16b^2}{6(a-8b)(a-2b)}\right)\\
&= 6\left(\frac{(a-8b)(a+2b)}{6(a-8b)(a-2b)}\right)\\
&= 6\left(\frac{\cancel{(a-8b)}(a+2b)}{6\cancel{(a-8b)}(a-2b)}\right)\\
&= 6\left(\frac{a+2b}{6(a-2b)}\right)\\
&= \frac{6(a+2b)}{6(a-2b)}\\
&= \frac{\cancel{6}(a+2b)}{\cancel{6}(a-2b)}
\end{aligned}$$

$$\boxed{B = \frac{a+2b}{a-2b}}$$

reemplazando:

$$\frac{4(2ab)^{\frac{3}{4}}(a+2b)^{-1}}{\sqrt{a}-\sqrt{2b}} : \frac{\sqrt{2b\sqrt{2ab}}+\sqrt[4]{2a^3b}}{\sqrt{2ab}} - 6\left(\frac{a}{6a-48b} - \frac{2b}{3a-6b} - \frac{8b^2}{a^2-10ab+16b^2}\right) =$$

$$\begin{aligned}
&= \frac{8ab}{(a+2b)(a-2b)} - \frac{a+2b}{a-2b}\\
&= \frac{8ab-(a+2b)^2}{(a+2b)(a-2b)}\\
&= \frac{8ab-(a^2+4ab+4b^2)}{(a+2b)(a-2b)}\\
&= \frac{8ab-a^2-4ab-4b^2}{(a+2b)(a-2b)}\\
&= \frac{-a^2+4ab-4b^2}{(a+2b)(a-2b)}\\
&= \frac{-(a^2-4ab+4b^2)}{(a+2b)(a-2b)}\\
&= \frac{-(a-2b)^2}{(a+2b)(a-2b)}\\
&= \frac{-(a-2b)^{\cancel{2}}}{(a+2b)\cancel{(a-2b)}}
\end{aligned}$$

$$= \frac{-(a-2b)}{a+2b}$$

$$= \frac{-a+2b}{a+2b}$$

$$= \boxed{\frac{2b-a}{2b+a}}$$